Twisted compactification of matrix models for superstrings

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Based on:
A.C., Phys. Rev. D84 (2011) 106010 (arXiv:1108.1107 [hep-th])
A.C. and Larisa Jonke, Phys. Rev. D (2012) (arXiv:1202.4310 [hep-th])

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Introduction and Motivation

Superstring / M theory: excellent candidates for unified description of our current knowledge of Nature.

Most of our knowledge within perturbative string theory (especially regarding compactifications).

Access to non-perturbative regime?

Why Matrix Models?

Matrix Models as non-perturbative definitions of string / M theory.

Banks, Fischler, Shenker, Susskind '96, Ishibashi, Kawai, Kitazawa, Tsuchiya '96

- Framework to address profound conceptual problems, study brane dynamics, test symmetries-dualities, analytically and numerically.

- Ask questions about low-energy physics.
  - ✔ Particle physics models, new approach to model building.

    Aoki '10, A.C., Steinacker, Zoupanos '11
  - ✔ Cosmological implications, expanding universe.

    Kim, Nishimura, Tsuchiya '11

Compactifications of MM.

Banks, Fischler, Shenker, Susskind '96, Taylor '96

Matrix compactifications on tori $\rightsquigarrow$ striking relations to non-commutative geometry Connes, Douglas, Schwarz '97

$\rightsquigarrow$ NC deformations in correspondence to supergravity fluxes.

Douglas, Hull '97, Brace, Morariu, Zumino '98, Kawano, Okuyama '98

Can we learn more from MM compactifications?

Why twisted?

Twisted tori:

- ✔ supersymmetric backgrounds rich in fluxes.
- ✔ vacua of heterotic, type II, M theory.

Kaloper, Myers '99, Kachru, Schutz, Tripathy, Trivedi '02,
Hull, Reid-Edwards '05,'06, Grana, Minasian, Petrini, Tomasiello '06

One step closer to non-geometric fluxes / unconventional backgrounds. Dabholkar, Hull '02, Flournoy, Wecht, Williams '04,...

Q: Matrix compactifications on twisted tori?

Lowe, Nastase, Ramgoolam '03, A.C. '11, A.C., Jonke '12

Overview

Twisted tori as nilmanifolds

Matrix compactifications on (twisted) tori

Conclusions and open questions

Nilmanifolds Mal'cev '51

Smooth manifolds $\mathcal{M}$ of the form A/Γ
A: Nilpotent Lie group, Γ: Discrete subgroup of A

Classification of nilpotent Lie algebras $\mathcal{A}_d$:
Morozov '58, Mubarakzyanov '63, Patera et.al. '75

- 1 in 3D
- 1 in 4D
- 6 in 5D
- 22 in 6D

Nilpotency $\rightsquigarrow$ upper triangular matrices...Facilitates construction of $\mathcal{M}$.

Compactness criterion for $\mathcal{M}$: unimodularity of $\mathcal{A}$ ($f^a{}_{ab} = 0$)
f^a_{bc} being the structure constants $\sim$ geometric fluxes.

Well-known example: 3D

3D nilpotent Lie algebra $\mathcal{A}_3$: $[X_2, X_3] = X_1$.

Upper triangular basis:

$$X_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad X_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad X_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Group element: $\quad g = \begin{pmatrix} 1 & x^2 & x^1 \\ 0 & 1 & x^3 \\ 0 & 0 & 1 \end{pmatrix}, x^i \in \mathbb{R}.$

Restriction to Γ: $g|_\Gamma = \begin{pmatrix} 1 & \gamma^2 & \gamma^1 \\ 0 & 1 & \gamma^3 \\ 0 & 0 & 1 \end{pmatrix}, \gamma^i \in \mathbb{Z}.$

Compact nilmanifold: A_3/Γ.

Why "twisted torus"?

For a torus T^3, with covering group $\mathbb{R}^3$ and lattice $\mathbb{Z}^3$,

$$(x^1, x^2, x^3) \sim (x^1 + 1, x^2, x^3) \sim (x^1, x^2 + 1, x^3) \sim (x^1, x^2, x^3 + 1),$$

x^i the toroidal coordinates.

For the 3D nilmanifold:

$$(x^1, x^2, x^3) \sim (x^1{+}1, x^2, x^3) \sim (x^1, x^2, x^3{+}1) \sim (x^1{+}x^3, x^2{+}1, x^3).$$

$\rightsquigarrow$ twisted fibration of a T^2 fiber over an S^1 base.

- ✔ The T^2 geometry changes as it traverses S^1 $\rightsquigarrow$ twisted $\tilde{T}^3$.

T-duality approach

Alternatively, square T^3 with N units of NS-NS flux $H = dB$:

- ✔ Metric: $ds^2 = \delta_{ab} dx^a dx^b$.
- ✔ B-field: $B_{31} = Nx^2$.

Perform a T-duality along x^1 using the Buscher rules.

In the T-dual frame:

- ✔ Metric: $ds^2 = \delta_{ab} e^a e^b \rightsquigarrow e^a$: the globally well-defined 1-forms of the twisted torus.
- ✔ B-field: $B = 0$.

$$H_{123} \xrightarrow{T_1} f^1_{23}.$$

Matrix Models

Matrix theory: suggested as non-perturbative definition of M-theory.

Banks, Fischler, Shenker, Susskind '96

Action:

$$\mathcal{S}_{BFSS} = \frac{1}{2g}\int dt \left[Tr\left(\dot{\mathcal{X}}_a\dot{\mathcal{X}}_a - \frac{1}{2}[\mathcal{X}_a,\mathcal{X}_b]^2\right) + 2\psi^T\dot{\psi} - 2\psi^T\Gamma^a[\psi,\mathcal{X}_a]\right],$$

$\mathcal{X}_a(t)$: 9 time-dependent $N \times N$ Hermitian matrices,
ψ: fermionic superpartners, Γ^a: rep. of $SO(9)$.

EOM: $\ddot{\mathcal{X}}_a + [\mathcal{X}_b,[\mathcal{X}^b,\mathcal{X}_a]] = 0.$

IKKT: non-perturbative type IIB superstring.

Ishibashi, Kawai, Kitazawa, Tsuchiya '96

10 Hermitian matrices...and action:

$$\mathcal{S}_{IKKT} = \frac{1}{2g} Tr\left(-\frac{1}{2}[\mathcal{X}_a,\mathcal{X}_b]^2 - \bar{\psi}\Gamma^a[\mathcal{X}_a,\psi]\right).$$

Toroidal compactification (no twist) Connes, Douglas, Schwarz '97

Specific restriction of the matrix action...For a T^3 compactification:

$$
\begin{aligned}
\mathcal{X}_1 + R_1 &= U_1 \mathcal{X}_1 U_1^{-1}, \\
\mathcal{X}_2 + R_2 &= U_2 \mathcal{X}_2 U_2^{-1}, \\
\mathcal{X}_3 + R_3 &= U_3 \mathcal{X}_3 U_3^{-1}, \\
\mathcal{X}_a &= U_i \mathcal{X}_a U_i^{-1}, \quad a \neq i, \quad a = 1, \ldots, 9, \quad i = 1, 2, 3.
\end{aligned}
$$

Determine:

- ✔ The form of the Hermitian matrices $\mathcal{X}$ and...
- ✔ ...the set of unitary matrices U

defining a consistent background of the compactified model.

The compactification conditions are generally solved by:

$$\begin{aligned} \mathcal{X}_i &= iR_i\mathcal{D}_i, \quad \mathcal{X}_m = \mathcal{A}_m, \quad m = 4, \dots, 9, \\ U_i &= e^{i\hat{x}^i}, \end{aligned}$$

$\mathcal{D}_i$: covariant derivatives $\mathcal{D}_i = \partial_i - i\mathcal{A}_i(\hat{U})$,
U-algebra: $U_i U_j = \lambda_{ij} U_j U_i, \quad \lambda_{ij} = e^{2\pi i \theta^{ij}}$.

$\theta^{ij} = 0 \rightarrow$ standard torus.
But $\theta^{ij} \neq 0$ is fully legitimate too and Us are then non-commuting operators...

Compactification on non-commutative T^3_θ.

Coordinates $x^i \to$ Operators $\hat{x}^i$ with $[\hat{x}^i, \hat{x}^j] = -2\pi i\theta^{ij}$.

- ✔ $\mathcal{A}$-fields depend on a set of operators $\hat{U}$, commuting with U:

  $$\hat{U}_i = e^{i\hat{x}^i - 2\pi\theta^{ij}\hat{\partial}_j},$$

  satisfying dual relations $\hat{U}_i\hat{U}_j = e^{2\pi i\hat{\theta}^{ij}}\hat{U}_j\hat{U}_i, \quad \hat{\theta}^{ij} = -\theta^{ij}$.

  Brace, Morariu, Zumino '98
- ✔ Results into a sYM theory living on the dual NC torus with deformation parameter $\hat{\theta}$.

Connes-Douglas-Schwarz correspondence

Deformation parameters θ_{ij} of MM on $\mathrm{T}^d_\theta \leftrightarrow$ moduli of 11D sugra

11D sugra: 3-form potential C_{IJK}. Claim: $\theta_{ij} \propto \int dx^i dx^j C_{ij-}$.

In IIA language: $\theta_{ij} \propto \int dx^i dx^j B_{ij}$.

$\rightsquigarrow$ Deform tori in MM $\overset{CDS}{\longleftrightarrow}$ Turn on background in sugra

Twisted compactifications

Restrict the action by imposing conditions corresponding to the twisted identifications for nilmanifolds... For the twisted $\tilde{\mathrm{T}}^3$ case:

$$
\begin{aligned}
\mathcal{X}_1 + R_1 &= U_1 \mathcal{X}_1 U_1^{-1}, \\
\mathcal{X}_2 + R_2 &= U_2 \mathcal{X}_2 U_2^{-1}, \\
\mathcal{X}_3 + R_3 &= U_3 \mathcal{X}_3 U_3^{-1}, \\
\mathcal{X}_1 + R_2 \mathcal{X}_3 &= U_2 \mathcal{X}_1 U_2^{-1}, \\
\mathcal{X}_a &= U_i \mathcal{X}_a U_i^{-1}, \quad a \neq i, \quad (a,i) \neq (1,2).
\end{aligned}
$$

As before, there exists a solution on a commutative twisted torus and sets of solutions on NC deformations.

Non-commutative case

A general configuration has:

$$\begin{aligned} U_1 U_3 &= \lambda_{13} U_3 U_1, \quad \lambda_{13} = e^{-2\pi i \theta^{13}}, \\ U_1 U_2 &= \lambda_{12} U_2 U_1, \quad \lambda_{12} = e^{-2\pi i \theta^{12}}, \end{aligned}$$

but also some $\hat{x}$-dependence:

$$U_2 U_3 = e^{-2\pi i \theta^{23} - iR\hat{x}^1} U_3 U_2, \quad R \equiv \frac{R_2 R_3}{R_1}$$

The non-commutative solution may be written in a closed form admitting a direct generalization for any higher-dimensional nilmanifold.

The general solution now is simply given by:

$$\mathcal{X}_i = iR_i\hat{\mathcal{D}}_i, \quad \mathcal{X}_m = \mathcal{A}_m, \quad U_i = e^{i\hat{x}^i},$$

with commutation relations:

$$\begin{aligned}
[\hat{x}^i, \hat{x}^j] &= iR_{(ij)} f^{ij}{}_k \hat{x}^k + 2\pi i \theta^{ij}, \\
[\hat{\partial}_i, \hat{\partial}_j] &= 0, \\
[\hat{\partial}_i, \hat{x}^j] &= \delta_i^j + iR_{(jk)} f^{jk}{}_i \hat{\partial}_k, \quad j < k \ .
\end{aligned}$$

Note that $\hat{\mathcal{D}}_i = \hat{\partial}_i - i\hat{\mathcal{A}}_i(\hat{U})$, with $\hat{\mathcal{A}}_i = \mathcal{A}_i(\hat{U}) + iRf^{jk}{}_i\mathcal{A}_k(\hat{U})\hat{\partial}_j$.

- ✔ Mixed non-commutativity (constant and non-constant).
- ✔ The last relation guarantees associativity.
- ✔ The dual $\hat{U}$ can be written down explicitly.

Relation to sugra à la CDS: Geometric fluxes (f) + constant B-field (θ).

Towards more general backgrounds

So far:

- ✔ Constant non-commutativity ($\rightsquigarrow$ constant B-field).
- ✔ $\hat{x}$-type non-commutativity ($\rightsquigarrow$ geometric flux).

...or both.

How is NS-NS H-flux implemented (non-constant B-field)?

$$\begin{aligned} [\hat{x}^i, \hat{x}^j] &= i f^{ij}{}_k \hat{x}^k + N\, H^{ijk} \hat{\partial}_k , \\ [\hat{\partial}_i, \hat{\partial}_j] &= 0, \\ [\hat{\partial}_i, \hat{x}^j] &= \delta_i^j + i f^{jk}{}_i \hat{\partial}_k , \quad j < k \; . \end{aligned}$$

This provides a solution to the compactification conditions for the twisted torus.

The phase-space algebra has a non-associative structure,

$$[\hat{x}^i, \hat{x}^j, \hat{x}^k] = N \ H^{ijk}.$$

This should be related to the presence of N units of H-flux...
Lüst '10, '12

Non-associativity in H-flux backgrounds also in CFT computations.
Cornalba, Schiappa '01, Blumenhagen, Plauschinn '10, Lüst '10
Blumenhagen, Deser, Lüst, Plauschinn, Rennecke '11

Q: More fluxes? Further toroidal deformations $\rightsquigarrow$ non-geometry...
Answers soon..(work in progress with L. Jonke)

Conclusions and open questions

Main message

- ✔ A class of new Matrix Model (flux) compactifications was described.
  - (Deformed, quantum) Twisted tori in diverse dimensions.
- ✔ Non-commutative deformations tantamount to sugra fluxes.
  - Constant B-field background, geometric flux, NS-NS flux.

Main questions

- ✔ Could unconventional compactifications be described? (non-geometric, winding modes, T-folds...)
- ✔ What can we learn about non-perturbative dualities?
- ✔ How does gravity operate? Lessons for properties of gravity in string theory?
- ✔ Are there phenomenologically interesting configurations?