The Fermion Mass Hierarchy in Models with Warped Extra Dimensions and a Bulk Higgs

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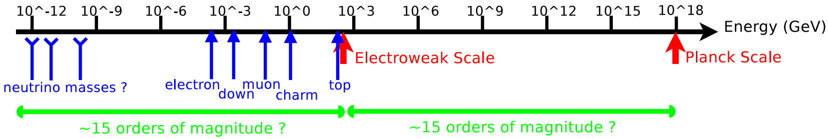
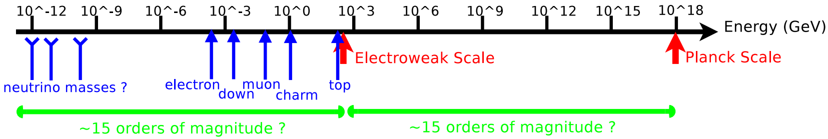
Hierarchies in High Energy Physics

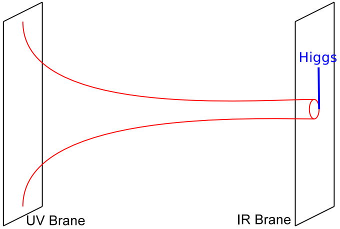


Hierarchies in High Energy Physics



(Randall & Sundrum '99)

The Randall and Sundrum model considers a slice of AdS_5, cut off by two branes.



$$ds^2 = e^{-2A(r)}\eta^{\mu\nu}dx_\mu dx_\nu - dr^2$$

$A(r) = kr$

- Fundamental scale of theory is Planck scale $k \sim M_{\rm Pl} \sim 10^{18}$ GeV.
- Higgs is localised on IR brane ($r_{\rm IR} = R$).
- 4D effective Higgs mass (and EW scale) warped down from fundamental value at Planck scale.

$$m_{4D} = e^{-A(R)} m_{5D}$$

- Potential resolution to gauge hierarchy problem exists when space can be stabilised such that

$$\Omega \equiv e^{A(R)} \approx 10^{15}$$

The Fermion Mass Hierarchy

When SM fermions propagate in bulk, model also generates large possible hierarchy in fermion mass spectrum.

$$S = \int d^5x\, \bar{\Psi} \left(i\Gamma^M \Delta_M - M \right) \Psi$$

with $M = ck$ and $c \sim \mathcal{O}(1)$. After KK decomposition

$$\psi_{L,R} = \sum_n e^{-2A} f^{(n)}_{L,R}(r) \psi^{(n)}_{L,R}(x)$$

fermion zero modes (associated with SM particles) exponentially peaked towards either IR or UV brane.

$$\boxed{f^{(0)}_{L,R} = N e^{\mp ckr}}$$

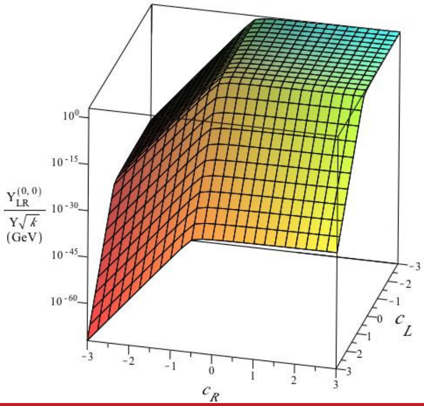
Effective Yukawa couplings to Higgs exponentially sensitive to bulk mass parameters.

(Grossman & Neubert '99, Gherghetta & Pomarol '00, Huber & Shafi '00)

$$\boxed{\mathcal{L} = Y\, f^{(n)}_L(R) f^{(m)}_R(R)\, \Phi \bar{\psi}^{(n)}_L \psi^{(m)}_R}$$

with order one Yukawa couplings $\tilde{Y} = Y\sqrt{k}$

$$m_{\text{fermion}} \approx \frac{\tilde{Y}}{\sqrt{2}} v_4 \sqrt{\frac{(1-2c_L)(1+2c_R)}{(\Omega^{1-2c_L}-1)(\Omega^{1+2c_R}-1)}}\, \Omega^{1-c_L+c_R}$$



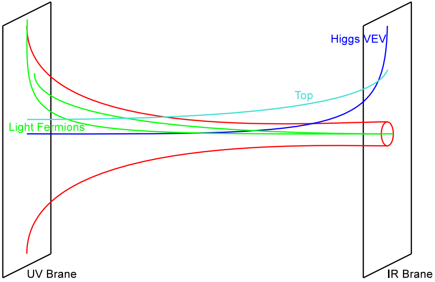
The Fermion Mass Hierarchy with a Bulk Higgs

When Higgs propagates in the Bulk the Higgs VEV is found to be

$$\langle \Phi \rangle = h(r) \approx N_h e^{(2+\alpha)kr}$$

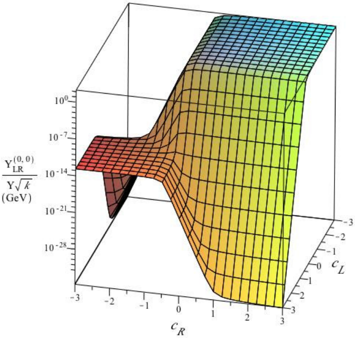
Fermion zero mode mass now approximately given by integral,

$$m_{\text{fermion}} \approx Y_{LR}^{(n,m)} = \frac{1}{\sqrt{2}} Y \int dr \; h f_L^{(0)} f_R^{(0)}.$$



(Agashe, Okui & Sundrum '08 PRA, Huber & Jäger '11)

With $\alpha = 0.01$ fermion mass range given by;

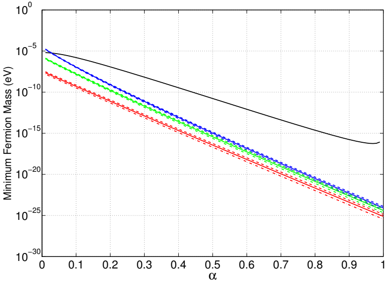


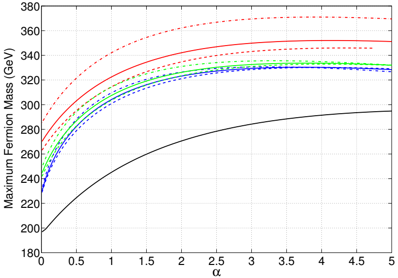
Assuming $\mathcal{O}(1)$ Yukawa couplings and non-split fermion scenario then

Dirac Mass term of SM fermions range, from EW scale, by a factor of $\Omega^{-1-\alpha}$.

'Maximum' and 'Minimum' Dirac Mass.

After fitting to $\widehat{\alpha}$, $\widehat{M}_Z$ and $\widehat{G_f}$ and setting $\Omega = 10^{15}$, $M_{\rm KK} = 2$TeV. Assuming $\mathcal{O}(1)$ Yukawa couplings and $c_L = -c_R$.





Colours refer to deformations in geometry (described later).

When $\alpha \approx 0$, allowed mass range remarkably close to observed masses.

However α is essentially a free parameter. This work seeks to investigate the physical implications of changing the parameter α.

The Model.

Consider minimal scenario first, I.e. just SM with all fields in the bulk. Many possibilities for extending this scenario.

Higgs Sector

$$S = \int d^5x\sqrt{G}\left[|D_M\Phi|^2 - V(\Phi)\right] + \int d^4x\sqrt{g_{\mathrm{IR}}}\left[-V_{\mathrm{IR}}(\Phi)\right] + \int d^4x\sqrt{g_{\mathrm{UV}}}\left[-V_{\mathrm{UV}}(\Phi)\right]$$

with

$$V(\Phi) = M_\Phi^2|\Phi|^2 \qquad V_{\mathrm{IR}}(\Phi) = -M_{\mathrm{IR}}|\Phi|^2 + \lambda_{\mathrm{IR}}|\Phi|^4 \qquad V_{\mathrm{UV}}(\Phi) = M_{\mathrm{UV}}|\Phi|^2$$

Only mass dimension 6 operator included is $|\Phi|^4$ term on the IR brane. Other operators assumed to be suppressed by mass scale greater than KK scale.

With these assumptions, models largely fixed. Only remaining 'input' is the geometry.

The Geometry

$$\boxed{ds^2 = e^{-2A(r)}\eta^{\mu\nu}dx_\mu dx_\nu - dr^2}$$

- In the original RS model, just gravity propagated in the bulk, consider purely AdS_5 spaces.

$$A(r) = kr$$

- Must at least also include a bulk GW scalar to stabilise space. Typically many other bulk fields. Such additional bulk fields will have an (assumed small) back reaction on the space. (Goldberger & Wise '99)
- Important to ask if RS phenomenology is robust against small deformations in geometry. Hence also consider a class of asymptotically AdS spaces arising from a solution of scalar-gravity systems. (Cabrer, von Gersdorff & Quiros '09 '11)

$$A(r) = kr + \frac{1}{\nu^2}\ln\left(1 - \frac{r}{R+\Delta}\right)$$

- Space is singular at $r = R + \Delta$ but here we cut off space at $r = R$.

The Higgs VEV

Expanding the Higgs field

$$\Phi(x,r) = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi_1(x,r) + i\pi_2(x,r) \\ h(r) + H(x,r) + i\pi_3(x,r) \end{pmatrix},$$

For AdS_5 the Higgs VEV can be solved for (expression for modified metrics more complicated)

$$\boxed{h(r) = N_h e^{2kr}\left(e^{\alpha kr} + Be^{-\alpha kr}\right)}$$

where $\alpha = k^{-1}\sqrt{4k^2 + M_\Phi^2}$ and $B = -\frac{2k+\alpha k - M_{\mathrm{UV}}}{2k-\alpha k - M_{\mathrm{UV}}}$.

- Note, without fine tuning Higgs VEV will be heavily peaked towards IR. Still offers a potential resolution to gauge hierarchy problem.
- Also for $\alpha \approx 0$ requires $M_\phi^2 < 0$.

The Breitenlohner-Freedman Bound (Breitenlohner & Freedman '82)

In AdS spaces (without IR cut-off) flux of energy momentum tensor must vanish at AdS boundary ($r \to \infty$).
For $AdS_5 \Rightarrow M_\Phi^2 \geqslant -4k^2 \Rightarrow \boxed{\alpha \geqslant 0}$

The Holographic Picture

(Arkani-Hamed, Porrati & Randall '00 Rattazzi & Zaffaroni '00)

- A bulk Higgs is conjectured to be dual to a partially composite Higgs, i.e. an elementary Higgs mixing with a composite sector.
- In AdS_5 the scaling dimensions of 4D dual Higgs operator ($\mathcal{O}$) is found to be $2+\alpha$.
- In order to offer a potential resolution to gauge hierarchy problem one requires the corresponding dual Higgs mass operator $\mathcal{O}^\dagger\mathcal{O}$ have a scaling dimensions $\geqslant 4$.

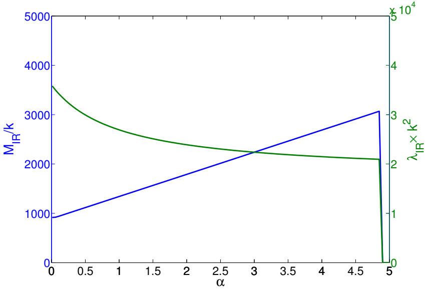
$$\boxed{\Rightarrow \quad \alpha \geqslant 0}$$

(Luty & Okui '06)

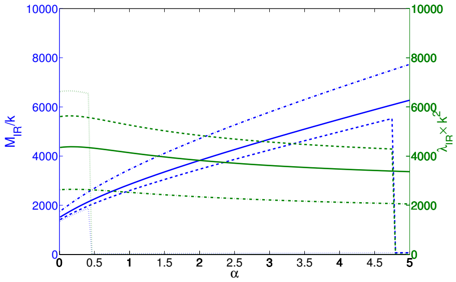
- Would expect this to approximately hold for the modified metrics, but this is challenging to verify.

Breaking EW symmetry

Results largely insensitive to $V_{\rm UV}$, so fix $M_{\rm UV} = k$ (as well $M_{\rm KK} = 2$ TeV).
Assuming $m_H = 125$ GeV, one can find values of $M_{\rm IR}$ and $\lambda_{\rm IR}$, required to break EW symmetry.



AdS$_5$



Modified metrics with $v = 3$, $k\Delta = 0.5$ (dash-dot), $k\Delta = 1$ (solid), $k\Delta = 1.5$ (dash-dash) and $k\Delta = 2$ (dot).

Effective $|\Phi|^2$ term receives additional positive contribution proportional to α.
For large values of α, EW symmetry is not broken. (Should really consider other dim. 6 operators).

Electroweak Corrections

- To compute EW corrections, convenient to expand in holographic basis and not KK basis (Cabrer, von Gersdorff, Quiros '11)

$$Z_\mu(p,r) = G^{(Z)}(p,r)\tilde{Z}_\mu(p) \quad \text{s.t.} \quad Z_\mu(p,r_{\rm UV}) = \tilde{Z}_\mu(p) \;\; \text{and define} \;\; P_Z(p,r) \equiv \frac{e^{-2A}\partial_5 G^{(Z)}(p,r)}{G^{(Z)}(p,r)}$$

- Expanding in 4 momentum $P_{W,Z}(p,r) = \sum_{n=0} \frac{p^{2n}}{n!} P^{(n)}_{W,Z}(r)$, corrections to W/Z propagators are

$$\Pi_{W,Z}(0) = P^{(0)}_{W,Z}\big|_{r=r_{\rm UV}} - m^2_{W,Z} \quad \text{and} \quad \Pi'_{W,Z}(0) = P^{(1)}_{W,Z}\big|_{r=r_{\rm UV}} + 1.$$

- $\Rightarrow$ Dominant contribution to T parameter determined by

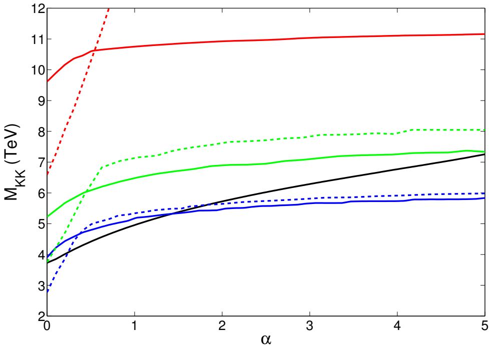
$$\boxed{P^{(0)\,2}_{W,Z}\big|_{r=r_{\rm UV}} = \partial_5 P^{(0)}_{W,Z}\big|_{r=r_{\rm IR}} - \partial_5 P^{(0)}_{W,Z}\big|_{r=r_{\rm UV}} + M^2_{W,Z}|_{r=r_{\rm UV}} - \Omega^{-2} M^2_{W,Z}|_{r=r_{\rm IR}}}$$

where $M_W(r) = \frac{g h(r)}{2}$ and $M_Z(r) = \frac{\sqrt{g^2+g'^{\,2}}\,h(r)}{2}$.

- Holds for generic geometries and generic Higgs potentials.
- **The flatter the Higgs VEV (i.e. the smaller α), the smaller the correction to the T parameter.**

Electroweak Corrections (cont.)

After fitting to $\hat{\alpha}$, $\hat{G}_f$ and $\hat{M}_Z$ and comparing to latest $S-T$ ellipse at 95% C.L. constraints on KK scale found to be



For AdS_5 (black) and modified metrics with $k\Delta = 1$ (solid), $k\Delta = 0.5$ (dash-dash) and $v = 10$ (blue), $v = 5$ (green) and $v = 3$ (red).

- Here KK scale is defined by

$$M_{\mathrm{KK}} = \frac{\partial_5 A(r)|_{r=R}}{\Omega}$$

- Mass of first KK gauge field, $m_1/M_{\mathrm{KK}} =$

$k\Delta$	RS	$v = 10$	$v = 5$	$v = 3$
0.5	2.45	2.23	1.68	0.87
1	2.45	2.27	1.82	1.07
1.5	2.45	2.30	1.89	1.18

Additional Scalar Degrees of Freedom.

(Falkowski & Perez-Victoria '08 Cabrer, von Gersdorff, Quiros '11)

A clear phenomenological distinction between a bulk Higgs and a brane Higgs is that models with a bulk Higgs have additional scalars. Considering $\mathrm{SU}(2)\times\mathrm{U}(1)$.

	5D theory	Compactification	Spont. Sym. breaking
KK modes	4 massless 5D gauge fields 4×3 transverse DoF. plus complex Higgs 4 DoF.	4 massive 4D gauge fields $\rightarrow 4\times 2$ transverse DoF. $+4$ longitudinal DoF. plus complex Higgs 4 DoF.	4 massive 4D gauge fields $4\times(2+1)$ gauge DoF. 1 Higgs field $+3$ **Scalar DoF.**
Zero mode	-"-	4 massless 4D gauge fields $\rightarrow 4\times 2$ transverse DoF. plus complex Higgs 4 DoF.	3 massive & 1 massless gauge field $3\times(2+1)$ gauge DoF. 1×2 transverse DoF. 1 Higgs field

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$\Rightarrow W_5/Z_5$ mix with π_i. Gauge independent combination forms physical pseudo-scalars

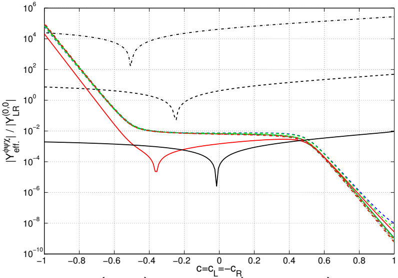
$$\phi_W^{1,2} = \partial_5(M_W^{-1}\pi_{2,1}) \mp A_5^{1,2} \qquad \text{and} \qquad \phi_Z = \partial_5(M_Z^{-1}\pi_3) + Z_5$$

- If A_μ has NBC's then scalars have DBC's (i.e. no zero mode) regardless of Higgs.
- As $\alpha \rightarrow \infty$ (i.e. brane Higgs), scalars become infinitely heavy (unphysical).

Scalar-mediated FCNC's

- Higgs mediated FCNC's due to Higgs Profile differing from Higgs VEV found to be numerically small. (Azatov, Toharia & Zhu '09)
- Additional pseudo-scalars couple to fermion scalar current ($\bar{\psi}\psi$) via the Yukawa couplings and fermion pseudo-scalar currents ($\bar{\psi}\gamma^5\psi$) via gauge couplings.

- However only scalar currents contain purely SM fermions.
- Possibility of scalar mediated FCNC's arise due to misalignment of effective Yukawa couplings and fermion masses.



for $\alpha = 0.01$ (solid), $\alpha = 0.5$ (dash-dash), $\alpha = 1.01$ (dash-dot)

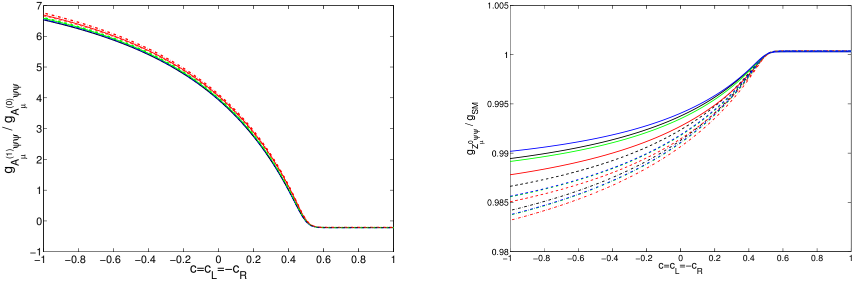
- Effective Yukawa coupling given by

$$Y^{(\phi\psi\psi)}_{\text{eff.}} = -\frac{gYN_{\psi_L}N_{\psi_R}}{2\sqrt{2}\, m_n^{(\phi)\,2}} \int dr \;\left(2A'h - h' - (c_L - c_R)kh\right) e^{-2A-(c_L-c_R)kr} f_n^{\phi}W$$

- For AdS$_5$, loose perturbative control of theory for large values of α.

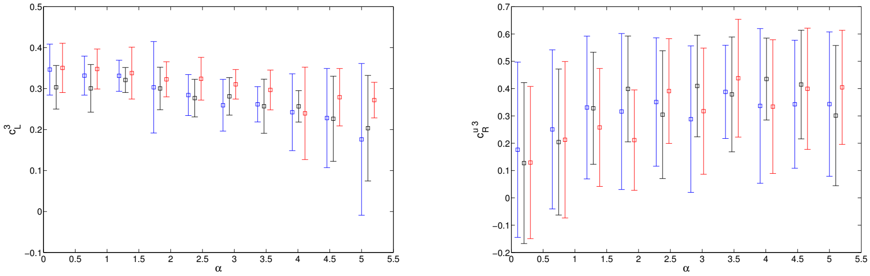
Gauge-mediated FCNC's

Couplings of first KK gauge field and Z zero mode to fermions, for $\alpha = 0.01$ (solid), $\alpha = 1.01$ (dash-dash), $\alpha = 5$ (dash-dot) with $v = 10$ (blue), $v = 5$ (green) $v = 3$ (red) and AdS_5 (black).



- Warped Extra dimensions have a natural mechanism to suppress FCNC's due to approximately universal couplings to light fermions peaked towards UV ($c_L > 0.5$).
- Problems (particularly with ϵ_K) arise since heavier quarks have to sit towards IR brane.
- Having a partially split fermion scenario helps, but is constrained by $Z \to b\bar{b}$.

Gauge-mediated FCNC's (cont.)



- Assuming anarchic Yukawa couplings ($\tilde{Y}$), it is found preferred fermion positions are shifted further towards UV for smaller values of α.
- Relatively small shifts in fermion position result in significant reductions in constraints from flavour physics. See (Agashe, Azatov & Zhu '08 PRA, Huber & Jäger '11 Cabrer, von Gersdorff & Quiros '11)

Conclusions

- Central result: With a bulk Higgs and order one Yukawa couplings, Fermion Dirac mass term extends by a factor of $\Omega^{-1-\alpha}$ from EW scale.
- Models with a bulk Higgs offers a concrete model for investigating changing the scaling dimension of the Higgs operator.
- Numerous theoretical and phenomenological motivations for considering $\alpha \approx 0$ to be optimal value (i.e. a scaling dimension of 2).
- Existence of a heavy scalar for every W' and Z' is important falsifiable prediction of model.
- Many possible extensions to this scenario. Model presented here does not include neutrino masses.