Higgs inflation and Loop Quantum Cosmology

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Outline

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- Motivation

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- LQC and non-minimal coupling to gravity

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- Summary

Motivation

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- Matching the data

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- Matching the data
- Theoretical advantages

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- New physics at gravitational level

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- Matching the data
- Theoretical advantages
- New physics at gravitational level
- Strong quantum gravity effects

Non-minimally coupled scalar field

The action in Jordan frame is of the form of

$$S = \int d^4x\sqrt{-g}\Big[- U(\phi)R + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi)\Big] \ , \qquad (1)$$

where $U = \frac{1}{2} + f(\phi)$ and $f(\phi)R$ is a non-minimal coupling to gravity.

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where $U = \frac{1}{2} + f(\phi)$ and $f(\phi)R$ is a non-minimal coupling to gravity. Einstein equations and the inflaton's equation of motion:

$$\begin{aligned} UH^2 + HU'\dot{\phi} &= \frac{1}{6}\rho \ , & (2)\\ 2U'\ddot{\phi} + 2U(\frac{2\ddot{a}}{a} + H^2) + 2U''\dot{\phi}^2 + 4HU'\dot{\phi} &= -P \ , & (3)\\ \ddot{\phi} + 3H\dot{\phi} = \frac{2U'V - UV' - U'\dot{\phi}^2(3U'' + \frac{1}{2})}{U + 3U'^2} &= F(\phi,\dot{\phi}) \ , & (4)\end{aligned}$$

where $F = -V'_{eff}$ is the effective „rolling" force of the inflaton.

Inflation!

The slow-roll approximation gives

$$H^2 \simeq \frac{V}{6U} \,, \qquad \dot{H} \simeq -\frac{FU'}{2U} \,, \qquad 3H\dot{\phi} \simeq F(\phi) \tag{5}$$

$$F \simeq \frac{2U'V - UV'}{U + 3U'^2} = -\frac{U^3}{U + 3U'^2}\left(\frac{V}{U^2}\right)' \,. \tag{6}$$

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Let us assume, that $U = \frac{1}{2}\left(1 + \xi\phi^2\right)$ and $V(\phi) = \frac{1}{4}\lambda\phi^4 - \frac{1}{2}m^2\phi^2$.

For $\xi\phi^2 \gg 1$ and $\lambda \gg \xi m^2$ one obtains

$$F \simeq -\frac{2\lambda}{3\xi^2}\phi = -m_{eff}^2\phi\,, \qquad -\frac{\dot{H}}{H^2} \sim \frac{1}{\xi\phi^2} \ll 1\,, \qquad \xi = 47000\sqrt{\lambda}$$

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One obtains accelerated expansion of space-time!

Inflation ends for $\phi \sim \xi^{-1/2} \ll M_{pl}$

Einstein frame

Let us redefine $g_{\mu\nu}$ and $\phi(t)$, so new scalar field would be canonical and minimally coupled

$$\tilde{g}_{\mu\nu} = 2U(\phi)g_{\mu\nu}\ , \qquad \left(\frac{d\tilde{\phi}}{d\phi}\right)^2 = \frac{1}{2}\frac{U + 3U'^2}{U^2}\ . \tag{7}$$

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New action $\tilde{S}\left[\tilde{g}_{\mu\nu}, \tilde{\phi}\right] = S[g_{\mu\nu}, \phi]$ has a minimal coupling $\tilde{U} = \frac{1}{2}$. The effective potential is of the form of

$$\tilde{V}(\tilde{\phi}) = \frac{1}{4}\frac{V(\phi)}{U^2(\phi)} \qquad \leftarrow \qquad \phi = \phi(\tilde{\phi}) \qquad (8)$$

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Easier to calculate evolution of background and perturbations!
Perhaps Quantum Gravity corrections shall be calculated in this frame?

Loop Quantum Gravity corrections

One can describe gravity by the Hamiltonian and Hamilton equations. Let us define Ashtekar canonical variables c, p and their Hamiltonian by

$$p = a^2 \ , \qquad c = \gamma \dot{a} \ , \qquad \mathcal{H} = -\frac{3}{\gamma^2} c^2 \sqrt{p} + \mathcal{H}_{matt} \ . \tag{9}$$

Classical equations of motion can be obtained from

$$\dot{c} = \{c, \mathcal{H}\} \ , \qquad \dot{p} = \{p, \mathcal{H}\} \ , \qquad \{c, p\} = \frac{\gamma}{3} \tag{10}$$

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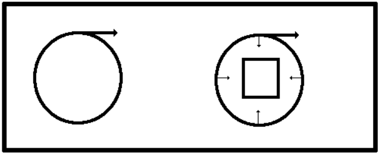
The loop correction



Loop Quantum Gravity corrections

Assumption: only LQC holonomy correction is important. Then

$$\pi_\phi \to \pi_\phi \ , \qquad p \to p \ , \qquad c \to \frac{1}{\bar{\mu}} \sin\left(\bar{\mu} c\right) \ , \tag{11}$$

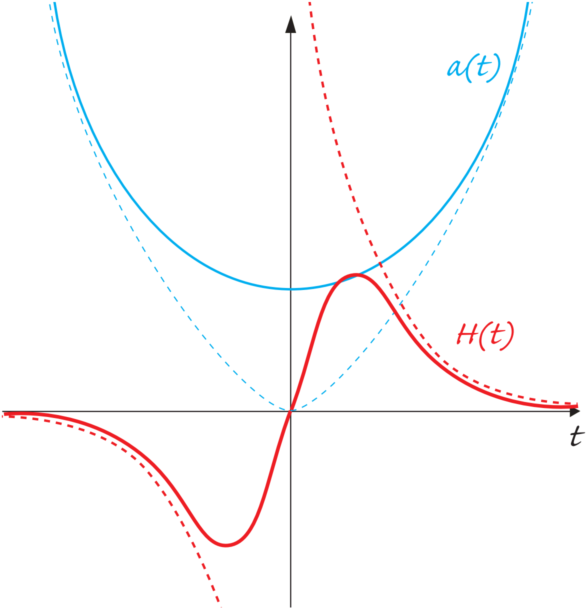
where $\bar{\mu} = \sqrt{\Delta / p}$ and Δ is a minimal value of area operator. This gives Friedmann equations of the form of

$$3H^2 = \rho \left(1 - \frac{\rho}{\rho_{cr}}\right) \ , \qquad 2\dot{H} = -(\rho + P)\left(1 - 2\frac{\rho}{\rho_{cr}}\right) \ , \tag{12}$$

where $\rho_{cr} = \frac{3}{\Delta\gamma^2}$ is a critical (maximal) energy density.

This leads to the Big Bounce for $\rho = \rho_{cr}$

LQC in FRW Universe



LQC with non-minimal coupling: Einstein frame

Let us use the Einstein frame. Redefinition of the metric tensor leads to

$$a \to \tilde{a} = \sqrt{2U}a\ , \qquad dt \to d\tilde{t} = \sqrt{2U}dt \tag{13}$$

Thus one can rewrite GR Friedmann equations into

$$3\tilde{H}^2 = \tilde{\rho} = \frac{1}{2}\left(\frac{d\tilde{\phi}}{d\tilde{t}}\right)^2 + \tilde{V}\ , \qquad 2\frac{d\tilde{H}}{d\tilde{t}} = -\left(\tilde{\rho} + \tilde{P}\right)\ . \tag{14}$$

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Together with LQC holonomy correction one finds

$$3\tilde{H}^2 = \tilde{\rho}(1 - \frac{\tilde{\rho}}{\rho_{cr}}) \ , \qquad 2\frac{d\tilde{H}}{d\tilde{t}} = -\left(\tilde{\rho} + \tilde{P}\right)\left(1 - 2\frac{\tilde{\rho}}{\rho_{cr}}\right) \ . \tag{15}$$

Big Bounce for $\tilde{\rho} = \rho_{cr}$!

Scale of a bounce

Using $a(t)$ and $\phi(t)$ one can rewrite the first Friedmann equation

$$6UH^2 + 6U'H\dot{\phi} = \rho - \frac{\rho}{4U^2\rho_{cr}}\left(\rho + \frac{3U'^2}{2U}(\rho + P)\right) . \qquad (16)$$

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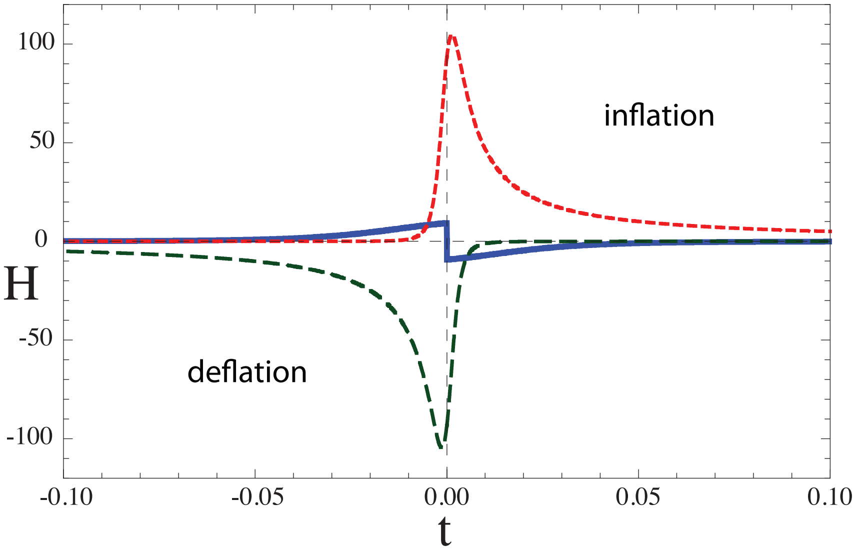
The Big Bounce appears for

$$\tilde{\rho} = \frac{1}{4U^2}\rho + \frac{3U'^2}{8U^3}(\rho + P) = \rho_{cr} \ . \qquad (17)$$

In general $\rho \neq \rho_{cr}$ during the BB $\Rightarrow$ scale of a bounce may be beyond Planck scale

Evolution of Hubble parameters



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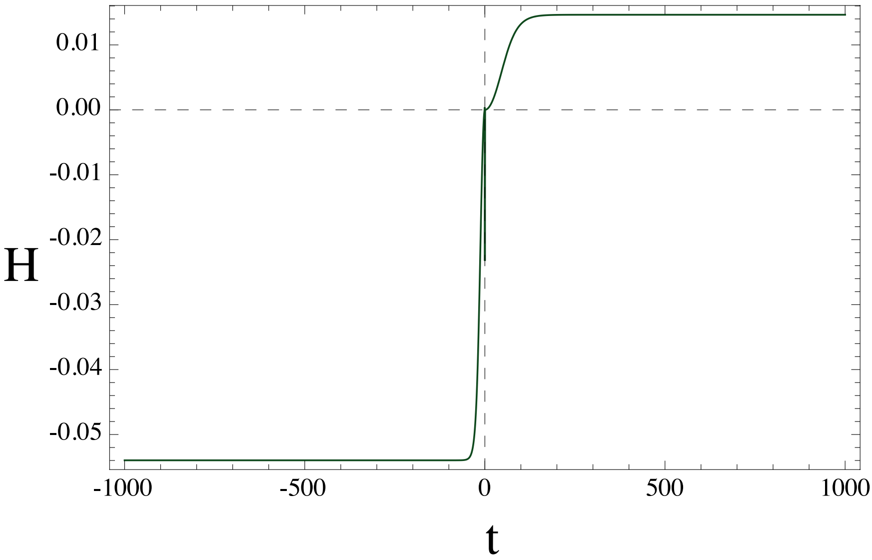
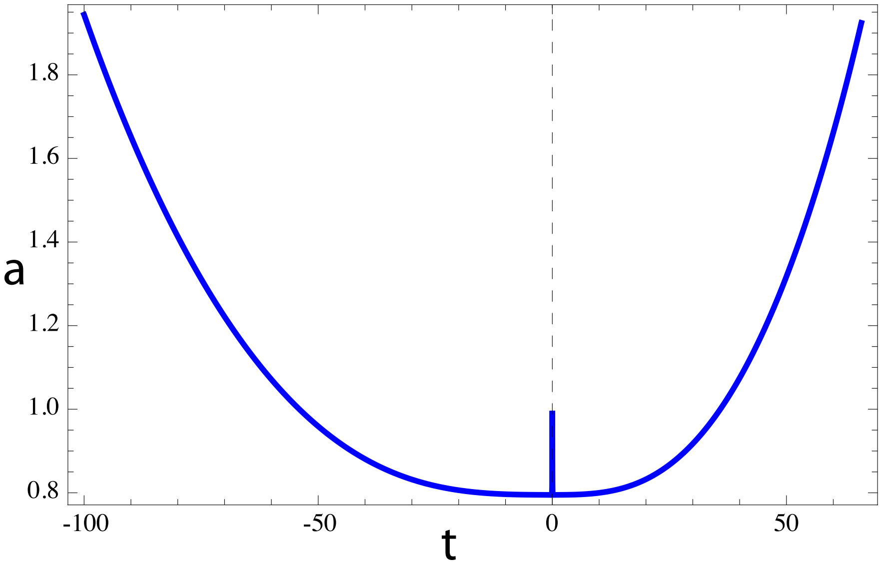
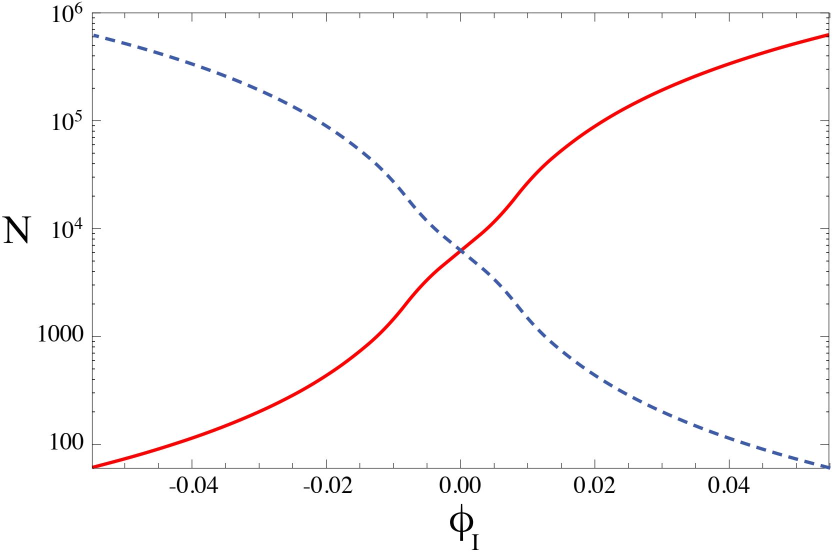


Evolution of a scale factor



e-folds



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- Solution: non-minimal coupling of Higgs field
- Very small ϵ, η - long and stable inflation
- Strong quantum gravity effects: multiple bounces and anadiabatic evolution
- Natural initial conditions for inflation