Restoring hidden local symmetries with SUSY

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based on *S.Abel, JB* - 1202.2863

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Hidden local symmetries (HLSs) are an old idea and a great way of understanding low energy interactions of Goldstone bosons (GBs).

Most famous example is the chiral Lagrangian for QCD:

- QCD with two massless flavours has chiral flavour symmetry $G = SU(2)_L \times SU(2)_R$
- Quark condensate breaks it to $H = SU(2)_{\rm diag}$
- Leads to an HLS description of a broken $SU(2)$ gauge theory
- Provides successful model of pions and ρ-mesons

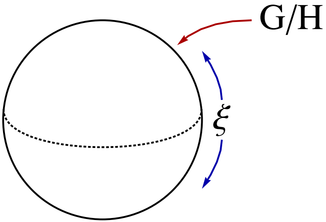
Gauged $SU(2)$ is the HLS: only emerges from low energy dynamics.

More generally[1] start from some theory with flavour symmetry G.

Break it (somehow) to subgroup H at fixed scale v.

- Symmetry breaking $G \to H$ fully determines GB interactions
- Any effective action realising this will do
- Choose a non-linear σ-model on G/H

Artist's impression of a non-linear σ-model



[1] See *Bando, Kugo, Yamawaki* - Phys.Rept. 164 for a comprehensive review, including the supersymmetric generalisation

Matter fields $\xi = e^{i\pi^a T^a}$ contain all real GBs π^a and satisfy the constraint $\xi^\dagger \xi = \mathbb{1}$ (radius of sphere fixed).

Transform non-linearly under G as

$$\xi \longrightarrow g\,\xi\,h^{-1}(g,\pi)$$

But corresponding effective action also invariant under

$$\xi \longrightarrow g\,\xi\,h^{-1}(x)$$

H gauge transformation $\implies$ HLS.

VEV $\xi^\dagger \xi = \mathbb{1}$ spontaneously breaks:

- gauge group H completely
- global G to global H (often mixing with HLS generators)

Now add massive gauge fields for broken H gauge group (no effect on low energy physics).

Non-linear σ-model then recast as linear model with symmetry

$$G_{\text{global}} \times H_{\text{local}}$$

under which

$$\xi \longrightarrow g\, \xi\, h^{-1}(x)$$

This is the basis of the HLS description – actually just gauge equivalent to the σ-model description.

But is the HLS always broken?

SUSY says NO!

Aim of this talk: sketch out why, and consider what we can learn along the way.

SUSY $\implies$ each GB has a real scalar and Weyl fermion partner.

- Superpartners are necessarily massless
- Must appear in any effective theory

The scalar partners are especially important.

- Could be GBs themselves
- Otherwise provide additional massless scalars: quasi-GBs
- Quasi-GBs not just massless – they are flat directions

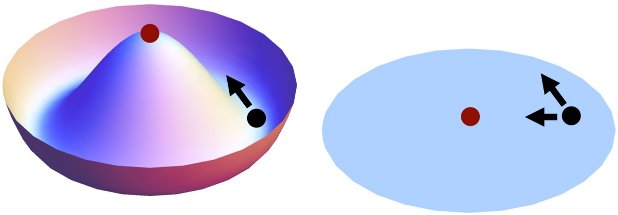
(Technical aside: use complexified symmetries $G \to G^c$, $H \to \hat{H}$.)

Crucial observation

Quasi-GBs may provide the order parameter for breaking $G \to H$.

The symmetry breaking scale v is then able to fluctuate.

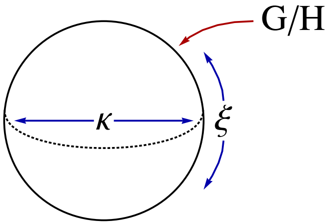
Non-supersymmetric vs. supersymmetric vacua



Left: non-supersymmetric theory – order parameter is stable.

Right: supersymmetric theory – order parameter is flat direction, moduli space contains a point of enhanced symmetry

In σ-model language



Have additional 'radial' degree of freedom.

Looks a lot like a dilaton...

(In SQCD the corresponding degree of freedom is the dilaton.)

More precisely: define matter chiral superfield $\xi = e^{\Pi^a \hat{T}^a}$

Supersymmetric σ-model description

$$K_\sigma = v^2 \,\mathrm{Tr}\left[\ln\left(\xi^\dagger \xi\right)\right]$$

Invariant under $\xi \to g\,\xi\,\hat{h}^{-1}(x)$ for holomorphic function $\hat{h}^{-1}(x)$.

Theory has $\hat{H}$ gauge symmetry – add $\hat{H}$ gauge superfield V

Supersymmetric HLS description

$$K_H = v^2 \,\mathrm{Tr}\left[\left(\frac{\xi^\dagger \xi}{d^\dagger d}\right) e^{-V} + V\right]$$

for auxiliary chiral superfield d.

No kinetic terms for V – looks like Lagrange multiplier.

Solve vector superfield EoM

$$\xi^\dagger \xi = d^\dagger d e^V$$

Substitute back into K_H to find $K_H \to K_\sigma$.

In the Wess-Zumino gauge, the scalar component is

$$\xi^\dagger \xi = d^\dagger d$$

Hence:

- non-supersymmetric constraint $\xi^\dagger \xi = \mathbb{1}$ is modified
- $d \neq 0$ is order parameter for all symmetry breaking
- d vital to keep quasi-GB VEVs unfixed

Is there a limit with $\xi^\dagger\xi \to 0$ such that gauge symmetry is restored?

Yes! When the order parameter is a quasi-GB VEV we can write

$$\xi^\dagger\xi = e^{2\bar{\kappa}}\,(\text{some complex matrix})$$

for real $\bar{\kappa}$.

Hence:

- $\bar{\kappa}$ rescales the VEV of $\xi^\dagger\xi$
- Symmetry is restored by scaling the order parameter to zero

If G contains a broken $U(1)_R$ the superpartner of its GB gives $\bar{\kappa}$ (another dilaton connection).

How to take the $\xi^\dagger\xi \to 0$ limit:

- $\xi^\dagger\xi$ fluctuations equivalent to fluctuations in order parameters
- Symmetry unbroken when $\xi^\dagger\xi \to 0$ so should also have $v \to 0$
- Take limit $e^{\bar{\kappa}} \to 0$ with $e^{-\bar{\kappa}}v$ constant (again, cf the dilaton)

In this limit

$$K_H \longrightarrow \mathrm{Tr}\left[q^\dagger q e^{-V}\right]$$

for normalised chiral superfield $q = v\xi/d$.

Result

HLS description becomes canonically normalised, unbroken gauge theory with flavour symmetry G and gauge group $\hat{H} \supset H$.

Comments:

- Direct application of this idea to SQCD recovers Seiberg duality (see Steve Abel's talk)
- SUSY not the key ingredient – it was a symmetry breaking flat direction
- Similar behaviour should therefore be seen in spontaneously broken CFTs with broken global symmetries
- Non-linearly realised dilaton provides the scaling direction
- Can we use a similar idea to write down explicit conformal gauge theories out of non-supersymmetric HLS descriptions?
- Can we then find 'Seiberg dualities' for them?

Work in progress...

Summary

SUSY allows order parameters to be scaled by moving along quasi-GB directions.

Scaling to zero restores gauge symmetry in the HLS description.

Result is a canonically normalised, unbroken gauge theory.

Crucial feature is that symmetry is broken by flat direction VEV.

Approach may have interesting applications to non-supersymmetric CFTs.