Massive Gravity on Arbitrary Backgrounds

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L.A., A. Chamseddine and V. Mukhanov, arXiv:1008.5132

Linear Massive gravity in Minkowski spacetime

The linear action for metric perturbations $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[-R + \underbrace{\frac{m_g^2}{4}\left(h^2 - h_{\mu\nu}h^{\mu\nu}\right)}_{\text{Fierz-Pauli (FP) mass term (1939)}}\right]$$

- is ghost-free
- propagates 5 massive degrees of freedom

PROBLEMS

- breaks the diffeomorphism invariance of general relativity
- leads to unacceptable observational consequences (van Dam, Veltman, Zakharov 1970)
- propagates a ghost around any other backgrounds ${}^{(0)}g^{\mu\nu} \neq \eta^{\mu\nu}$

SOLUTIONS

- introduction of four Higgs (Stückelberg) fields
- non-linear modifications to FP mass term (Vainshtein 1972)
- define $h^{\mu\nu} \equiv g^{\mu\nu} -{}^{(0)} g^{\mu\nu}$

Higgs massive gravity

Any theory of massive gravity can be represented as Einstein gravity interacting with four scalar fields ϕ^A with $A = 0, 1, 2, 3$:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[-R + \frac{m_g^2}{4}\mathcal{L}_{FP}\left(\phi^A,\, g^{\mu\nu}\right)\right]$$

The interaction term $\mathcal{L}_{FP}$ is a function of a diffeomorphism invariant composite scalar

$$\bar{h}^{AB} = g^{\mu\nu}\partial_\mu\phi^A\partial_\nu\phi^B - \eta^{AB}$$

such that the quadratic action reduces to the FP mass term around the backgrounds

$$g^{\mu\nu} = \eta^{\mu\nu}, \qquad \phi^A = x^\mu\delta^A_\mu.$$

The minimal such generalization of the Fierz-Pauli action is

$$\mathcal{L}_{FP} = \bar{h}^2 - \bar{h}^A_B\bar{h}^B_A, \quad \bar{h}^A_B \equiv \bar{h}^{AC}\eta_{BC}$$

- diffeomorphism invariant
- exhibits Vainshtein mechanism
- propagates 6 degrees of freedom
- strongly coupled at very low scales

Gravitational field from a massive external source

Let us focus on the scalar metric and matter perturbations in the longitudinal gauge

METRIC

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$$

$$h_{00} = 2\phi$$
$$h_{0i} = 0$$
$$h_{ij} = 2\psi\delta_{ij}$$

SCALAR FIELDS

$$\phi^A = x^\mu \delta^A_\mu + \chi^A$$

$$\chi^0 = \chi^0$$
$$\chi^i = \partial_i \pi = \pi_{,i}$$

Hence the metric takes the form:

$$ds^2 = (1 + 2\phi)\, dt^2 - (1 - 2\psi)\, \delta_{ik} dx^i dx^k$$

In General Relativity, in the presence of a static spherically symmetric source M_0

$$\psi = \phi, \qquad \Delta\phi = T^{00}/2 \qquad \Rightarrow \qquad \phi \sim -\frac{M_0}{r}$$

Gravitational field from a massive external source

In Fierz-Pauli massive gravity

$$\mathcal{L}_{FP} = \bar{h}^2 - \bar{h}^A_B \bar{h}^B_A$$

the linearized constraints and equations of motion are:

$$\Delta\psi = \frac{m_g^2}{2}(3\psi + \Delta\pi) + \frac{T^{00}}{2} \qquad \Delta\left(\psi - \phi - m_g^2\pi\right) = 0,$$

$$\Delta\chi^0 = 0 \qquad \Delta\left(2\psi - \phi\right) = 0$$

This implies:

$$\psi = \phi/2 \qquad \Rightarrow \qquad \phi = -\frac{4}{3}\frac{M_0}{r}e^{-m_g r}$$

even when $m_g \to 0 \Rightarrow$ vDVZ discontinuity!

(idea of the picture: Creminelli 2011)

Vainshtein scale of the nonlinear expansion

Beyond linear order in matter perturbations the equations are modified as:

$$2\psi - \phi + O(1)\,\partial^4\pi^2 = 0$$
$$\psi - \phi - m_g^2\pi = 0$$

where $\pi_{,ik}$, $\Delta\pi \to \partial^2\pi \ll 1$. In the spherically symmetric case $\partial^n \sim r^{-n}$

$$\Rightarrow \psi + m_g^2\pi + O(1)\,r^{-4}\pi^2 \simeq 0$$

At the Vainshtein scale all the terms become comparable:

$$\psi \sim m_g^2\pi \sim O(1)\,r^{-4}\pi^2,$$

For $\psi \sim -M_0/r$ this gives the well known result for the Vainshtein scale (1972)

$$R_V \simeq \left(\frac{M_0}{M_P^2 m_g^4}\right)^{1/5}$$

Smooth limit to General Relativity

ABOVE THE VAINSHTEIN SCALE

$$r \gg R_V$$

$$\psi + m_g^2 \pi + \cancel{O(1)\, r^{-4} \pi^2} \simeq 0$$

$$\psi - \phi = -\psi \left[1 - O(1) \left(\frac{R_V}{r} \right)^5 \right]$$

$$\left(\Delta - m_g^2 \right) \phi = \frac{4}{3} \left(\frac{T^{00}}{2} \right)$$

Yukawa decay!

BELOW THE VAINSHTEIN SCALE

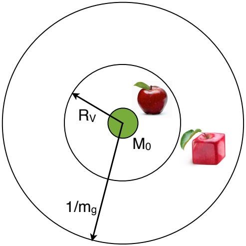
$$r \ll R_V$$

$$\psi + \cancel{m_g^2 \pi} + O(1)\, r^{-4} \pi^2 \simeq 0$$

$$\psi - \phi = O(1) \psi \left(\frac{r}{R_V} \right)^{5/2}$$

$$\Delta \phi = \frac{T^{00}}{2}$$

GR restored up to corrections $\frac{\delta\phi}{\phi} \sim \left(\frac{r}{R_V} \right)^{5/2}$



QUESTION REMAINS:
Does a single everywhere non-singular solution matching both asymptotics exist?
YES!

Babichev, Deffayet, Ziour (2010)

Strong coupling

In terms of the canonically normalized fields $\hat{\phi}$, $\hat{\psi}$ the dominant terms in the cubic action for the field $\hat{\pi}$ become

$$S_{\hat{\pi}} \supset \int d^4x \left\{ \Delta\hat{\pi}(2\hat{\psi} - \hat{\phi}) + \frac{1}{2}\frac{1}{M_P m_g^4}\left(\Delta\hat{\pi}\hat{\pi}_{,ik}\hat{\pi}_{,ik} - \hat{\pi}_{,ik}\hat{\pi}_{,kj}\hat{\pi}_{,ji}\right) + \dots \right\}$$

$\Rightarrow$ The theory becomes strongly coupled above energy cutoff $\Lambda_5 = (M_P m_g^4)^{1/5}$!

Arkani-Hamed, Georgi, Schwartz (2002)

$\Rightarrow$ The full quantum theory is needed to describe physics around spherically symmetric sources below the radius $r_* = \left(\frac{M_0}{M_P}\right)^{1/3}\frac{1}{\Lambda_5} \gg R_V = \left(\frac{M_0}{M_P}\right)^{1/5}\frac{1}{\Lambda_5}$!

NO REGION OF APPLICABILITY OF THE VAINSHTEIN MECHANISM?!

WAY OUT: Raise the energy cutoff by adding appropriate counterterms which eliminate the self-coupling. In this way the Vainshtein radius can be lowered order by order!

Λ_3 theories

For a Langrangian with the highest self coupling $\mathcal{L} \supset (\partial^2\pi)^n$, the corresponding Vainshtein radius is

$$R_V = \left(\frac{M_0}{M_P^2 m_g^{\frac{2(n-1)}{n-2}}} \right)^{\frac{n-2}{3n-4}}, \quad \Lambda^{(n)} = (M_P m_g^{\frac{2(n-1)}{n-2}})^{\frac{n-2}{3n-4}}$$

and the corrections to the gravitational potential within $r \ll R_V$ radius are

$$\frac{\delta\phi}{\phi} \sim \left(\frac{r}{R_V} \right)^{\frac{3n-4}{n-1}}$$

L.A., A. Chamseddine, V. Mukhanov (2010)

The minimal possible scale when $n \to \infty$: $R_V^\infty = \left(\frac{M_0}{M_P} \right)^{1/3} \frac{1}{\Lambda_3}$, $\Lambda_3 = (M_P m_g^2)^{1/3}$

Strong coupling scale in Λ_3 theories: $r_* = \frac{1}{\Lambda_3} \ll R_V \Rightarrow$ Vainshtein mechanism works!

Λ_3 is reached in the dRGT resummation of massive gravity in terms of variables

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\lambda}\partial_\lambda\phi^A\partial_\nu\phi^B\eta_{AB}}$$

de Rham, Gabadadze, Tolley (2010)

$$S_{\mathrm{dRGT}} = -\frac{1}{2}\int d^4x\sqrt{-g}R + \frac{m^2}{2}\int d^4x\sqrt{-g}\left([\mathcal{K}]^2 - \left[\mathcal{K}^2\right]\right)$$

What is "massive spin-2 particle" on curved background?

- In Lorentz invariant spacetime there is a rigid definition of the mass of the particle:

$$m^2 \equiv \eta_{\mu\nu} p^\mu p^\nu = E^2 - \vec{p}^2$$

$\Rightarrow$ Under the little group transformations a particle with spin j belongs to a representation of dimension $2j + 1$.

$\Rightarrow$ Spin-2 particle has 5 degrees of freedom.

- On the other hand consider a free scalar field obeying $(\Box + m^2)\phi = 0$. Expanded in the Fourier series $\sum_k \phi_k e^{ikx}$ its modes satisfy the dispersion relation $-\omega^2 + \vec{k}^2 + m^2 = 0$.

$\Rightarrow$ "Alternative" definition of mass!

- Combine the two! $\Rightarrow$ Define a massive spin-2 particle on curved background such that
  - it has 5 degrees of freedom and
  - they all obey the same equation of motion $(\Box_g + m^2)\phi = 0$ and thus have equal dispersion relations

Non-Minkowski solutions of dRGT

The scalar field part of the dRGT theory has its own effective EMT

$$T^{(\phi)}_{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = -\frac{m^2}{2} g_{\mu\nu}\left([\mathcal{K}]^2 - \left[\mathcal{K}^2\right]\right) + \frac{m^2}{2}\mathcal{K}^\alpha_\beta \frac{\delta \mathcal{K}^\lambda_\rho}{\delta g^{\mu\nu}}\left[\delta^\beta_\alpha \delta^\rho_\lambda - \delta^\rho_\alpha \delta^\beta_\lambda\right].$$

$\Rightarrow$ a whole zoo of different non-trivial solutions $\left\{{}^{(0)}g_{\mu\nu} \neq \eta_{\mu\nu},\ {}^{(0)}\phi^A\right\}$!

The field $\mathcal{K}^\mu_\nu$ can then be splitted as $\mathcal{K}^\mu_\nu =^{(0)} \mathcal{K}^\mu_\nu + \delta\mathcal{K}^\mu_\nu$ with

$$^{(0)}\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{^{(0)}g^{\mu\lambda}\partial_\lambda {}^{(0)}\phi^A \partial_\nu {}^{(0)}\phi^B \eta_{AB}}$$

If $^{(0)}\mathcal{K}^\mu_\nu \neq 0 \Rightarrow$ the Fierz-Pauli structure is lost!

Demand that

$$^{(0)}\mathcal{K}^\mu_\nu = 0 \quad \Rightarrow \quad {}^{(0)}g^{\mu\nu}(x)\frac{\partial^{(0)}\phi^A}{\partial x^\mu}\frac{\partial^{(0)}\phi^B}{\partial x^\nu} = \eta^{AB}$$

This is a coordinate transformation: curved $\rightarrow$ flat ↯

Generalization of Higgs massive gravity

L.A. (2011)

Replace the Minkowski metric by a set of scalar functions as

$$\eta^{AB} \to \bar{f}^{AB}(\phi)$$

Then define

$$\bar{h}^{AB} \equiv g^{\mu\nu}(x)\partial_\mu\phi^A\partial_\nu\phi^B - \bar{f}^{AB}(\phi)$$

with

$$\bar{f}^{AB}(\phi) \equiv^{(0)} g^{\mu\nu}(\phi)\delta^A_\mu\delta^B_\nu .$$

$$\Rightarrow \text{the old story:} \quad \mathcal{L}_{FP} = \bar{h}^2 - \bar{h}^A_B\bar{h}^B_A!$$

- The "distances" in the scalar field space are now measured by the "metric" $\bar{f}^{AB}$, and the scalar field space indices have to be raised and lowered as

$$\phi_B \equiv \bar{f}_{AB}\phi^A$$

- Lagrangian is invariant under the isometries of the "metric" $\bar{f}^{AB}$!
- The nonlinear dRGT completion can be written in terms of $\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\lambda}\partial_\lambda\phi^A\partial_\nu\phi^B\bar{f}_{AB}(\phi)}$.

Example: massive graviton on de Sitter spacetime

Consider spatially flat de Sitter metric ${}^{(0)}g^{\mu\nu} = a^{-2}(\eta)\eta^{\mu\nu}$ with $a(\eta) = -1/(H\eta)$.

$\Rightarrow$ The scalar field metric has to be chosen as $\bar{f}^{AB}(\phi^0) = (H\phi^0)^2\eta^{AB}$

$\Rightarrow$ The FP mass term

$$S_{FP} = \frac{m^2}{8}\int d^4x\sqrt{-g}\left\{g^{\mu\nu}g^{\alpha\beta}\partial_\mu\phi^A\partial_\nu\phi^B\partial_\alpha\phi^C\partial_\beta\phi^D\left[\eta_{AB}\eta_{CD} - \eta_{BC}\eta_{AD}\right] - \right.$$
$$\left. - 6(H\phi^0)^2 g^{\mu\nu}\partial_\mu\phi^A\partial_\nu\phi^B\eta_{AB} + 12(H\phi^0)^4\right\}$$

- diffeomorphism invariant
- NOT invariant under the shifts $\phi^A \to \phi^A + \lambda^A$
- invariant under $\phi^A \to \hat{\Lambda}^A_B\phi^B$ such that $\hat{\Lambda}^A_C\hat{\Lambda}^B_D\bar{f}_{AB}(\phi) \to \bar{f}_{CD}(\phi)$
- gives 5 d.o.f. to massive graviton q_i each of which satisfy

$$(\Box_{dS} + m^2)q_i = 0 \quad \Rightarrow \quad (\partial_\eta^2 + 2\mathcal{H}\partial_\eta + m^2)q_i = 0$$

In terms of physical time $dt = a(\eta)d\eta$:

$$\ddot{q}_i - \frac{\Delta}{a^2}q_i + m^2_{eff}q_i = 0, \qquad m^2_{eff} = m^2 - \frac{9}{4}H^2$$

Deser and Waldron (2001)

Conclusions

- Λ_3 is the highest strong coupling scale achievable in non-linear massive gravity
- The non-linear perturbative analysis shows that the Vainshtein mechanism in Λ_3 theories does work
- General relativity can be restored around static spherically symmetric massive sources up to corrections to the Newtonian potential $\delta\phi/\phi \sim (r/R_V)^3$

- A diffeomorphism invariant Fierz-Pauli mass term can be constructed on arbitrary background by the use of four scalar fields
- BUT: on each background the generally covariant theory is a fundamentally different theory with different symmetries
- No single unified theory of massive gravity exists such that the graviton always behaves as a massive spin-2 particle around any background metric

Thank you for your attention!

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