The Potential of Lepton Minimal Flavour Violation

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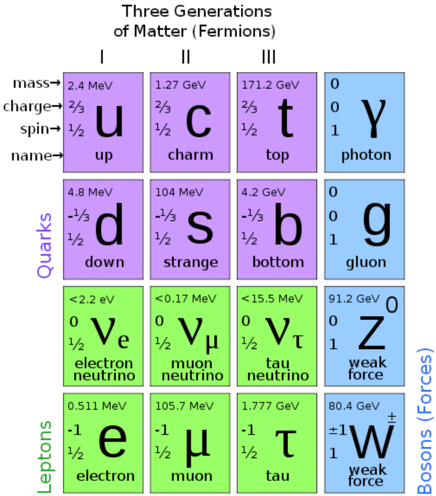
based on the work with Belén Gavela, Luca Merlo , Stefano Rigolin & Daniel Hernández

Outline

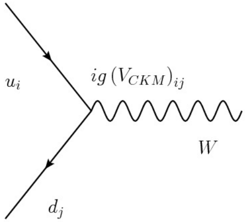
The Flavour Puzzle



- Why 3 generations? CP violation?
- Visible part of the universe $\rightarrow$ 1^{st} generation

The Three Generations

Generations connect with each other through mixing matrices



Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix}$$

Leptons

$$U_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 0.2 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

Why is the mixing patern so different for leptons and quarks?

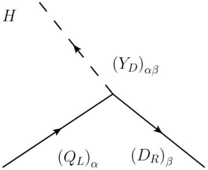
Minimal Flavour Violation (MFV)

...in our attempt to answer it we will adopt a predictive assumption

Minimal Flavour Violation

The MFV hypothesis: *The Yukawa couplings are the* ***only*** *sources of flavour violation in and* ***beyond*** *the Standard Model* [1].

$$-\mathcal{L}_{Yukawa} = \overline{Q}_L Y_U U_R \tilde{H} + \overline{Q}_L Y_D D_R H + \overline{\ell}_L Y_E E_R H + h.c. + (\nu \text{ mass})$$



[1] Georgi & Chivukula 1987; D'Ambrosio, Giudice, Isidori, & Strumia, 2002; Cirigliano, Grinstein, Isidori & Wise 2005.

Minimal Flavour Violation; Realization

- Generations are distinguished by masses; in the limit of zero mass ($\not{\mathcal{L}}_{Yuk}$) the SM presents an extended symmetry group :

$$G_f = \overbrace{SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R}}^{Quark} \times \overbrace{SU(3)_{\ell_L} \times SU(3)_{E_R}}^{Lepton} \times \cdots$$

$$D_R = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \qquad D_R \sim (1, 3, 1 \cdots)$$

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- The Yukawa couplings break the symmetry, unless

$$\overline{Q}_L Y_D D_R H \qquad Y_D \sim (3, \bar{3}, 1) \quad \text{'SPURIONS'}$$

An hypothesis well compatible with current data and NP @ the TeV

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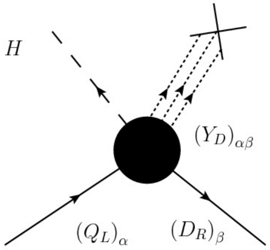
can one go further in this assumption's direction?...

The Dynamics Behind MFV

Transformation properties suggest that the Yukawa couplings have a *dynamical origin* .



The Yukawa couplings arise with the v.e.v.s of fields that transform for real under the flavour symmetry:

$$\boxed{Y = \langle\Sigma\rangle/\Lambda_f\,,\ \Sigma\sim(..3,\bar{3},..)}\,,\ \text{or } Y\sim\left\langle\Sigma^2\right\rangle/\Lambda_f^2,\ \text{or } Y\sim\left\langle\Sigma^{-1}\right\rangle/\Lambda_f^{-1}$$

Quarks

The Dynamics Behind MFV: Quarks

the flavour symmetry: $SU(3)_L \times SU(3)_{D_R} \times SU(3)_{U_R}$

Straightforward case : The Yukawas are the vev of 1 field $Y \sim \langle \Sigma \rangle$

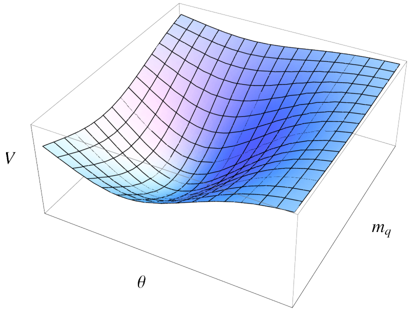
$$\Sigma_d \sim (3, \bar{3}, 1) \qquad \Sigma_u \sim (3, 1, \bar{3})$$

$$\boxed{\frac{\langle \Sigma_d \rangle}{\Lambda_f} = Y_D = V_{CKM} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}}, \quad \boxed{\frac{\langle \Sigma_u \rangle}{\Lambda_f} = Y_U = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}}.$$

... but these vevs are acquired somehow...

The Potential

the Potential $V(\Sigma_u, \Sigma_d)$
is the culprit of fixing $\langle \Sigma_{u,d} \rangle \Leftrightarrow V_{CKM}$ & m_q



unfortunately no mixing can come out of this potential
more on this later...

Gavela, Merlo, Rigolin, R.A. 2011

Leptons

The Dynamics Behind MFV: Leptons

For concreteness let's pick a constrained Inverse Seesaw Model with all the possible distinctive features of ν's:

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\, \bar{N}^c\,,\, \bar{N}'^c) \begin{pmatrix} 0 & \tilde{\phi} Y & \tilde{\phi} Y' \\ (\tilde{\phi} Y)^T & 0 & \Lambda \\ (\tilde{\phi} Y')^T & \Lambda & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

$$|Y'| \ll |Y| \;\Rightarrow \text{approx LN} \qquad \Lambda_{LN} = \Lambda / \, |Y'|\,, \quad \Lambda_{fl} = \Lambda\,.$$

the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2} + m_{\nu_3}}}\, U_{PMNS} \begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

Gavela, Hambye, P. Hernández, D. Hernández

LMFV

$$\text{when } Y, Y' \to 0 \;\Rightarrow \mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L \,,\, \bar{N}^c \,,\, \bar{N}'^c) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Lambda \\ 0 & \Lambda & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

the flavour symmetry is:

$$SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N \;.$$

Fully restored at high energies with the introduction of the scalar fields:

$$Y_E = \frac{\langle \Sigma_E \rangle}{\Lambda_f} \sim (3\,,\bar{3}\,,1)\,; \quad (Y\,,\; Y') = \frac{\langle \chi \rangle}{\Lambda} \sim (3\,,1\,,2)\,. \qquad (1)$$

whose vev's are

$$\langle \Sigma_E \rangle \propto \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \,, \quad \langle \chi \rangle \propto U_{PMNS} \begin{pmatrix} 0 & 0 \\ \sqrt{m_{\nu_2}} & 0 \\ 0 & \sqrt{m_{\nu_3}} \end{pmatrix} \begin{pmatrix} -iy & iy' \\ y & y' \end{pmatrix}$$

what is the potential for the mixing parameters now?

Invariant terms of the Potential: Mixing

Let's focus on the mixing parameters, the only invariant that depends on them is, at renormalizable level $(|Y'| << |Y|)$:

$$\mathrm{Tr}\left(\Sigma_E\Sigma_E^\dagger\chi\chi^\dagger\right)\propto\Big\{\sum_{l,i}|U_{PMNS}^{li}|^2 m_l^2 m_{\nu_i}+ \Big[i\,e^{2i\alpha}\sum_{l,i<j}U_{PMNS}^{li}(U_{PMNS}^{lj})^* m_l^2\sqrt{m_{\nu_i}m_{\nu_j}}+c.c.\Big]\Big\}\,.$$

whereas for quarks:

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\Sigma_d\Sigma_d^\dagger\right)\propto\sum_{i,j}|U_{CKM}^{ij}|^2 m_{u_i}^2 m_{d_j}^2\,,$$

Minimum of the Potential: 2 Generations

Leptons

The invariant containing the angle (2 family case & $(|Y'| << |Y|)$):

$$\mathrm{Tr}\left(\Sigma_E \Sigma_E^\dagger \chi \chi^\dagger\right) \propto (m_\mu^2 - m_e^2)\left((m_{\nu_2} - m_{\nu_1}) \cos 2\theta + 2\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\alpha \sin 2\theta\right) ,$$

Renormalizable level: $\partial_\theta V = 0$ yields:

$$\boxed{\mathrm{tg} 2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}} , \qquad \sin 2\theta \cos 2\alpha = 0 , \qquad \boxed{\alpha = \pm\pi/4}$$

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Quarks

Let's bring back the quark invariant (Bifundamental):

$$\mathrm{Tr}\left(\Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger\right) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

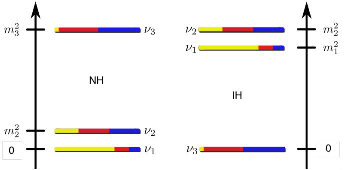
which yields

$$\boxed{(m_c^2 - m_u^2)(m_s^2 - m_d^2) \sin 2\theta = 0}.$$

3 Family Case

For the three family case:

- Only one angle can be set different from 0 as $m_{\nu_1} = 0$ imposes strong hierarchies with m_{ν_2}, m_{ν_3}. This is a peculiarity of the model.



- Such angle lies in the experimentally allowed region for an inverse Yukawa relation: $Y^{-1} \sim \Sigma$.
  - ! Suggesting: this happens in gauged flavour symmetry models [2] which also solve the problem of goldstones...

[2] Berezhiani, Khlopov 1990, Grinstein, Redi, Villadoro 2011, Feldmann 2011

Summary

1. The distinctive Majorana character of neutrinos within a Seesaw Model makes the potential of MFV different from that of quarks.
2. This difference allows for *maximal angles in the limit of degenerate neutrino masses* (but Majorana phase $\neq 0$!).
3. The realistic 3-family scenario points towards an inverse relation of Yukawas and scalar fields and degenerate neutrino mass spectrum.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