R-Symmetry and Emergent Symmetry

Planck 2012

Warsaw, Poland

Matthew Buican

CERN PH-TH

May 28, 2012

Overview

- Particle physics motivation
- RG generalities.
- R-symmetry, the R-current multiplet, the U multiplet, and the RG flow.
- A simple example.
- Correlation functions of the U multiplet and IR phases.
- Upper bound on emergent bosonic symmetry.
- Lower bound on emergent fermionic symmetry.
- Universality of emergent SUSY.

Particle physics motivation

• Based on LHC results, weakly-coupled Higgs looks likely... $\Rightarrow$ SUSY?

• If SUSY exists, then it is broken, and need hidden sector. Should have some type of R-symmetry (Nelson-Seiberg).

• If SUSY is broken dynamically, there will be some type of strong coupling involved $\Rightarrow$ study general non-perturbative aspects of R-symmetric theories.

• Also, we frequently have some emergent bosonic symmetries in such theories (ISS, etc.) $\Rightarrow$ Constraints on emergent symmetries should lead to constraints on DSB.

Particle physics motivation (cont...)

• ... But sparticles still haven't been observed. If SUSY is to remain "natural," we need light stops. This suggests a sector in which SUSY breaking is suppressed.

• We will suggest a new non-perturbative RG rule (an inequality in the spirit of the a-theorem) applicable in a broad class of R-symmetric theories. It will be related to the emergence of accidental symmetries (both bosonic and fermionic) in the IR.

RG Generalities

• Under rather general assumptions, UV-complete QFTs can be understood as interpolations between UV and IR scale-invariant limits (may also be gapped and hence empty in IR).

• Given well-defined operators and correlation functions of the UV theory, can we say something about the corresponding objects in the IR?

• What are the emergent symmetries of the IR fixed points? What are the broken symmetries?

• In general, new internal and space-time symmetries. What are they? How do we get a handle on them?

RG Generalities (cont...)

• Non-perturbative dynamics along the RG flow make these questions hard to answer.

• We will specialize to four-dimensional R-symmetric theories.

• As we will see SUSY, and, in particular R-symmetry give us strong handles to use to answer a lot of these questions in controlled settings.

The R-symmetry Current

• Since $[R,Q]\sim Q$, $\{Q,\bar{Q}\}\sim P$, the R-current transforms in a multiplet with $S_{\mu\alpha}$ and $T_{\mu\nu}$.

$$\bar{D}^{\dot{\alpha}}\mathcal{R}_{\dot{\alpha}\alpha}=\chi_\alpha\ ,\quad D^\alpha\chi_\alpha-\bar{D}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}=\bar{D}_{\dot{\alpha}}\chi_\alpha=0\ . \tag{1}$$

When $\chi_\alpha=0$, this is the superconformal R-symmetry.

• There is an ambiguity in the above equation under $\mathcal{R}_{\alpha\dot{\alpha}}\to\mathcal{R}_{\alpha\dot{\alpha}}+[D_\alpha,\bar{D}_{\dot{\alpha}}]\,J$ and $\chi_\alpha\to\chi_\alpha+\frac{3}{2}\bar{D}^2D_\alpha J$ for conserved J, i.e., $\bar{D}^2J=0$. This affects the supercurrent and stress tensor through improvements.

The R-symmetry Current (cont...)

• For the theories we will consider, can write

$$\chi_\alpha = \bar{D}^2 D_\alpha U \ , \tag{2}$$

for a well-defined (and away from the endpoints of the RG flow, non-conserved) U.

• Solving the above equations in the UV of an asymptotically free theory, we find

$$\begin{aligned} \mathcal{R}^{UV}_{\alpha\dot{\alpha}} &= \sum_i \Big(2 D_\alpha \Phi_i \bar{D}_{\dot{\alpha}} \bar{\Phi}^i - r_i [D_\alpha, \bar{D}_{\dot{\alpha}}] \Phi_i \bar{\Phi}^i\Big)\,, \\ U^{UV} &= -\frac{3}{2} \sum_i \left(r_i - \frac{2}{3}\right) \bar{\Phi}^i \Phi_i \ . \end{aligned} \tag{3}$$

More generally: $U^{UV}_\mu = \frac{3}{2}\Big(R^{UV}_\mu - \tilde{R}^{UV}_\mu\Big)$.

The R-symmetry Current and the RG Flow

• **Idea:** Study $\mathcal{R}$ and U along the flow.

• **Assumption**: The UV and IR fixed points are SCFTs (this can be made rigorous in SQCD-like theories **[1102.2294]**; see also recent work of `[Luty, Polchinski, and Rattazzi]`)

• At the IR fixed point, we know what should happen to $\mathcal{R}_{\alpha\dot{\alpha}}$. Indeed, either this multiplet flows to the superconformal R-multiplet or to an object that can be improved to the superconformal R-multiplet:

$$\tilde{\mathcal{R}}_{\alpha\dot{\alpha}} = \mathcal{R}^{IR}_{\alpha\dot{\alpha}} - [D_\alpha, \bar{D}_{\dot{\alpha}}]J \ , \quad \tilde{U} = U^{IR} - \frac{3}{2}J = 0 \ . \tag{4}$$

Determine $\tilde{\mathcal{R}}_{\alpha\dot{\alpha}}$ from duality or a-maximization.

• **Upshot:** Therefore, $U \to \frac{3}{2}J$, where $U^{IR}_\mu = \frac{3}{2}\left(R^{IR}_\mu - \tilde{R}^{IR}_\mu\right)$.

The R-symmetry Current and the RG Flow (cont...)

• J may be a conserved current of the full theory or an accidental symmetry of the IR. We will see an extreme version of this for SQCD in the free magnetic range.

• In the case of a free magnetic phase, we have

$$U^{IR} = -\frac{3}{2}\sum_i \left(r_i - \frac{2}{3}\right)\bar{\phi}^i\phi_i \ , \qquad (5)$$

for the "emergent" d.o.f's.

Example: SQCD in the Free Magnetic Range

- Consider $SU(N_c)$ with $N_c + 1 < N_f \leq 3N_c/2$: this is a flow between Gaussian fixed points

- The UV (electric) theory:

$$\begin{array}{ccccc} & SU(N_c) & SU(N_f)\times SU(N_f) & U(1)_R & U(1)_B \\ Q & \mathbf{N_c} & \mathbf{N_f}\times\mathbf{1} & 1-\frac{N_c}{N_f} & 1 \\ \tilde{Q} & \bar{\mathbf{N}}_\mathbf{c} & \mathbf{1}\times\bar{\mathbf{N}}_\mathbf{f} & 1-\frac{N_c}{N_f} & -1 \end{array} \tag{6}$$

- Some bilinears that we can write are $c_i^j Q^i Q_j^\dagger + \tilde{c}_i^j \tilde{Q}^i \tilde{Q}_j^\dagger$. What are they in the IR?

Example: SQCD in the Free Magnetic Range (cont...)

- We have the following IR (magnetic) theory [Seiberg]

	$SU(N_f - N_c)$	$SU(N_f) \times SU(N_f)$	$U(1)_R$	$U(1)_B$
q	$\mathbf{N_f - N_c}$	$\mathbf{\bar{N}_f \times 1}$	$\frac{N_c}{N_f}$	$\frac{N_c}{N_f - N_c}$
$\tilde{q}$	$\mathbf{\bar{N}_f - \bar{N}_c}$	$\mathbf{1 \times \bar{N}_f}$	$\frac{N_c}{N_f}$	$-\frac{N_c}{N_f - N_c}$
M	$\mathbf{1}$	$\mathbf{N_f \times N_f}$	$2 - 2\frac{N_c}{N_f}$	0

(7)

- Some objects are trivial to map, e.g. $QQ^\dagger - \tilde{Q}\tilde{Q}^\dagger \longrightarrow \frac{N_c}{N_f - N_c}\left(|q|^2 - |\tilde{q}|^2\right)$.

Example: SQCD in the Free Magnetic Range (cont...)

- But what about $J_A = QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger$? It is not conserved:

$$\bar{D}^2 J_A = \text{Tr} W_\alpha^2 \ . \tag{8}$$

- **Claim:** We can follow this operator using the $\mathcal{R}$ multiplet. Indeed, using the R-charge assignments in the electric table, we find

$$U^{UV} = \left(-\frac{1}{2} + \frac{3N_c}{2N_f}\right)\left(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger\right) \tag{9}$$

- Using the R-charge assignments in the IR, we find

$$U^{IR} = \left(1 - \frac{3N_c}{2N_f}\right)\left(qq^\dagger + \tilde{q}\tilde{q}^\dagger\right) - \left(2 - \frac{3N_c}{N_f}\right) MM^\dagger \tag{10}$$

The R-current Multiplet and IR Phases of Gauge Theories

• We have seen that the R-current multiplet gives us a handle on a particular long (spin zero) multiplet, U.

• **Question:** Does it also contain some global information? Encodes the phase of the IR theory? Is the deep IR an interacting or a free SCFT (perhaps below some confining scale, Λ)?

The R-current Multiplet and IR Phases of Gauge Theories (cont...)

- To understand this question, we will study $\langle U(x)U(0)\rangle$.

- But which U (and R_μ)? This is ambiguous.

- We will study the one defined (up to some caveats) by a-maximization in the deformed UV theory, $(\mathcal{R}^{UV}_{\mu,\text{vis}}, U^{UV}_{\text{vis}})$.

The R-current Multiplet and IR Phases of Gauge Theories (cont...)

• We will study τ_U:

$$\langle U_{\mu,\text{vis}}^{UV,IR}(x) U_{\nu,\text{vis}}^{UV,IR}(0)\rangle = \frac{\tau_U^{UV,IR}}{(2\pi)^4}\left(\partial^2\eta_{\mu\nu} - \partial_\mu\partial_\nu\right)\frac{1}{x^4}\ . \qquad (11)$$

• Note that in theories without accidental symmetries, $\tau_U^{UV} > 0 = \tau_U^{IR}$.

• We will empirically check $\tau_U^{UV} > \tau_U^{IR}$ more generally (new information not contained in $a_{UV} > a_{IR}$).

The R-current Multiplet and IR Phases of Gauge Theories (cont...)

• $\tau_U^{UV} > \tau_U^{IR}$ implies a UV "upper" bound on accidental bosonic symmetries.

• This statement has potential implications for the IR phase of the ISS theory (and DSB).

• τ_U^{UV} is a quantity in the UV SCFT, although it is not intrinsically defined in it (only defined once have in mind an R-symmetric relevant deformation and/or R-symmetry-preserving vev).

Defining τ_U

• We start by using a-maximization to find the UV superconformal R-current; consider $\mathcal{R}^{t*}_{\mu,UV} = \mathcal{R}^{(0)*}_{\mu,UV} + \sum_i t^i J^{UV*}_{\mu,i}$, where $J^{UV*}_{\mu,i}$ are the full set of non-R symmetries of the UV SCFT.

• Taking $\tilde{a}^t_{UV} = 3\mathrm{Tr}\ \left(\mathcal{R}^{t*}_{UV}\right)^3 - \mathrm{Tr}\ \mathcal{R}^{t*}_{UV}$, solve $\partial_{t^i}\tilde{a}^t_{UV}|_{t^i=t^i_*} = 0$, $\partial^2_{t^it^j}\tilde{a}^t_{UV}|_{t^{i,j}=t^{i,j}_*} < 0$. This defines $\tilde{R}^{UV}_\mu$.

• Deform the theory by turning on an R-symmetry-preserving relevant deformation and/or an R-symmetry-preserving vev. Now only $\left\{\hat{J}^{UV*}_{\mu,a}\right\} \subset \left\{J^{UV*}_{\mu,i}\right\}$ are still conserved currents that respect the vacuum.

Defining τ_U (cont...)

• Maximizing $\tilde{a}$ over this subset yields $\mathcal{R}_\mu^{UV} = \mathcal{R}_\mu^{(0),UV} + \sum_a \hat{t}_*^a \hat{J}_{\mu,a}^{UV}$ and U^{UV} which descend from a corresponding pair in the undeformed UV SCFT, $(\mathcal{R}_{\mu,\text{vis}}^{UV}, U_{\text{vis}}^{UV})$. See **[1109.3279]** for a slightly more general definition.

SQCD

• Our procedure fixes $\mathcal{R}^{UV}_{\text{vis}}(Q) = \mathcal{R}^{UV}_{\text{vis}}(\tilde{Q}) = 1 - \frac{N_c}{N_f}$ and $U^{UV}_{\text{vis}}(Q) = U^{UV}_{\text{vis}}(\tilde{Q}) = \frac{1}{2} - \frac{3N_c}{2N_f}$.

• Consider $N_f < 3N_c$, and start from the free UV theory.

• Begin with $N_f = N_c$ and work our way up. All the subtleties we have discussed in this talk are present in this class of theories (accidental symmetries, Goldstone bosons, interacting fixed points etc.).

SQCD (cont...)

• $N_f = N_c$; $\tau_U^{UV} = 2N_c^2$; in the IR have a deformed moduli space $\det M + B\tilde{B} = \Lambda^{2N_c}$ with $< N_c^2 + 2$ mesons, M, and baryons, $B, \tilde{B}$.

• Since $\mathcal{R}_{\text{vis}}^{IR}(M) = \mathcal{R}_{\text{vis}}^{IR}(B) = \mathcal{R}_{\text{vis}}^{IR}(\tilde{B}) = 0$ and $U_{\text{vis}}^{IR}(M) = U_{\text{vis}}^{IR}(B) = U_{\text{vis}}^{IR}(\tilde{B}) = -1$, we have $\tau_U^{IR} < N_c^2 + 2$.

$$\tau_U^{UV} = 2N_c^2 > N_c^2 + 2 > \tau_U^{IR}. \tag{12}$$

• $N_f = N_c + 1$; $\tau_U^{UV} = \frac{N_c(1-2N_c)^2}{2(1+N_c)}$; confinement without chiral symmetry breaking, $(N_c+1)^2$ mesons, M, and $2(N_c+1)$ baryons B and $\tilde{B}$.

SQCD (cont...)

• Have $\mathcal{R}_{\text{vis}}(M) = \frac{1-2N_c}{1+N_c}, \mathcal{R}_{\text{vis}}(B) = \mathcal{R}_{\text{vis}}(\tilde{B}) = \frac{N_c}{2}\frac{1-2N_c}{1+N_c}$, $U(M) = -1+\frac{3}{N_c+1}$, $U(B) = U(\tilde{B}) = \frac{N_c-2}{2(N_c+1)}$. Therefore, $\tau_U^{IR} = \frac{(N_c-2)^2(3+2N_c)}{2(1+N_c)}$ and

$$\tau_U^{UV} = \frac{N_c(1-2N_c)^2}{2(1+N_c)} > \frac{(N_c-2)^2(3+2N_c)}{2(1+N_c)} = \tau_U^{IR} \ . \qquad (13)$$

• Can see that fully conserved current two-point functions have no definite behavior along the RG flow. Therefore, a-maximization picks out a current, U, that has nice properties.

SQCD (cont...)

• $N_f = N_c{+}2$, confining description breaks down; $\tau_U^{UV} = \frac{2N_c(N_c-1)^2}{N_c+2}$ while $\tau_U^{\text{conf}} = \frac{5N_c^3-10N_c^2-4N_c+36}{N_c+2}$, and so conjecture would be violated in a hypothetical confining phase.

• Luckily, correct description is free magnetic with $\mathcal{R}_{\text{vis}}^{IR}(M) = 2\left(1-\frac{N_c}{N_f}\right), \mathcal{R}_{\text{vis}}^{IR}(q) = \mathcal{R}_{\text{vis}}^{IR}(\tilde{q}) = \frac{N_c}{N_f}$ and $U_{\text{vis}}^{IR}(M) = 2-\frac{3N_c}{N_f}$, $U_{\text{vis}}^{IR}(q) = U_{\text{vis}}^{IR}(\tilde{q}) = -1+\frac{3N_c}{2N_f}$. Therefore:

$$\tau_U^{UV} = \frac{N_c(N_f-3N_c)^2}{2N_f} > \frac{(3N_f-N_c)(3N_c-2N_f)^2}{2N_f} = \tau_U^{IR} \ . \quad (14)$$

SQCD (cont...)

• The above expressions are valid for $N_c + 1 < N_f \leq 3N_c/2$. The inequality holds up to $N_f \sim 1.79N_c$ (where the theory flows to an interacting conformal fixed point, and the above expressions don't apply). Comes close to predicting onset of conformal window.

• In conformal window, $3N_c/2 < N_f < 3N_c$, trivially have (from assumed lack of accidental symmetries)

$$\tau_U^{UV} > 0 = \tau_U^{IR} \ . \tag{15}$$

• Can do some more complicated tests of conformal window.

SQCD (cont...)

• Easy to generalize the above discussion to $SO(N_c)$ and $Sp(N_c)$ gauge groups

• Also other more exotic s-confining theories; SCFTs with accidental symmetries; $\mathcal{N} = 2$ SYM; Kutasov and Brodie theories; See **[1109.3279]** for details.

The IR Phase of ISS and Constraints on DSB

• Intriligator, Seiberg, and Shenker consider an $SU(2)$ gauge theory with a single field, Q, in the isospin 3/2 representation.

• They conjectured that the IR theory at the origin is described by a confined $u = Q^4$ field (classically, the Kähler potential is singular at the origin); indeed, since $\mathcal{R}_{\text{vis}}^{UV}(Q) = 3/5$ and $\mathcal{R}_{\text{vis}}^{UV}(u) = 12/5$, the $U(1)_R$ and $U(1)_R^3$ anomalies match.

• If the confining description is correct, then, upon deforming the theory by $W = \lambda u$, we would find a simple model of (dynamical) SUSY breaking. In this vacuum, there would be a preserved R-symmetry that is a mixture of the accidental non-R symmetry under which u transforms and $\mathcal{R}_{\text{vis}}$.

The IR Phase of ISS and Constraints on DSB (cont...)

• Other techniques have since pointed to the opposite conclusion [Intriligator], [Poppitz and Unsal], [Vartanov]

• Our criterion also suggests this is the case. Indeed, $U_{\text{vis}}^{UV}(Q) = -\frac{1}{10}$, $U_{\text{vis}}^{IR}(u) = \frac{13}{5}$ and so

$$\tau_U^{UV} = \frac{1}{25}, \quad \tau_U^{IR,\text{confining}} = \frac{169}{25} , \tag{16}$$

• Conjecture formalizes the intuition that the theory is too weak to produce confined d.o.f's (the 1-loop beta fn is $b = 6 - 5 = 1$).

• Can also check that our procedure is consistent with better understood misleading anomaly matchings [Brodie, Cho, Intriligator].

Emergent SUSY

• The statement $\tau_U^{UV} > \tau_U^{IR}$ has potential consequences for emergent SUSY.

• Indeed, suppose we add a probe deformation $\delta\mathcal{L}_{UV} = m^2 U^{UV}|$. In the IR, at leading order in the deformation, we have $\delta\mathcal{L}_{IR} = m^2 U^{IR}|$. It immediately follows that $\delta\mathcal{L}_{IR} < \delta\mathcal{L}_{UV}$, and so SUSY breaking can be suppressed (up to some caveats).

• This idea generalizes the reasoning in the specific example of **[Csaki, Randall, and Terning]**. They added

$$\delta\mathcal{L}_{UV} = m^2 U^{UV}| = \left(-\frac{1}{2} + \frac{3N_c}{2N_f}\right)\left(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger\right) \tag{17}$$

Emergent SUSY (cont...)

- Recall that this flows to

$$\delta\mathcal{L}_{IR} = m^2 \left(\left(1 - \frac{3N_c}{2N_f} \right) \left(qq^\dagger + \tilde{q}\tilde{q}^\dagger \right) - \left(2 - \frac{3N_c}{N_f} \right) MM^\dagger \right) \quad (18)$$

- For $N_f = 3N_c/2$, $U^{IR} = 0$. If we identify $m^2 U^{UV}|$ with the stop mass, we find that the leading order mass vanishes in the IR.

How Universal is Emergent SUSY?

• We wish to study

$$\delta\mathcal{L}_{UV} = m_a^2 \hat{J}^a| + m_U^2 U^{UV}| + m_A^2 J_A| \ , \qquad (19)$$

where these soft terms correspond to symmetries of the UV SCFT, and find out how generic $\delta\mathcal{L}_{IR} = 0$ is (the $\hat{J}^a$ are symmetries that are conserved throughout the RG flow).

• Clearly, we are interested in theories for which $U^{IR} \to 0$ (i.e., the most extreme manifestation of $\tau_U^{UV} > \tau_U^{IR}$). But we know we should be weary of terms proportional to $\hat{J}^a$.

How Universal is Emergent SUSY? (cont...)

• **Theorem:** A necessary and sufficient condition for the $\widehat{J}_a \to 0$ in the deep IR is that all the 't Hooft anomalies involving these currents vanish: $\mathrm{Tr}\widehat{J}_a\widehat{J}_b\widehat{J}_c = \mathrm{Tr}R\widehat{J}_a\widehat{J}_b = \mathrm{Tr}R^2\widehat{J}_a = \mathrm{Tr}\widehat{J}_a = 0$.

• The proof follows from the fact that $U^{IR} \to 0$, and so $R_\mu^{IR} = \tilde{R}_\mu^{IR}$. Unitarity (in the guise of positivity of the IR current two point functions) then provides the non-trivial (sufficient) direction of the theorem.

• **Corollary:** In asymptotically free theories with simple gauge group and $W = 0$, there will generically be soft terms remaining in the IR unless we impose some discrete symmetries (and non-abelian flavor symmetries) on the UV soft terms.

How Universal is Emergent SUSY? (cont...)

• For small perturbations of such theories (i.e, small δW and additional weak gauging), the result still holds at leading order in the perturbations.

• Remarkably, this result follows from a simple exercise in linear algebra and does not depend on detailed knowledge of strong dynamics.

• Of course, there are many ways to get around this result: important contributions from W, interacting UV fixed point, no RG-conserved currents, etc.

Conclusions

• We have seen that the $(\mathcal{R}_\mu, U)$ multiplets contain a great deal of physics.

• We can potentially use this pair to learn things about emergent symmetries and DSB.

• We may also learn about the role that certain theories can and cannot play in particle physics!