Precise Conformal Window & Realization of Miransky Scaling

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Antipin,SDC,Mojaza,Mølgaard,Sannino; arXiv:1205.6157

CP^3 - Origins

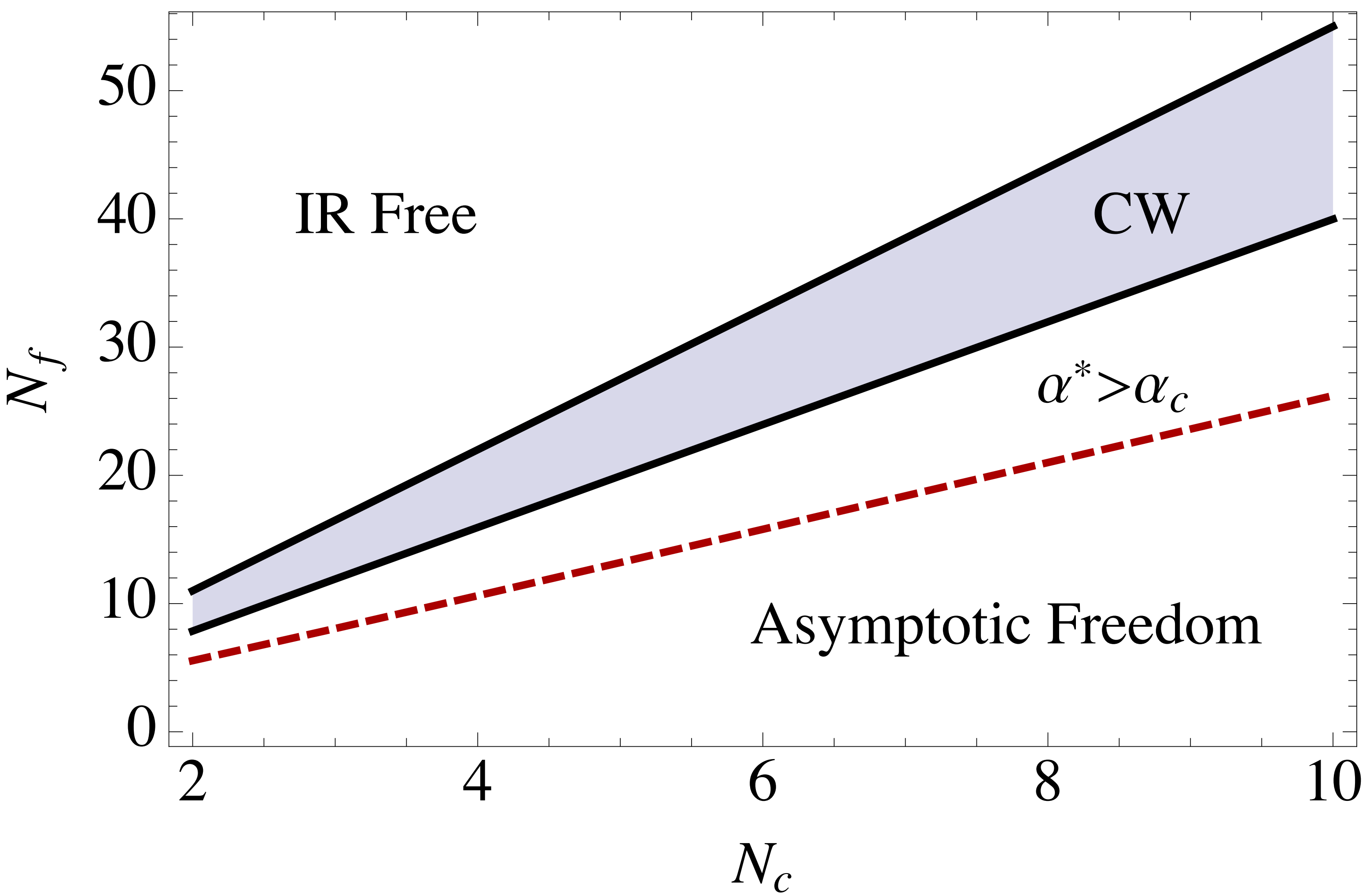
Particle Physics & Origin of Mass

Planck 2012, Warsaw

Conformal Window

Conformal Window (CW) $\equiv$ region of the $N_c - N_f$ plane featuring (at least) a zero in the beta functions:

$$\beta(g) = -\beta_0 \frac{\alpha^2}{4\pi} - \beta_1 \frac{\alpha^3}{(4\pi)^2}, \; \alpha^* = -4\pi \frac{\beta_0}{\beta_1}, \; \alpha_c = \frac{\pi}{3C_2(R)}$$



CW determined by requiring asymptotic freedom, existence of a fixed point (FP), and conformality to arise before chiral symmetry breaking:

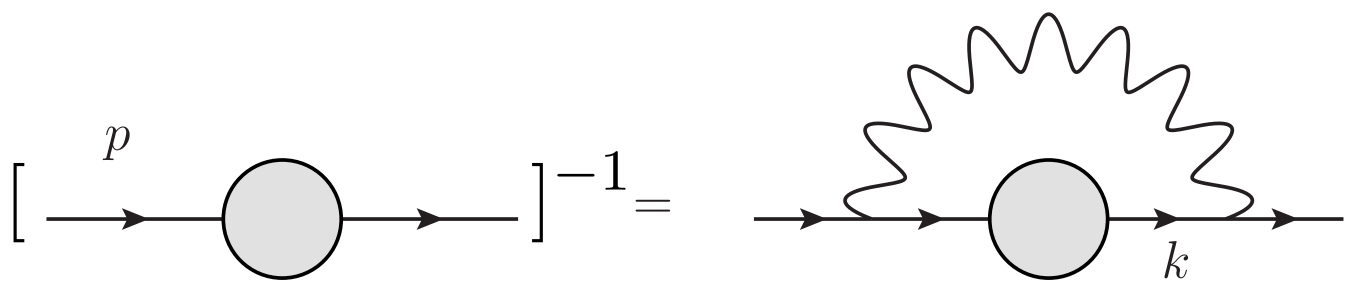
$$\beta_0 > 0, \; \beta_1 < 0, \; \alpha_* < \alpha_c$$

with α_c = critical coupling.

* Banks, Zaks '85

Miransky Scaling

If $\alpha^* > \alpha_c$ the theory is believed to generate a fermion condensate at an IR scale Λ. From the Schwinger-Dyson equation for a fermion self-energy in the "rainbow" approximation:



$$\Leftrightarrow \quad \Sigma(p^2) = 3C_2(R) \int \frac{d^4q}{(2\pi)^4} \frac{\alpha((q-p)^2)}{(q-p)^2} \frac{\Sigma(k^2)}{Z(q^2)q^2 + \Sigma^2(q^2)} ,$$

$$\alpha^* > \alpha_c \Rightarrow \quad \Lambda \sim \Sigma(0) \approx \mu_0 \exp\left(\frac{-\pi}{K\sqrt{N_f^*/N_f - 1}} \right) ,$$

with $N_f = N_f^*$ triggering chiral symmetry breaking and K a constant. This is the Miransky scaling of Λ in function of μ_0.

* Miransky '85

Motivations

Near conformal dynamics applications:

- Strongly coupled gauge theories
- Lattice
- Scalar massive resonance $a.k.a.$ Higgs boson

The couplings at the FPs are usually large: perturbative and approximate non-perturbative methods are not completely reliable.

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We would like a 4D theory where the conformal phase transition can be studied perturbatively…

AMS Model

Fields	$[SU(N_c)]$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_V$	$U(1)_{AF}$
λ	Adj	1	1	0	1
q	$\square$	$\overline{\square}$	1	$\frac{N_f-N_c}{N_c}$	$-\frac{N_c}{N_f}$
$\widetilde{q}$	$\overline{\square}$	1	$\square$	$-\frac{N_f-N_c}{N_c}$	$-\frac{N_c}{N_f}$
H	1	$\square$	$\overline{\square}$	0	$\frac{2N_c}{N_f}$
G_μ	Adj	1	1	0	0

We consider QCD with the addition of a meson-like scalar as well as an adjoint Weyl fermion $a.k.a.$ Antipin-Mojaza-Sannino (AMS) model:

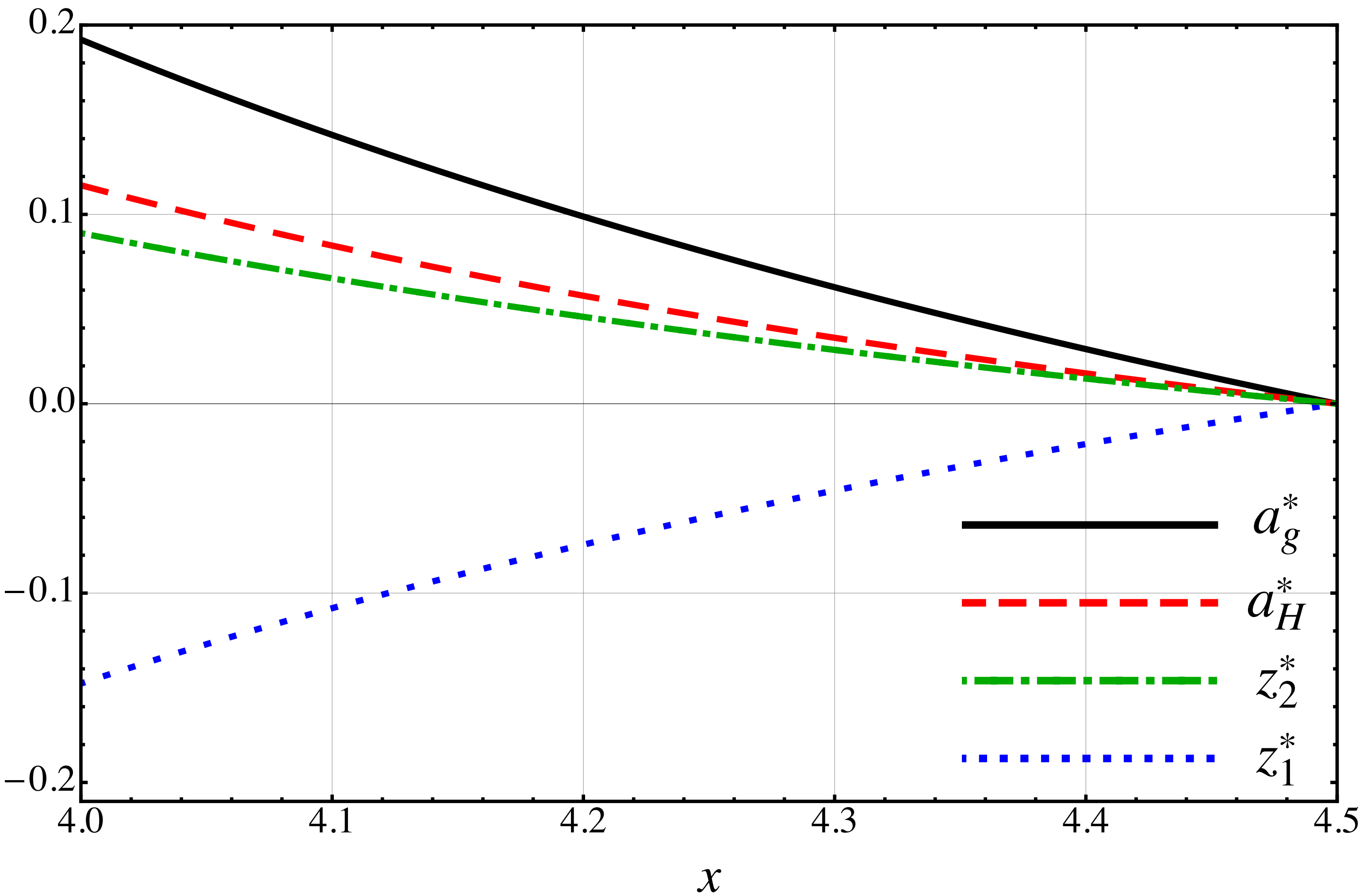
$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}\left[-\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + i\bar{\lambda}\not{D}\lambda + \overline{Q}i\not{D}Q + \partial_\mu H^\dagger \partial^\mu H + y_H \overline{Q}HQ\right] \\ &- u_1(\mathrm{Tr}[H^\dagger H])^2 - u_2\mathrm{Tr}(H^\dagger H)^2 \ .\end{aligned}$$

* Antipin,Mojaza,Sannino '11

Perturbative Fixed Point

Rescaled couplings for Veneziano limit, i.e. $N_c, N_f \to \infty$ while keeping $x \equiv N_f/N_c$ fixed:

$$a_g = \frac{g^2 N_c}{(4\pi)^2}\ , \quad a_H = \frac{y_H^2 N_c}{(4\pi)^2}\ , \ z_1 = \frac{u_1 N_f^2}{(4\pi)^2}\ , \ z_2 = \frac{u_2 N_f}{(4\pi)^2}\ .$$



The theory has a perturbative, infrared (IR) stable FP near the asymptotically free boundary $\overline{x} = 9/2$.
Chiral symmetry can be broken either by a fermion or a scalar condensate (Coleman-Weinberg mechanism): in the latter case a massive dilaton arises in the theory.

Two-Loop Analysis

The two-loop beta functions for the AMS model with ℓ gauginos in the Veneziano limit are:

$$\beta_{a_g} = -\frac{2}{3}a_g^2\left[11 - 2\ell - 2x + (34 - 16\ell - 13x)\,a_g + 3x^2 a_H\right]$$

$$\beta_{a_H} = a_H\left[2(x+1)a_H - 6a_g + (8x+5)a_g a_H + \frac{20(x+\ell)-203}{6}a_g^2 - 8xz_2a_H - \frac{x(x+12)}{2}a_H^2 + 4z_2^2\right]$$

$$\beta_{z_1} = 4(z_1^2 + 4z_1z_2 + 3z_2^2 + z_1a_H)$$
$$+2\left(5z_1a_ga_H + 2x^2a_H^3 + x(4z_2 - 3z_1)a_H^2 - 4z_1^2a_H - 12z_2^2a_H - 16z_1z_2a_H - 48z_2^3 - 20z_1z_2^2\right)$$

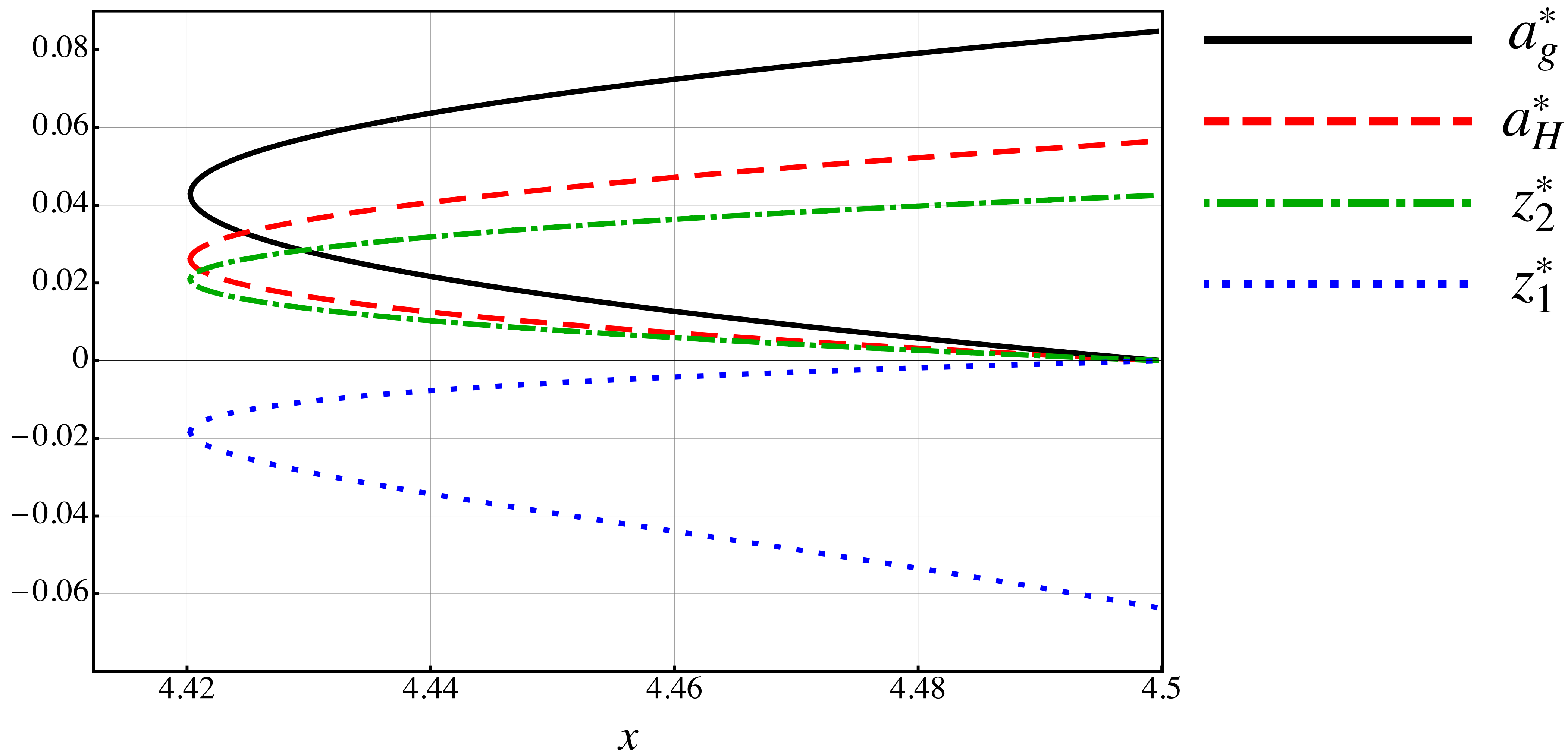
$$\beta_{z_2} = 2\left(2z_2a_H + 4z_2^2 - xa_H^2\right) + 2\left(5z_2a_ga_H - 2xa_ga_H^2 + 2x^2a_H^3 - 3xz_2a_H^2 - 8z_2^2a_H - 12z_2^3\right)$$

This expressions are consistent with the Callan-Symanzik equation for the two loop effective potential. We set $\ell = 1$; for $\ell = 0$ there is no perturbative FP.

* Antipin,SDC,Mojaza,Mølgaard,Sannino '12

Merging Fixed Points

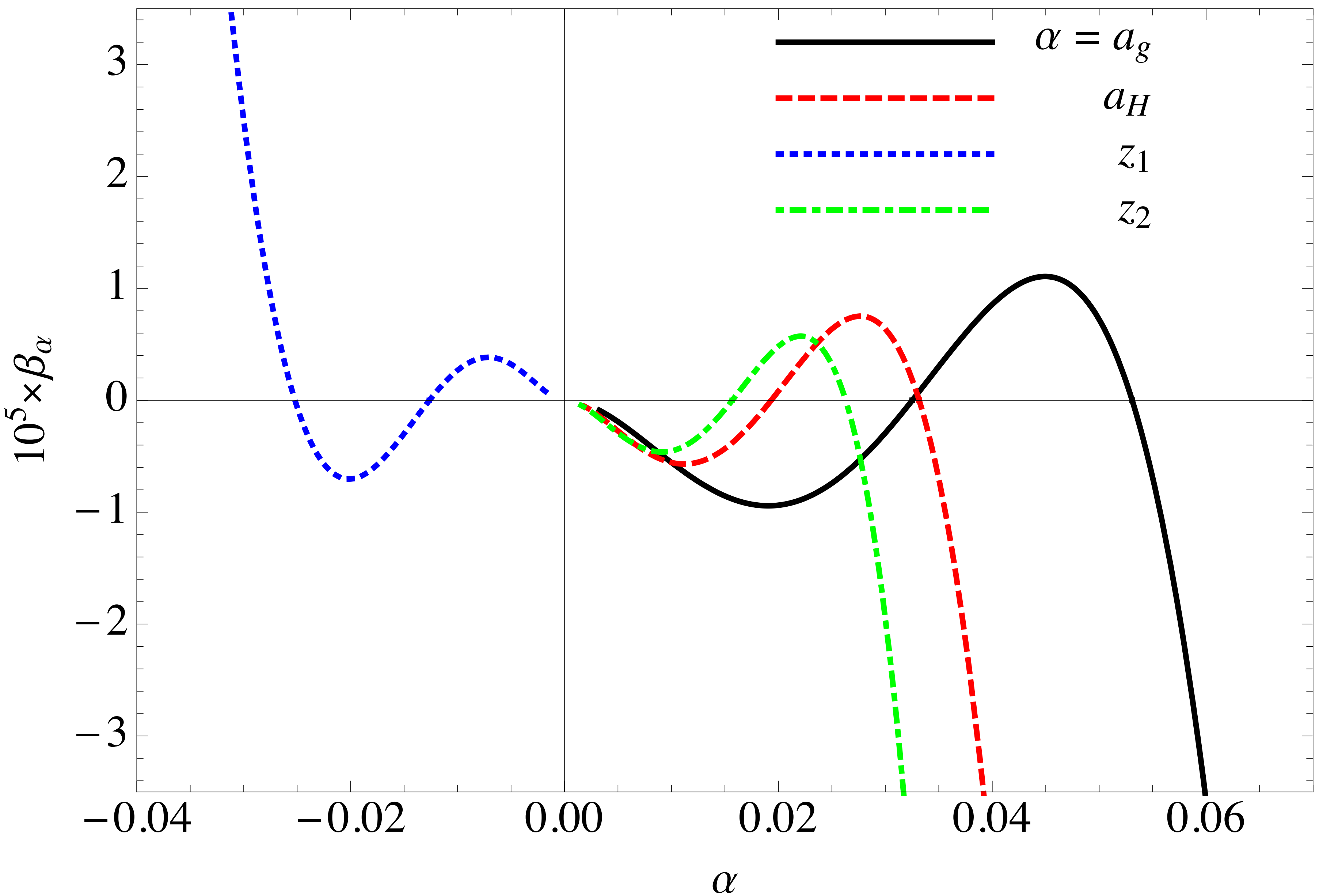
There are at most twelve real solutions giving zero two-loop beta functions. One of them is numerically close to the one loop IR FP solution. The IR FP merges with a UV FP for decreasing $x = N_f/N_c$:



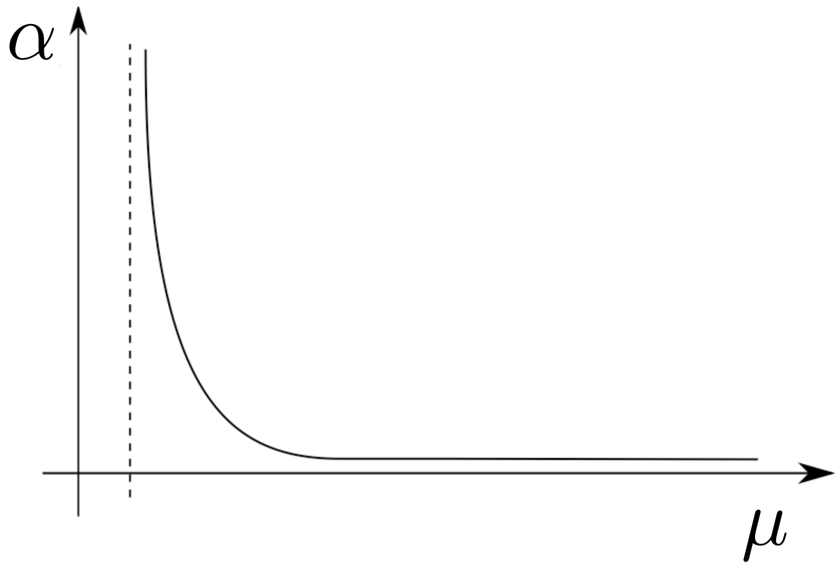
IR and UV FP merger at $x^* = 4.42$; asymptotic freedom lost for $x > 4.5$.

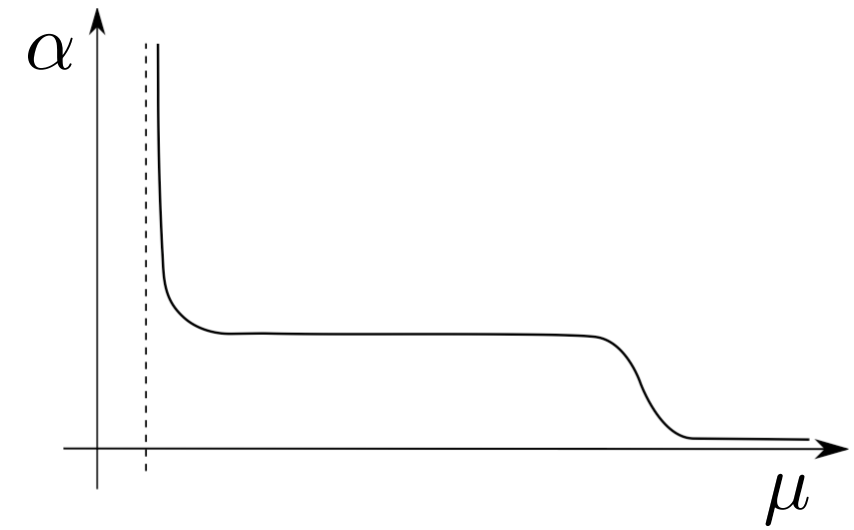
Betas in the Conformal Window

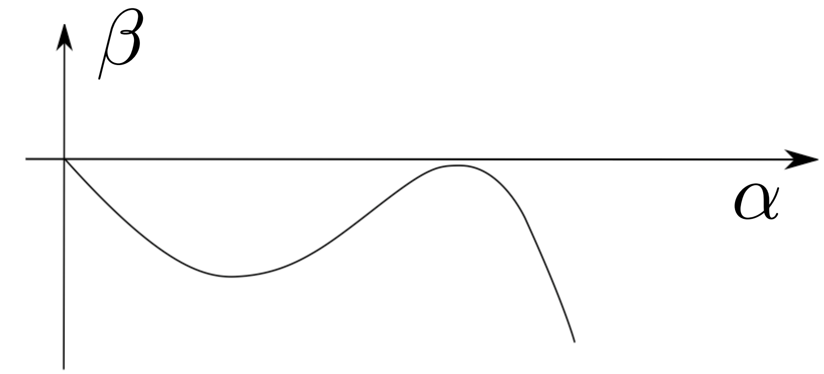
The UV FP is unstable under small changes in the values of the couplings, while the IR FP is stable. Beta functions plot for $x = 4.425$:



Running vs Walking Dynamics







$$\langle \overline{Q}Q \rangle_{UV} = \langle \overline{Q}Q \rangle_{IR} \exp\left(\int_{\Lambda_{IR}}^{\Lambda_{UV}} \frac{d\mu}{\mu} \gamma_m(\mu) \right)$$

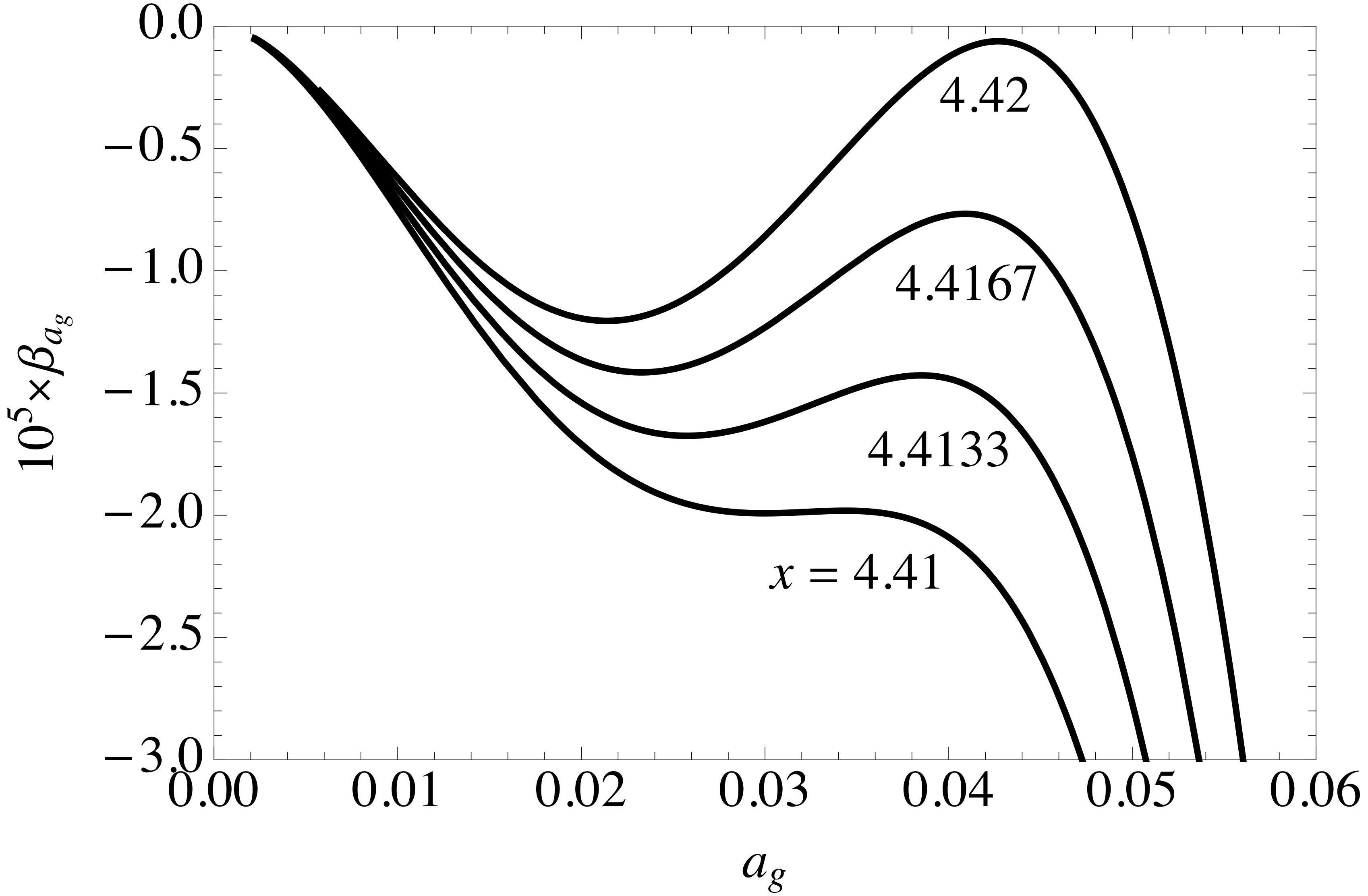
for $\Lambda_{UV} > \mu > \Lambda_{IR}$:

- Running Dynamics: $\alpha(\mu) \propto \frac{1}{\ln \mu}, \Rightarrow \langle \overline{Q}Q \rangle_{UV} \simeq \langle \overline{Q}Q \rangle_{IR}$
- Walking Dynamics: $\beta(\alpha_*) = 0 \Rightarrow \langle \overline{Q}Q \rangle_{UV} \simeq \langle \overline{Q}Q \rangle_{IR} \left(\frac{\Lambda_{UV}}{\Lambda_{IR}} \right)^{\gamma_m(\alpha_*)}$

* Holdom '81, Yamawaki et al. '86, Appelquist et al '86

Walking Dynamics Regime

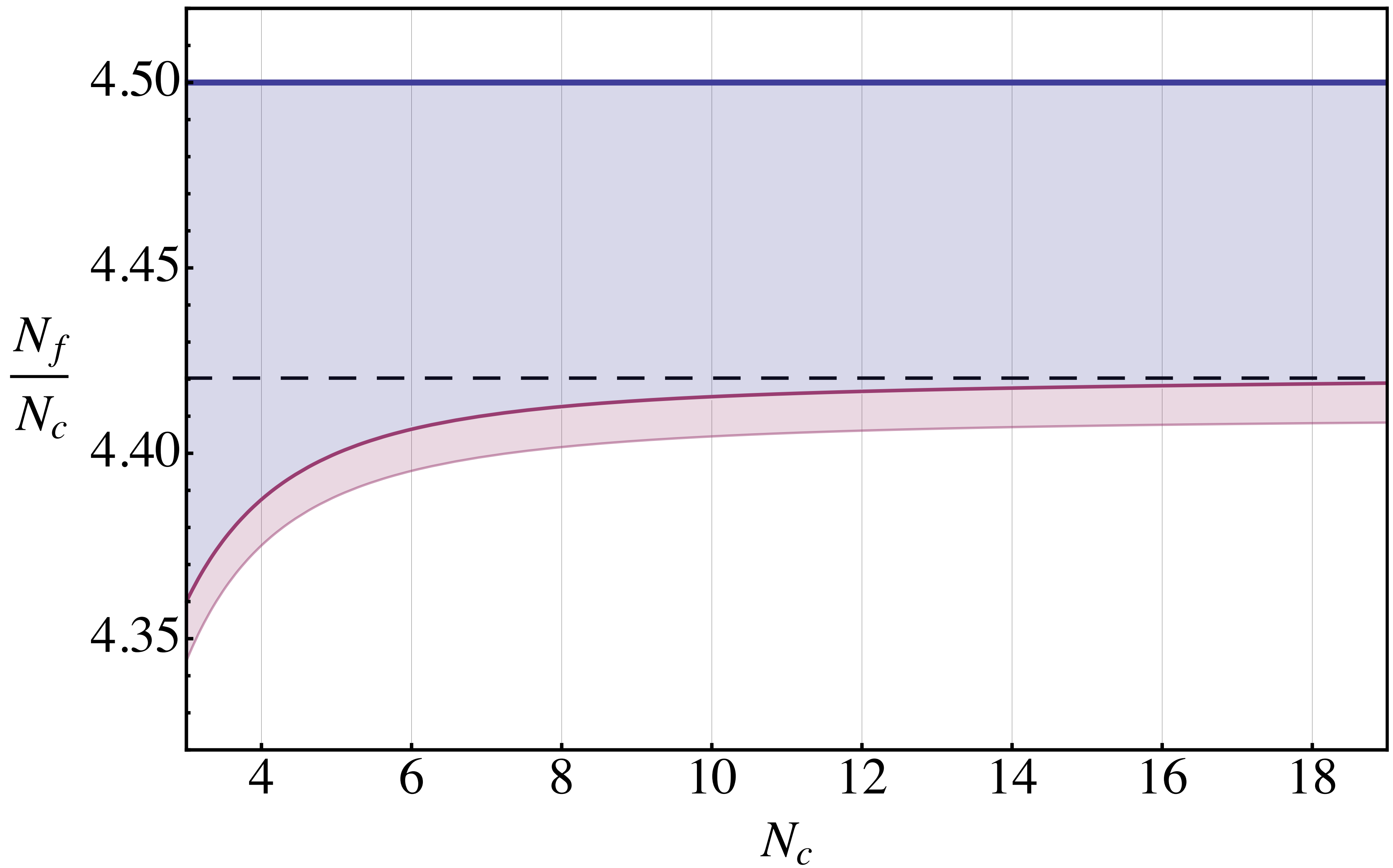
The range of values of $x < x^*$ for which β_{a_g} features a local maximum close to the quasi-FP defines concretely the walking dynamics regime. For $x < 4.41$ the theory runs.



* Antipin,SDC,Mojaza,Mølgaard,Sannino '12

Precise Conformal Window

Perturbative phase diagram including the conformal window (light blue shaded region) and walking region (pink shaded region) of the theory as a function of the number of flavors and colors.



Deviations from the Veneziano limit (dashed line) are sizable at small N_c.

Scaling

Below the FPs merger the theory must develop a physical scale Λ determined in terms of a UV scale μ_0 and initial conditions k_i as follows:

$$\log \frac{\Lambda}{\mu_0} = \int_{\mu_0}^{\Lambda} \frac{\mathrm{d}\mu}{\mu} = \int_{a_g(\mu_0)}^{a_g(\Lambda)} \frac{\mathrm{d}a_g}{\beta_{a_g}(a_g, k_1, k_2, k_3)} \; .$$

Natural choices are $a_g(\mu_0) = a_g^*$, the quasi-FP, and $a_g(\Lambda) = \infty$. The integration is performed numerically. An approximate analytical solution can be obtained by expanding β_{a_g} around its local maximum:

$$\int_{a_g^*}^{\infty} \frac{1}{\beta_0 + \frac{\beta_2 \left(a_g - a_g^*\right)^2}{2!}} \, da_g = -\frac{\pi}{\sqrt{\beta_0 \beta_2}},$$

with

$$\partial_{a_g} \beta_{a_g}\Big|_{a_g^*} = 0, \quad \beta_0 \equiv \beta_{a_g}(a_g^*), \quad \beta_2 \equiv \partial_{a_g, a_g} \beta_{a_g}\Big|_{a_g^*} \, .$$

Miransky Scaling

The leading order contribution in x we obtain for the RG time is

$$t = \log \frac{\Lambda}{\mu_0} \simeq -\frac{\pi}{c_1 \sqrt{x^* - x}} \ , \quad c_1 = 2.99 \times 10^{-2} \ .$$

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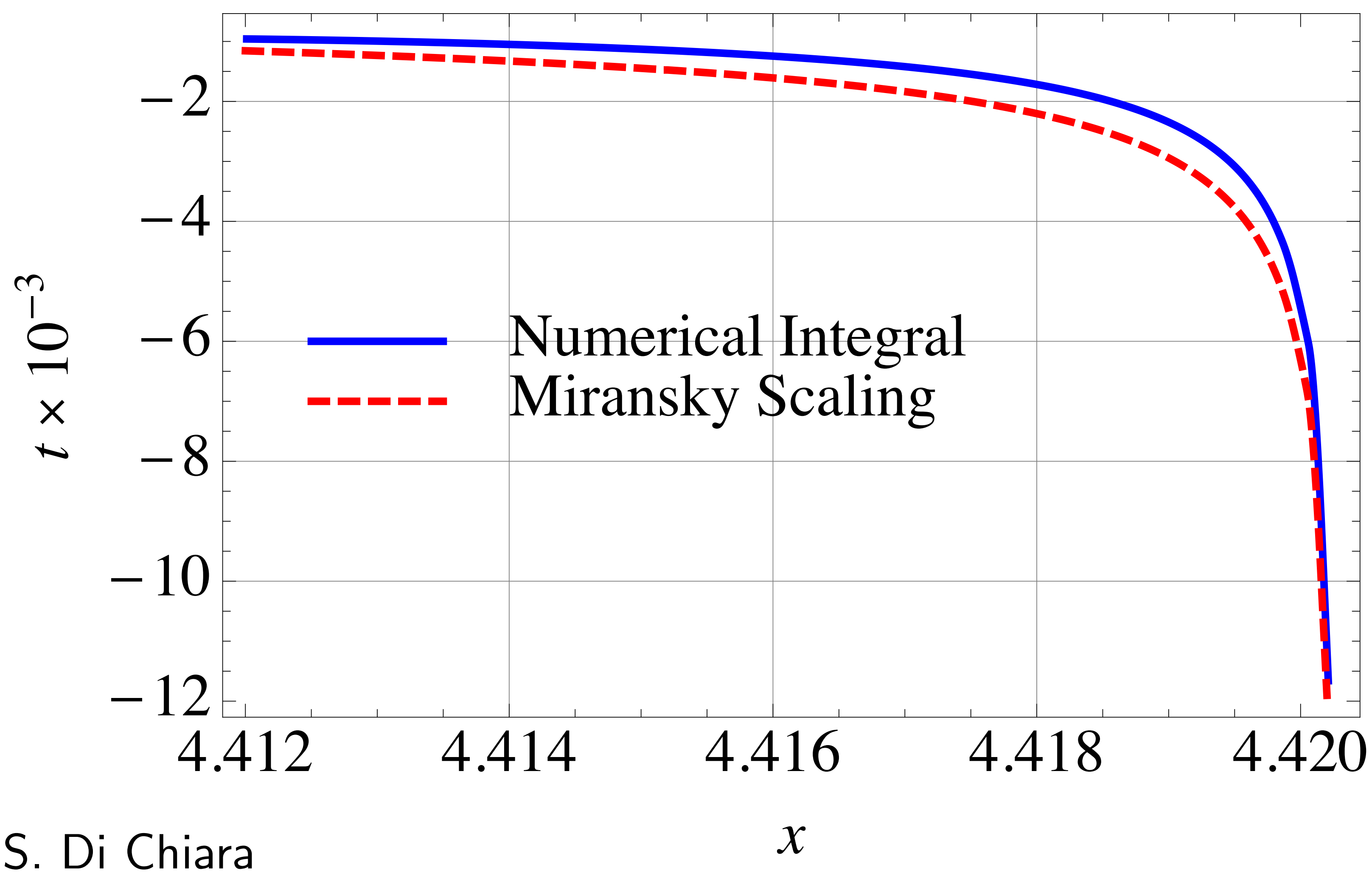

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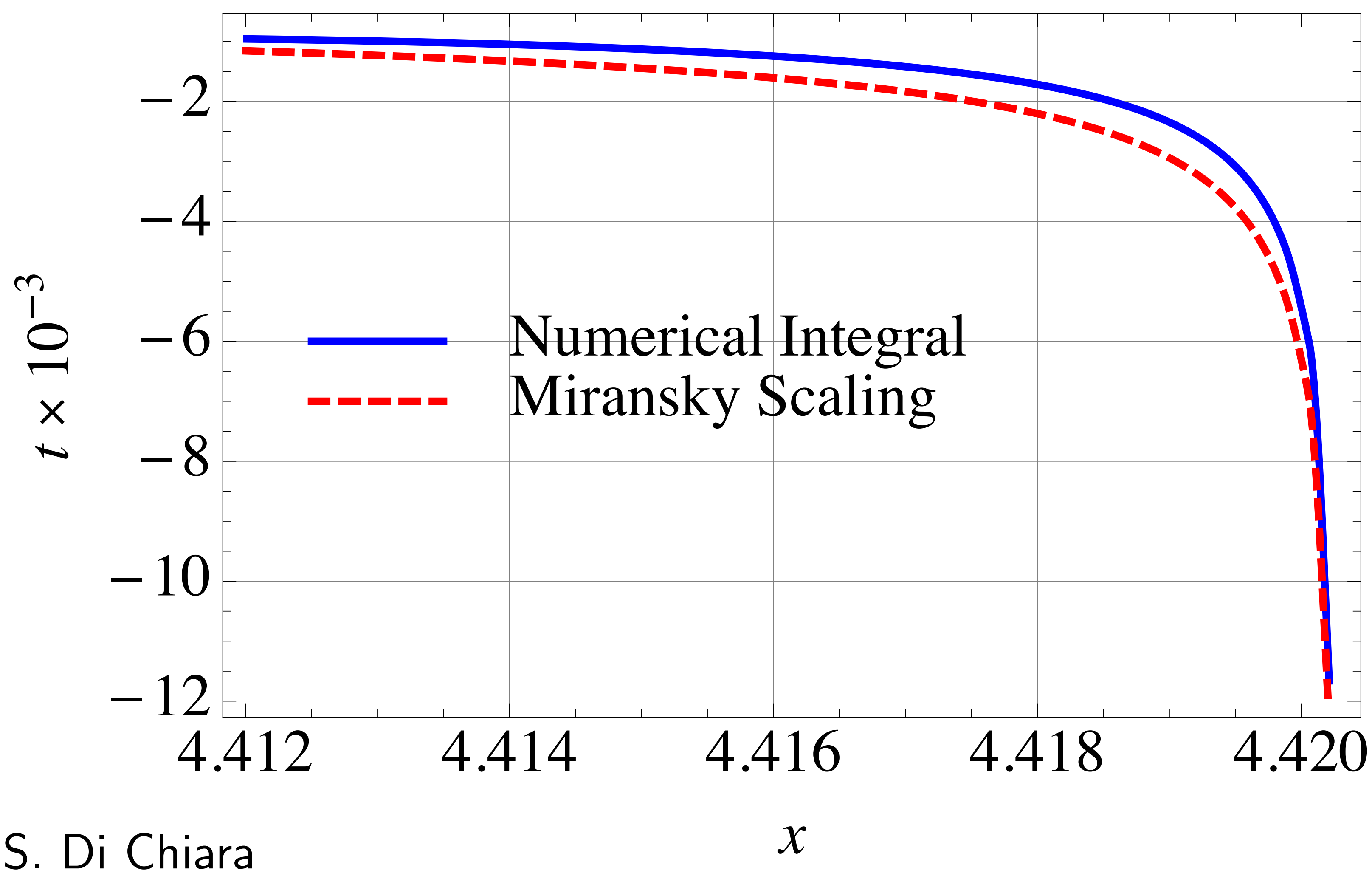


The Miransky scaling approximation becomes increasingly accurate as x gets closer to x^*.

Conclusions

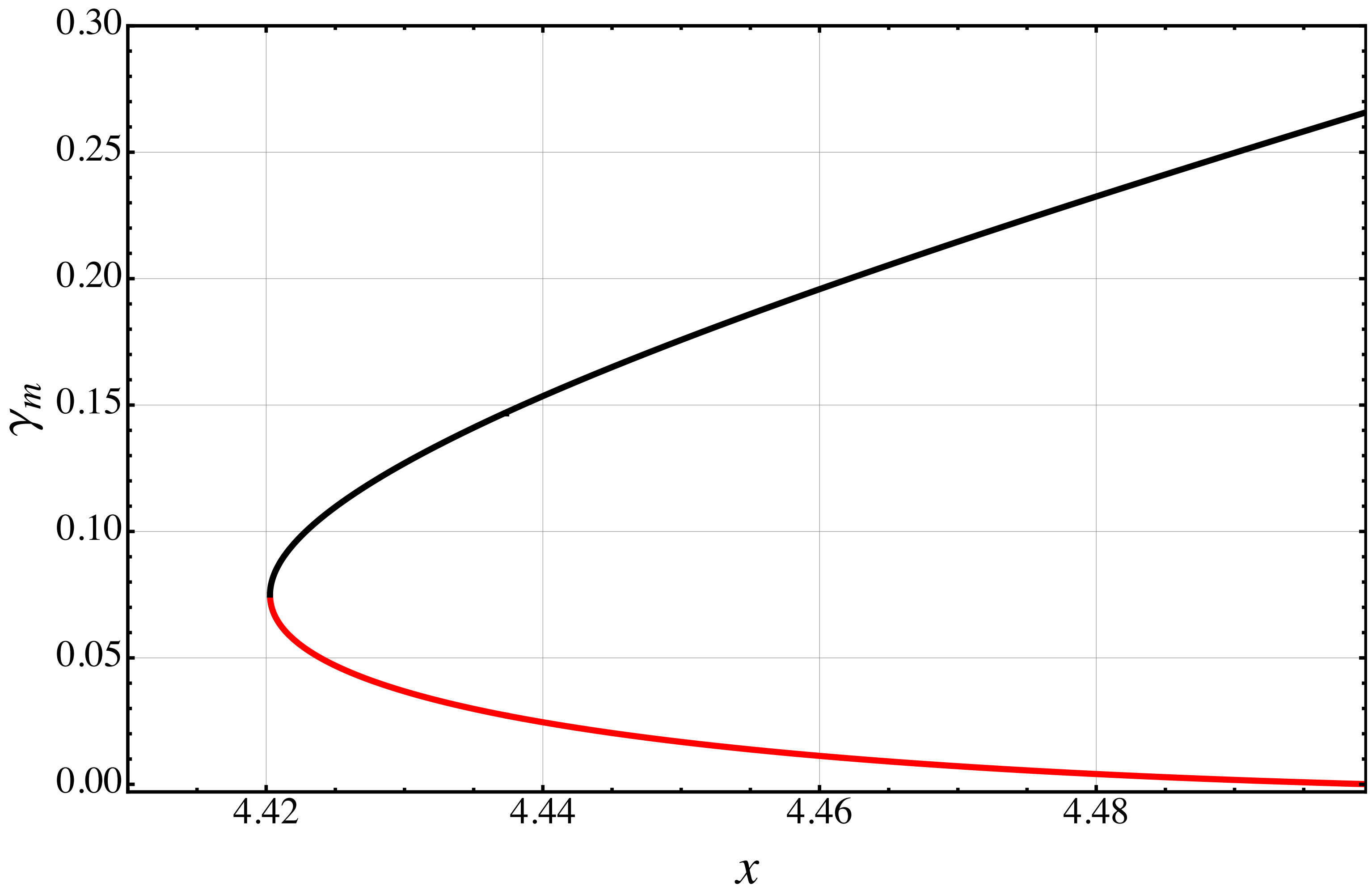
- Conformal window and walking region as functions of N_f determined within perturbatively trustable regime
- Merger of an IR and UV fixed points leads to Miransky scaling

Backup Slides

Anomalous Dimension

Anomalous dimension of the fermion mass operator:

$$\gamma_m \equiv -\frac{d\log m}{d\log\mu}\ , \qquad \Delta_{\bar{\psi}\psi} = 3 - \gamma_m\ .$$

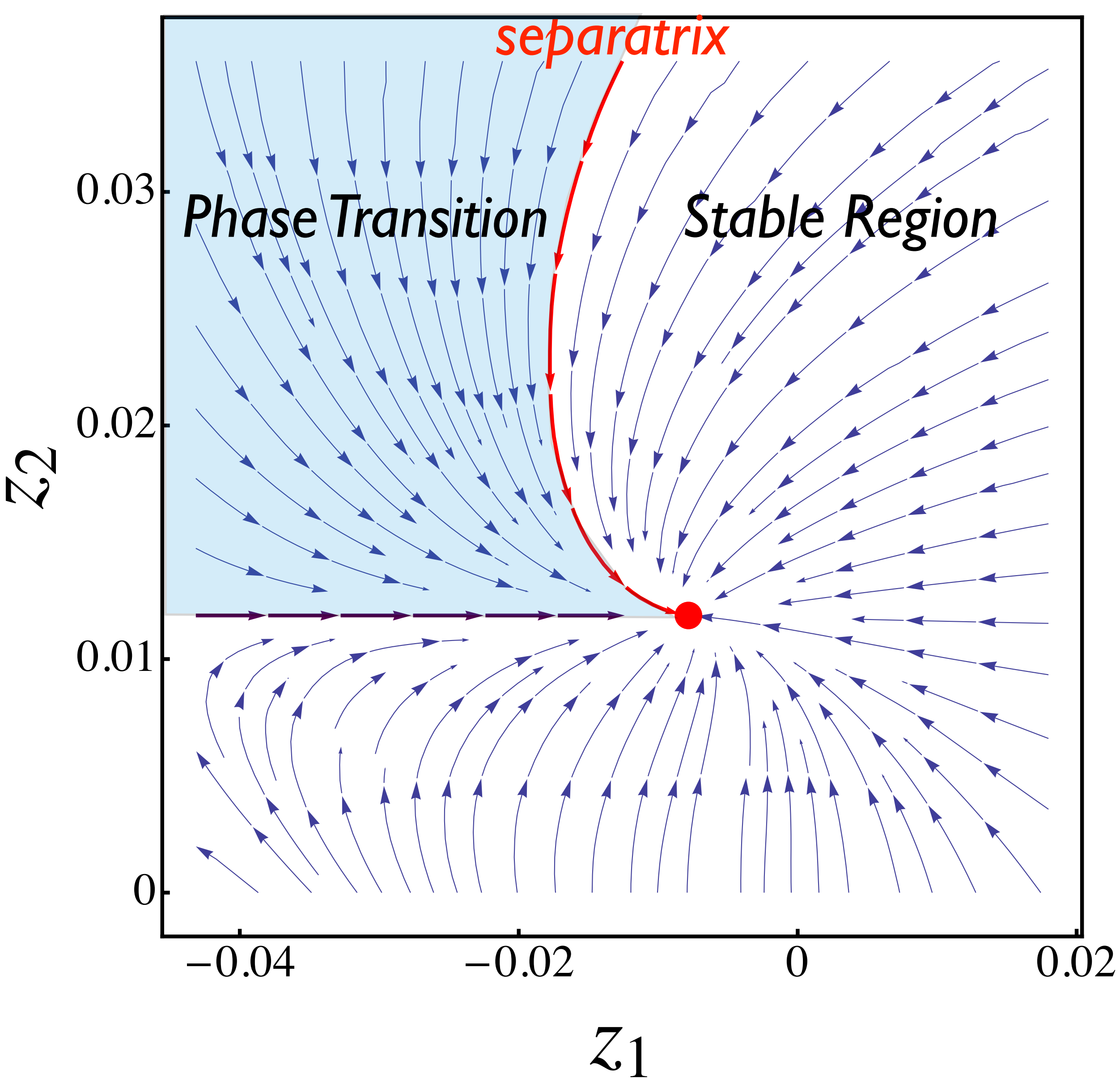


In the walking regime the physics is dominated by the nearby FP and therefore the relevant anomalous dimension is:

$$\gamma_{\rm m} \approx 0.08$$

In strongly coupled gauge theories γ_m is conjectured to be as large as 1.

Dilaton



Chiral symmetry can be broken either by a fermion or scalar condensate (Coleman-Weinberg mechanism).

In the latter case the pseudo-NG boson associated to spontaneous conformal symmetry breaking $a.k.a.$ dilaton acquires, through one-loop corrections, a mass m_d:

$$m_d^2\Big|_{z_1=-z_2} = 4\mu_0^2\left[4z_2^2 - xa_H^2\right]\exp\left(-\frac{1}{2} - \frac{4z_2^2\ln(4z_2) - xa_H^2\ln(xa_H)}{4z_2^2 - xa_H^2}\right),$$

Phase Diagram

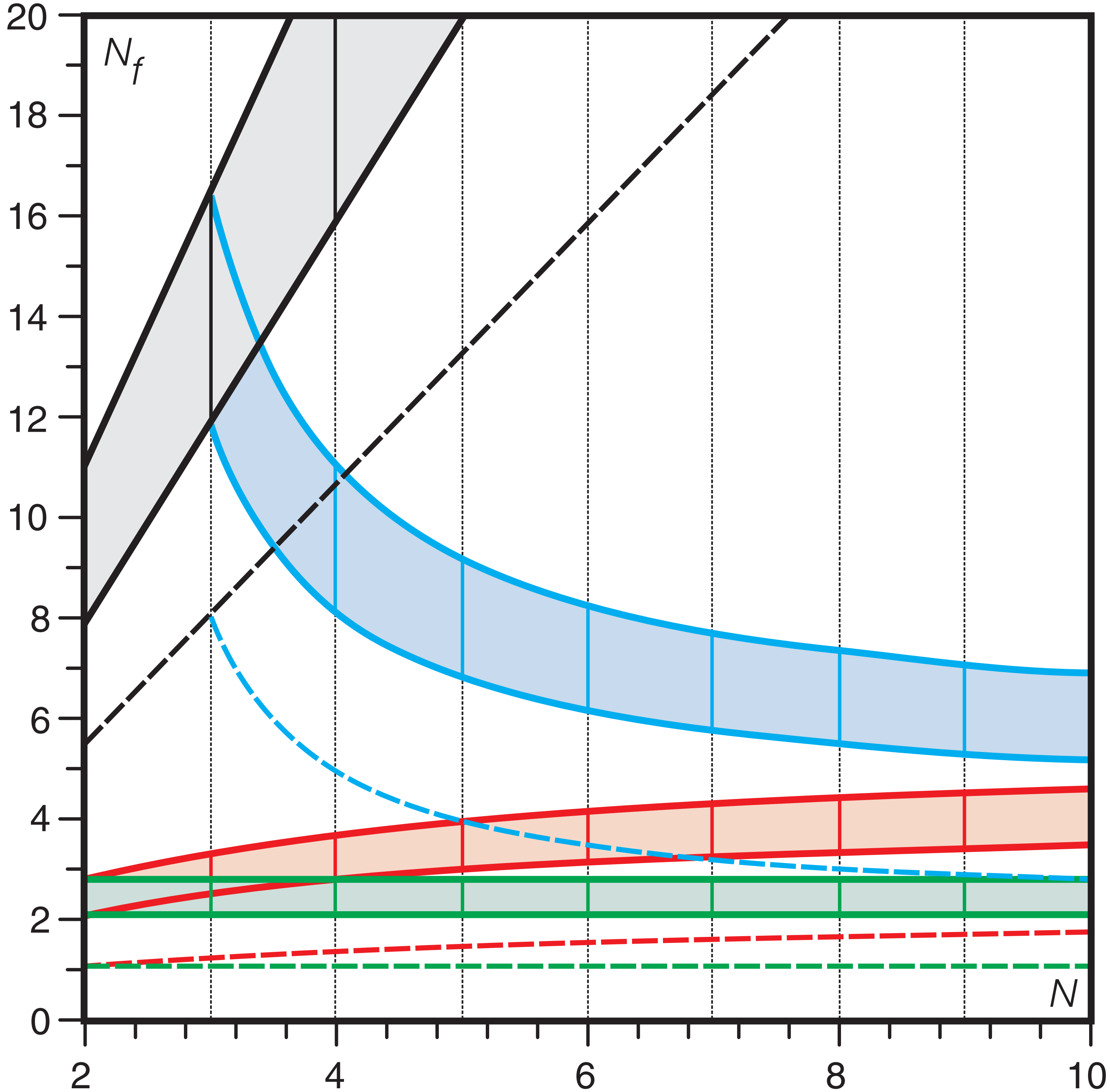
Phase diagram in theory space: Representation (R), Number of colors (N), Number of flavors (N_f);

$$\beta(g) = -\beta_0 \frac{\alpha^2}{4\pi} - \beta_1 \frac{\alpha^3}{(4\pi)^2}, \; \alpha_* = -4\pi \frac{\beta_0}{\beta_1}, \; \beta_0 = \frac{11}{3} C_2(\mathrm{G}) - \frac{4}{3} T(\mathrm{R}),$$
$$\beta_1 = \frac{34}{3} {C_2}^2(\mathrm{G}) - \frac{20}{3} C_2(\mathrm{G}) T(\mathrm{R}) - 4 C_2(\mathrm{R}) T(\mathrm{R}).$$

The conformal window is defined by requiring asymptotic freedom, existence of a Banks-Zaks fixed point, and conformality to arise before chiral symmetry breaking:

$$\beta_0 > 0 \quad \Rightarrow \quad N_f > \frac{11}{4} \frac{d(\mathrm{G}) C_2(\mathrm{G})}{d(\mathrm{R}) C_2(\mathrm{R})},$$
$$\beta_1 < 0 \quad \Rightarrow \quad N_f < \frac{d(\mathrm{G}) C_2(\mathrm{G})}{d(\mathrm{R}) C_2(\mathrm{R})} \frac{17 C_2(\mathrm{G})}{10 C_2(\mathrm{G}) + 6 C_2(\mathrm{R})}$$
$$\alpha_* < \alpha_c \quad \Rightarrow \quad N_f > \frac{d(\mathrm{G}) C_2(\mathrm{G})}{d(\mathrm{R}) C_2(\mathrm{R})} \frac{17 C_2(\mathrm{G}) + 66 C_2(\mathrm{R})}{10 C_2(\mathrm{G}) + 30 C_2(\mathrm{R})}.$$

Walking in the SU(N)



Phase diagram for theories with fermions in the:

- fundamental representation (grey)
- two-index antisymmetric (blue)
- two-index symmetric (red)
- adjoint representation (green)

 * Dietrich, Sannino '06