LFV Processes in TeV Scale See-Saw Mechanisms

Dinh Nguyen Dinh

Based on: D. N. Dinh, A. Ibarra, E. Molinaro, S. T. Petcov, **hep-th/1205.4671**

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Outline

Introduction

- Neutrino oscillations are convincing evidences to show that neutrinos are massive and flavor lepton numbers do not conserve in weak processes. Beside neutrino oscillation, there is no any other has been detected.

- Experimental status on LFV:

- The $\mu \to e\gamma$ decay: $BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$ (MEG, 2011).
- The $\mu \to 3e$ decay: $BR(\mu \to 3e) < 1.0 \times 10^{-12}$ (SINDRUM, 1988).
- The $\mu - e$ conversion in nuclei: $CR(\mu Ti \to eTi) < 4.3 \times 10^{-12}$ (SINDRUM II, 1993).

Introduction

- Future experiments:

- Ongoing MEG can detect $BR(\mu \to e\gamma) > 1.0 \times 10^{-13}$.
- $BR(\mu \to 3e) > 1.0 \times 10^{-15}$, MuSIC facilities at Osaka University, Japan.
- Number of sensitive experiments searching for $\mu - e$ conversion:
  - COMET (KEK), Mu2e (Fermilab)$\to$ $CR(\mu Al \to eAl) = 10^{-16}$,
  - PRISM/PRIME (KEK) and project-X (Fermilab) probe the value of $\mu - e$ conversion rate on Ti, $CR(\mu Ti \to eTi) = 10^{-18}$.

Introduction

- Purpose of this research:

- Study the FLV processes in scenarios of low scale (TeV) Seesaw type models.
- Discuss the predictions and experimental constraints on the relevant parameters arising from FLV processes.

Brief review of type I see-saw

- The weak interaction in lepton sector can be written as:

$$\begin{aligned}
\mathcal{L}^{\nu}_{CC} &= -\frac{g}{\sqrt{2}}\,\bar{\ell}\,\gamma_{\alpha}\,((1+\eta)U)_{\ell i}\,\chi_{iL}\,W^{\alpha} + \text{h.c.}\,,\\
\mathcal{L}^{\nu}_{NC} &= -\frac{g}{2c_w}\,\overline{\chi_{iL}}\,\gamma_{\alpha}\,\left(U^{\dagger}(1+2\eta)U\right)_{ij}\,\chi_{jL}\,Z^{\alpha}\,,\\
\mathcal{L}^{N}_{CC} &= -\frac{g}{2\sqrt{2}}\,\bar{\ell}\,\gamma_{\alpha}\,(RV)_{\ell k}(1-\gamma_5)\,N_k\,W^{\alpha} + \text{h.c.}\\
\mathcal{L}^{N}_{NC} &= -\frac{g}{4c_w}\,\overline{\nu_{\ell L}}\,\gamma_{\alpha}\,(RV)_{\ell k}\,(1-\gamma_5)\,N_k\,Z^{\alpha} + \text{h.c.}\,,
\end{aligned}$$

where we have used the parametrization in hep-ph/1007.2378 and hep-ph/1103.6217.
$U_{PMNS} = ((1+\eta)U)$, $\eta = -\frac{1}{2}RR^{+} = -\frac{1}{2}(RV)(RV)^{+} = \eta^{+}$

Brief review of type I see-saw

- In this research, we consider a model with two RH neutrinos N_k having masses in range of $(100 - 1000)$GeV.
- To satisfy the constraints of experiments on neutrino oscillations, electroweak processes, etc..., and the observability of heavy neutrinos at TeV-scale, one has following relations and upper limits (see hep-ph/1007.2378, hep-ph/1103.6217):

$$
\begin{aligned}
(RV)_{\ell 2} &= \pm i\,(RV)_{\ell 1}\sqrt{\frac{M_1}{M_2}}\,,\ M_1 \approx M_2\,, \\
\left|(RV)_{\ell 1}\right|^2 &= \frac{1}{2}\frac{y^2 v^2}{M_1^2}\frac{m_3}{m_2+m_3}\left|U_{\ell 3} + i\sqrt{m_2/m_3}\,U_{\ell 2}\right|^2\,,\mathrm{NH}\,, \\
\left|(RV)_{\ell 1}\right|^2 &= \frac{1}{2}\frac{y^2 v^2}{M_1^2}\frac{m_2}{m_1+m_2}\left|U_{\ell 2} + i\sqrt{m_1/m_2}\,U_{\ell 1}\right|^2\,,\mathrm{IH}\,,
\end{aligned}
$$

Brief review of type I see-saw

$$
\begin{aligned}
|(RV)_{e1}|^2 &\lesssim 2 \times 10^{-3}\,, \\
|(RV)_{\mu 1}|^2 &\lesssim 0.8 \times 10^{-3}\,, \\
|(RV)_{\tau 1}|^2 &\lesssim 2.6 \times 10^{-3}\,.
\end{aligned}
$$

The $\mu \to e\gamma$ decay

- The weak interaction in lepton sector can be written as:

$$\begin{aligned} \mathrm{BR}(\mu \to e\gamma) &= \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e + \nu_\mu + \overline{\nu}_e)} = \frac{3\alpha_{\mathrm{em}}}{32\pi} \, |T|^2 \,, \\ |T| &\cong 2 \left| (RV)^*_{\mu 1} \, (RV)_{e1} \right| \left| G(X) - G(0) \right| \,, \\ G(x) &= \frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \log(x)}{3(x-1)^4} \,, \end{aligned}$$

where $x = M_1^2 / M_w^2$.

- Doing analysis we have:

$$\left| (RV)^*_{\mu 1} \, (RV)_{e1} \right| < 0.8 \times 10^{-4} \, (0.3 \times 10^{-4}) \,,$$

The $\mu \to e\gamma$ decay

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where $x = M_1^2/M_w^2$.

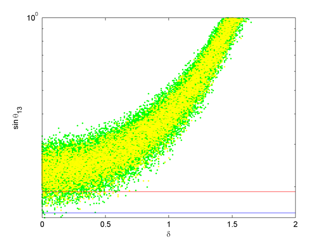
- Doing analysis we have:

$$\begin{aligned} &\left| (RV)^*_{\mu 1} (RV)_{e1} \right| < 0.8 \times 10^{-4} \, (0.3 \times 10^{-4}) , \\ &y \lesssim 0.035 \; (0.21) \quad \mathrm{NH}; \; y \lesssim 0.025 \; (0.15) \quad \mathrm{IH}, \end{aligned}$$

for $M_1 = 100 \, \mathrm{GeV} \, (1000 \, \mathrm{GeV})$ and $\sin\theta_{13} = 0.1$.

The $\mu \to e\gamma$ decay

- NH, $|(RV)^*_{\mu 1}(RV)_{e1}| \sim |U_{e3} + i\sqrt{m_2/m_3}U_{e2}| = 0$ for $\sin\theta_{13}$ out of 3σ.
- IH, $|(RV)^*_{\mu 1}(RV)_{e1}| \sim |U_{\mu 2} + iU_{\mu 1}| = 0$ for suitable values of $\sin\theta_{13}$, δ and α_{21}. $\sin\theta_{13}$ is in the current 3σ interval, if $0 < \delta < 0.7$.



The $\mu \to 3e$ decay

- Let us write down the branching ratio for the decay:

$$
\begin{aligned}
\mathrm{BR}(\mu \to 3e) &= \frac{4\alpha_{em}^2}{\pi^2} \left|(RV)^*_{\mu 1}(RV)_{e1}\right|^2 \left|C_{\mu 3e}(M_1^2/M_W^2)\right|^2 , \\
\left|C_{\mu 3e}(y)\right|^2 &= 3\bar{Z}^2(y) + 3\,\bar{Z}(y)H(y) + H^2(y)\left(\log\frac{m_\mu}{m_e} - \frac{11}{8}\right) \\
&+ \frac{1}{2\sin^4\theta_W}\bar{Y}_e^2 - \frac{2}{\sin^2\theta_W}\bar{Z}\,\bar{Y}_e - \frac{1}{\sin^2\theta_W}H(y)\,\bar{Y}_e .
\end{aligned}
$$

where

$$
H(y) = \frac{1}{4}\left(G(y) - G(0)\right), \quad \bar{Y}_e(y) \simeq Y_0(y) = \frac{y}{8}\left[\frac{y-4}{y-1} + \frac{3y}{(y-1)^2}\ln y\right],
$$

$$
\bar{Z}(y) = \frac{1}{4}\left(D_0(y) - \frac{2E_0(y)}{3}\right), \quad C_0(y) = \frac{y}{8}\left[\frac{y-6}{y-1} + \frac{3y+2}{(y-1)^2}\log y\right],
$$

$$
D_0(y) - \frac{2}{3}E_0(y) = \frac{y(18+y-7y^2)}{12(y-1)^3} - \frac{y^2(12-10y+y^2)}{6(y-1)^4}\log y .
$$

The $\mu \to 3e$ decay

- From current limit on $BR(\mu \to 3e)$, we have:

$$\left|(RV)^*_{\mu 1}(RV)_{e1}\right| \lesssim 2.1 \times 10^{-4} \; (5.3 \times 10^{-6}) \quad \text{for} \quad M_1 = 100 \; (1000) \quad \text{GeV} \, .$$

The $\mu - e$ conversion in nuclei

- In type I seesaw, the conversion rate has form:

$$\mathrm{CR}(\mu\mathcal{N} \to e\mathcal{N}) = \frac{\alpha_{\rm em}^5}{2\,\pi^4}\,\frac{Z_{eff}^4}{Z}\,\left|F(-m_\mu^2)\right|^2\,\frac{G_F^2 m_\mu^5}{\Gamma_{\rm capt}}\,\left|(RV)^*_{\mu 1}(RV)_{e1}\right|^2\,\left|\mathcal{C}_{\mu e}\right|^2$$

where:

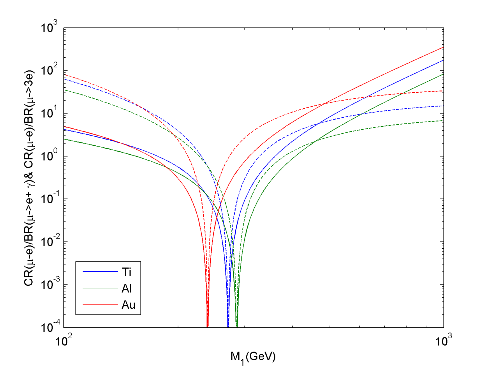
$$\mathcal{C}_{\mu e} \cong \left[Z\,(4R_0(X) + H_0'(X)) - (2Z+N)\,\frac{X_0(X)}{\sin^2\theta_W} + (Z+2N)\,\frac{Y_0(X)}{\sin^2\theta_W}\right],$$

$$R_0(y) = \frac{y(6y^3 - 55y^2 + 79y - 24)}{48(y-1)^3} + \frac{y(8y^3 - 2y^2 - 15y + 6)}{24(y-1)^4}\log y\,,$$

$$H_0'(y) = \frac{y(2y^2 + 5y - 1)}{4(y-1)^3} - \frac{3y^3\log y}{2(y-1)^4}\,,\quad X_0(y) = \frac{y}{8}\left[\frac{y+2}{y-1} + \frac{3y-6}{(y-1)^2}\ln y\right],$$

$$Y_0(y) = \frac{y}{8}\left[\frac{y-4}{y-1} + \frac{3y}{(y-1)^2}\ln y\right].$$

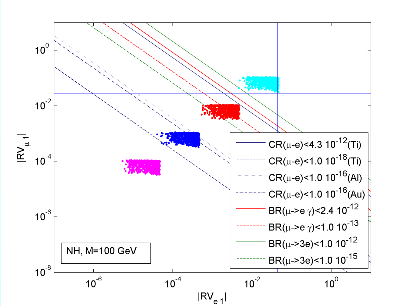
The $\mu - e$ conversion in nuclei

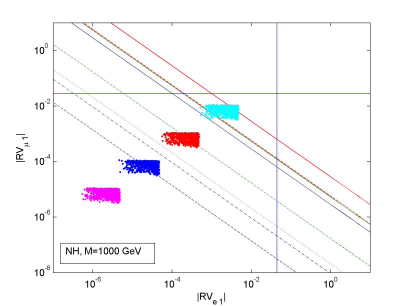


Comments:

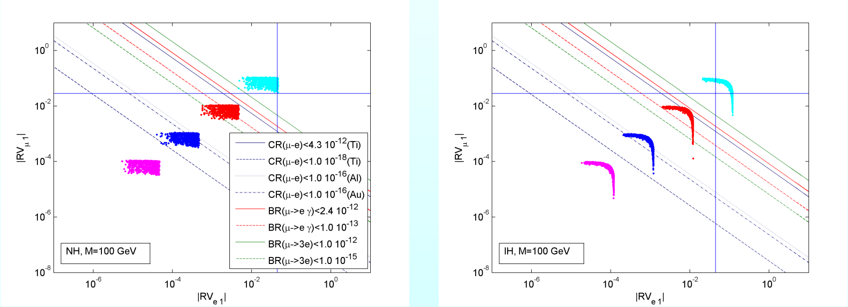
- For each nuclear, in Type I Seesaw scenario, there is suppression at suitable M_1. For Ti, Al and Au, these values are in range of (200 - 300) GeV.
- In principle, if two of the processes $\mu - e$ conversion, $\mu \to e\gamma$ or $\mu \to 3e$ are detected, M_1 is determined.

The $\mu - e$ conversion in nuclei

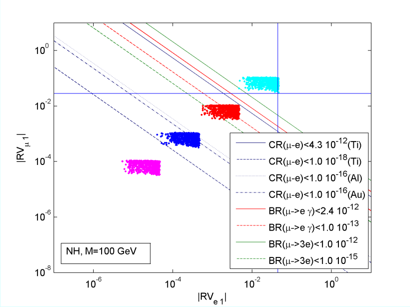


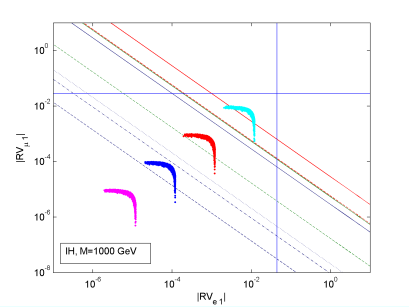


The $\mu - e$ conversion in nuclei



The $\mu - e$ conversion in nuclei





Review of type II see-saw

- The Lagrangian of the type II seesaw

$$\mathcal{L}^{\rm II}_{\rm seesaw} = -M_\Delta^2 \,{\rm Tr}\left(\Delta^\dagger \Delta\right) - \left(h_{\ell\ell'}\, \overline{\psi^C}_{\ell L}\, i\tau_2\, \Delta\, \psi_{\ell' L} + \mu_\Delta\, H^T\, i\tau_2\, \Delta^\dagger\, H + {\rm h.c.}\right) ,$$

- After SYM is spontaneously breaking, Majorana mass matrix is produced:

$$(m_\nu)_{\ell\ell'} \equiv m_{\ell\ell'} \simeq 2\, h_{\ell\ell'}\, v_\Delta \,,$$
$$h_{\ell\ell'} \equiv \frac{1}{2v_\Delta}\left(U^*\,{\rm diag}(m_1, m_2, m_3)\, U^\dagger\right)_{\ell\ell'} .$$

- Using $\rho = M_w^2/M_z^2 \cos^2\theta_w \approx 1$ in SM, and in HTM

$$\rho \equiv 1 + \delta\rho = \frac{1+2x^2}{1+4x^2}\,, \quad x \equiv v_\Delta/v,$$

we have a bound $v_\Delta/v \leq 0.03$, or $v_\Delta < 5$ GeV ($v = 174$GeV).

Review of type II see-saw

- One-loop correction of the form factor from the exchanges of the singly- and doubly-charged physical Higgs scalars contributes to LFV processes has form:

$$A_R = -\frac{1}{\sqrt{2}\,G_F}\frac{\left(h^\dagger h\right)_{e\mu}}{48\pi^2}\left(\frac{1}{8\,m^2_{\Delta^+}} + \frac{1}{m^2_{\Delta^{++}}}\right),$$

$$A_L(q^2) = -\frac{1}{\sqrt{2}\,G_F}\frac{h^*_{le}h_{l\mu}}{6\pi^2}\left(\frac{1}{12\,m^2_{\Delta^+}} + \frac{1}{m^2_{\Delta^{++}}}f\left(\frac{-q^2}{m^2_{\Delta^{++}}}, \frac{m_l^2}{m^2_{\Delta^{++}}}\right)\right),$$

where

$$f(r, s_l) = \frac{4s_l}{r} + \log(s_l) + \left(1 - \frac{2s_l}{r}\right)\sqrt{1 + \frac{4s_l}{r}}\,\log\frac{\sqrt{r} + \sqrt{r + 4s_l}}{\sqrt{r} - \sqrt{r + 4s_l}},$$

$$S_\ell = \frac{m_\ell^2}{M^2_{\Delta^{++}}}, \quad r = -\frac{q^2}{M^2_{\Delta^{++}}} \cong \frac{m_\mu^2}{M^2_{\Delta^{++}}}.$$

The $\mu \to e\gamma$ decay

The $\mu \to e\gamma$ decay branching ratio in this case is

$$\mathrm{BR}(\mu \to e\gamma) \quad = \quad \frac{\alpha_{\mathrm{em}}}{192\,\pi} \frac{\left|\left(h^\dagger h\right)_{e\mu}\right|^2}{G_F^2} \left(\frac{1}{m_{\Delta^+}^2} + \frac{8}{m_{\Delta^{++}}^2}\right)^2 .$$

For $m_{\Delta^+} \simeq m_{\Delta^{++}} \equiv M_\Delta$, using upper limit from MEG experiment, we have

$$\left|\left(h^\dagger h\right)_{e\mu}\right| \; < \; 5.8 \times 10^{-6} \left(\frac{M_\Delta}{100\,\mathrm{GeV}}\right)^2 .$$

The lower limit for v_Δ is also obtained

$$v_\Delta > 2.1 \times 10^2 \, \left|s_{13}\, s_{23}\, \Delta m_{31}^2\right|^{\frac{1}{2}} \left(\frac{100\,\mathrm{GeV}}{M_\Delta}\right) \cong 3.0\,\mathrm{eV} \left(\frac{100\,\mathrm{GeV}}{M_\Delta}\right) .$$

The $\mu \to e\gamma$ decay

We can rewrite the ratio

$$\mathrm{BR}(\mu \to e\gamma) \cong 2.7 \times 10^{-10} \left(\frac{1\,\mathrm{eV}}{v_\Delta}\right)^4 \left(\frac{100\,\mathrm{GeV}}{M_\Delta}\right)^4 ,$$

can be sensitive for future experiments ($BR(\mu \to e\gamma) > 1.0 \times 10^{-13}$) in some ranges of v_Δ and M_1.

The $\mu \to 3e$ decay

- In the Seesaw type II, $\mu \to 3e$ decay can occur at tree level, its amplitude is easy to be obtained:

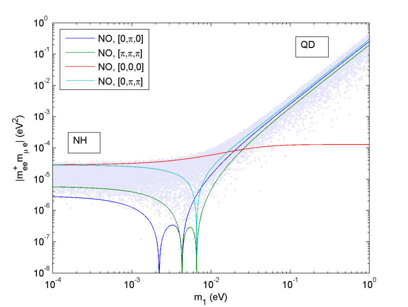
$$\mathrm{BR}(\mu \to 3e) = \frac{1}{G_F^2} \frac{|(h^\dagger)_{ee}(h)_{\mu e}|^2}{m_{\Delta^{++}}^4} = \frac{1}{G_F^2\, m_{\Delta^{++}}^4} \frac{|m_{ee}^*\, m_{\mu e}|^2}{16\, v_\Delta^4},$$

- The $\mathrm{BR}(\mu \to 3e)$ depends on the oscillation parameters, CP-Violation phases and the lightest neutrino mass.

- From the present limit $\mathrm{BR}(\mu \to 3e) < 10^{-12}$, we have constraint

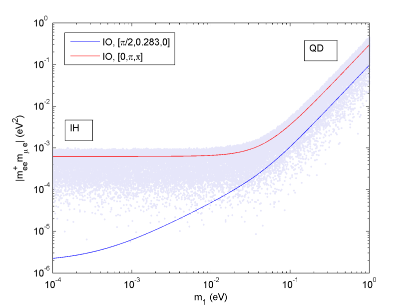
$$|(h^\dagger)_{ee}(h)_{\mu e}| < 1.2 \times 10^{-7} \left(\frac{m_{\Delta^{++}}}{100\,\mathrm{GeV}}\right)^2.$$

The $\mu \to 3e$ decay - NH



$$\mathrm{BR}(\mu \to 3e) \leq 6 \times 10^{-9} \left(\frac{1\ \mathrm{eV}}{v_\Delta}\right)^4 \left(\frac{100\ \mathrm{GeV}}{m_{\Delta^{++}}}\right)^4, \quad \mathrm{NH}.$$

The $\mu \to 3e$ decay - IH



$$\mathrm{BR}(\mu \to 3e) \leq 2.4 \times 10^{-6} \left(\frac{1\ \mathrm{eV}}{v_\Delta}\right)^4 \left(\frac{100\ \mathrm{GeV}}{m_{\Delta^{++}}}\right)^4, \ \mathrm{IH}$$

The $\mu - e$ conversion in nuclei

- The conversion rate can be written as:

$$\mathrm{CR}(\mu\,\mathcal{N} \to e\,\mathcal{N}) \cong \frac{\alpha_{\rm em}^5}{36\,\pi^4}\,\frac{m_\mu^5}{\Gamma_{\rm capt}}\,Z_{eff}^4\,Z\,F^2(-m_\mu^2)\times$$

$$\times\left|\left(h^\dagger h\right)_{e\mu}\left[\frac{5}{24\,m_{\Delta^+}^2}+\frac{1}{m_{\Delta^{++}}^2}\right]+\frac{1}{m_{\Delta^{++}}^2}\sum_{l=e,\mu,\tau} h_{el}^\dagger\,f(r,s_l)\,h_{l\mu}\right|^2 .$$

- Take an approximation $m_{\Delta^+} \approx m_{\Delta^{++}}$, we rewrite the formula as:

$$\mathrm{CR}(\mu\,\mathcal{N} \to e\,\mathcal{N}) \cong \frac{\alpha_{\rm em}^5}{36\,\pi^4}\,\frac{m_\mu^5}{\Gamma_{\rm capt}}\,Z_{eff}^4\,Z\,F^2(-m_\mu^2)\left|C_{\mu e}^{(II)}\right|^2 .$$

$$C_{\mu e}^{(II)} \equiv \frac{1}{4v_\Delta^2}\left[\frac{29}{24}\left(m^\dagger\,m\right)_{e\mu}+\sum_{l=e,\mu,\tau} m_{el}^\dagger\,f(r,s_l)\,m_{l\mu}\right],$$

The $\mu - e$ conversion in nuclei

- Upper limit from experiment:

$$|C_{\mu e}^{(II)}| < 1.24 \times 10^{-4} \left(\frac{M_\Delta}{100\,\mathrm{GeV}}\right)^2 .$$

The $\mu - e$ conversion in nuclei - Some remarks on function $f(r, s_l)$

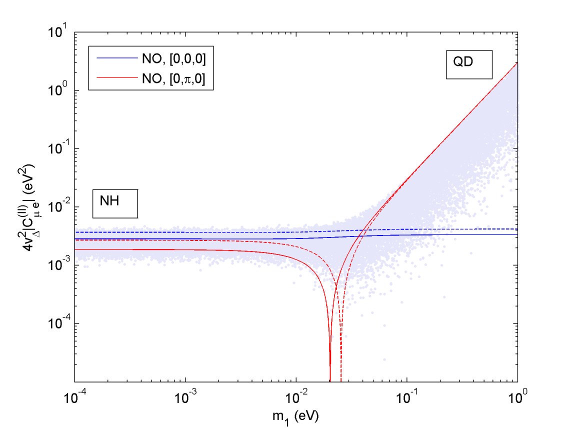
- Let us rewrite the function $f(r, s_l)$

$$f(r, s_l) = \frac{4s_l}{r} + \log(s_l) + \left(1 - \frac{2s_l}{r}\right)\sqrt{1 + \frac{4s_l}{r}}\log\frac{\sqrt{r} + \sqrt{r + 4s_l}}{\sqrt{r} - \sqrt{r + 4s_l}},$$

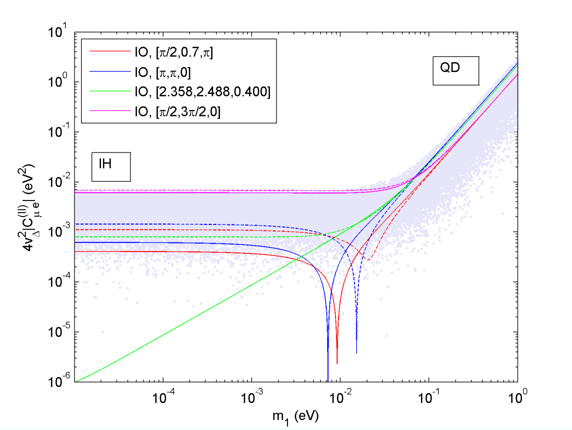
$$S_\ell = \frac{m_\ell^2}{M_{\Delta^{++}}^2}, \quad r = -\frac{q^2}{M_{\Delta^{++}}^2} \cong \frac{m_\mu^2}{M_{\Delta^{++}}^2}.$$

- $\lim_{s_l \to 0} f(r, s_l) = \log r$. For $M_{\Delta^{++}} > 10^4$ GeV, this is a good approximation for any leptons. However, for TeV scale seesaw scenario, it is only relevant for electron but not for μ and τ.
- Following this fact, the $CR(\mu - e)$ strongly depends on CP-violation phases.

The $\mu - e$ conversion in nuclei



The $\mu - e$ conversion in nuclei



Review of type III see-saw

- The interaction and mass terms in type III seesaw read:

$$\mathcal{L}^{\rm III}_{\rm seesaw} \;=\; -\lambda_{\ell j}\,\overline{\psi}_{\ell L}\,\boldsymbol{\tau}\,\widetilde{H}\cdot\,\mathbf{F}_{jR} \,-\, \frac{1}{2}\,(M_R)_{ij}\,\overline{\mathbf{F}^C_{iL}}\cdot\mathbf{F}_{jR} \,+\,\mathrm{h.c.}\,,$$

- The charged and neutral fermions are defined as:

$$E_j \;\equiv\; F^-_{jR} \,+\, F^{+C}_{jL} \qquad N_j \;\equiv\; F^{0C}_{jL} \,+\, F^0_{jR}\,.$$

- The weak interactions of the light Majorana neutrinos in mass eigenstates:

$$\mathcal{L}^{\nu}_{CC} \;=\; -\,\frac{g}{\sqrt{2}}\,\bar{\ell}\,\gamma_\alpha\,\left((1-\eta)U\right)_{\ell i}\,\chi_{iL}\,W^\alpha \,+\,\mathrm{h.c.}\,,$$

$$\mathcal{L}^{\nu}_{NC} \;=\; -\,\frac{g}{2c_w}\,\overline{\chi_{iL}}\,\gamma_\alpha\,\left(U^\dagger(1+2\,\eta)U\right)_{ij}\,\chi_{jL}\,Z^\alpha\,.$$

Review of type III see-saw

- and for heavy Majorana mass eigenstates:

$$\mathcal{L}^{N}_{CC} = \frac{g}{2\sqrt{2}}\,\overline{\ell}\,\gamma_\alpha\,(RV)_{\ell k}(1-\gamma_5)\,N_k\,W^\alpha + \text{h.c.}\,,$$
$$\mathcal{L}^{N}_{NC} = -\frac{g}{4c_w}\,\overline{\nu}_\ell\,\gamma_\alpha\,(RV)_{\ell j}(1-\gamma_5)\,N_j\,Z^\alpha + \text{h.c.}\,.$$

- Neutral current interactions => FCNCs:

$$\mathcal{L}^{\ell}_{NC} = \frac{g}{2c_w}\left(\overline{\ell}_L\,\gamma_\alpha\,(\mathbf{1}-4\,\eta)_{\ell\ell'}\,\ell'_L - 2\,s_w^2\,\overline{\ell}\,\gamma_\alpha\,\ell\right) Z^\alpha\,.$$

- Interactions of heavy charged leptons:

$$\mathcal{L}^{E}_{CC} = -g\,\overline{E}_j\,\gamma_\alpha\,N_j\,W^\alpha + g\,\overline{E}_j\,\gamma_\alpha\,(RV)_{\ell j}\,\nu^C_{\ell R}\,W^\alpha + \text{h.c.}\,,$$
$$\mathcal{L}^{E}_{NC} = g\,c_w\,\overline{E}_j\,\gamma_\alpha\,E_j\,Z^\alpha - \frac{g}{2\sqrt{2}c_w}\left(\overline{\ell}\,\gamma_\alpha\,(RV)_{\ell j}(1-\gamma_5)\,E_j\,Z^\alpha + \text{h.c.}\right)$$

The $\mu \to e\gamma$ Decay

The $\mathrm{BR}(\mu \to e\gamma)$:

$$\mathrm{BR}(\mu \to e\gamma) \quad = \quad \frac{3\alpha_{\mathrm{em}}}{32\pi}\,|T|^2\,,$$

where

$$T \;\cong\; -2\left(\frac{13}{3} + \mathcal{C}\right)\eta_{\mu e} + \sum_k (RV)_{ek}(RV)^*_{\mu k}\,\left[A(x_k) + B(y_k) + C(z_k)\right]\,,$$

$$x_k = \left(\frac{M_k}{M_W}\right)^2\,,\; y_k = \left(\frac{M_k}{M_Z}\right)^2\,,\; z_k = \left(\frac{M_k}{M_H}\right)^2\,, \mathcal{C} \simeq -6.56\,,$$

$$A(x) = \frac{-30 + 153\,x - 198\,x^2 + 75\,x^3 + 18\,(4 - 3\,x)\,x^2\,\log x}{3(x-1)^4}\,,$$

$$B(y) = \frac{33 - 18\,y - 45\,y^2 + 30\,y^3 + 18\,(4 - 3\,y)\,y\,\log y}{3(y-1)^4}\,,$$

$$C(z) = \frac{-7 + 12\,z + 3\,z^2 - 8\,z^3 + 6\,(3\,z - 2)\,z\,\log z}{3(z-1)^4}\,.$$

The $\mu \to e\gamma$ Decay

- We have upper limits from present experiment constraint:

$$|\eta_{\mu e}| \ < \ 9\ (20) \times 10^{-6}\,, \quad \text{for} \quad \overline{M} = 100\ (1000)\ \text{GeV}\,.$$

- Ongoing experiment can probe:

$$|\eta_{\mu e}| \ > \ 2\ (4) \times 10^{-6}\,, \quad \text{for} \quad \overline{M} = 100\ (1000)\ \text{GeV}\,.$$

The $\mu \to 3\,e$ and $\mu - e$ Conversion in Nuclei

The $\mu \to 3\,e$ decay rate:

$$\mathrm{BR}(\mu \to 3e) \simeq 16\,|\eta_{\mu e}|^2 \left(3\sin^4\theta_W - 2\sin^2\theta_W + \frac{1}{2}\right).$$

$$|\eta_{\mu e}| < 5.6 \times 10^{-7}.$$

The $\mu - e$ conversion rate:

$$\mathrm{CR}(\mu\,\mathcal{N} \to e\,\mathcal{N}) \cong \frac{32\,G_F^2}{\Gamma_{\mathrm{capt}}}\,|\eta_{\mu e}|^2 \left|\left(2\,g_{LV(u)} + g_{LV(d)}\right) V^{(p)} + \left(g_{LV(u)} + 2\,g_{LV(d)}\right)\right.$$

$$|\eta_{\mu e}| \lesssim 2.6 \times 10^{-7}.$$

where

$$V^{(n)} \simeq \frac{N}{Z}\,V^{(p)}, \quad g_{LV(u)} = 1 - \frac{8}{3}s_W^2 \quad \text{and} \quad g_{LV(d)} = -1 + \frac{4}{3}\,s_W^2.$$

Conclusion and Discussion

- We have considered and discussed the LFV processes, $\mu - e$ conversion in nuclei, $\mu \to e\gamma$ and $\mu \to 3e$ decays in the scenarios of TeV scale seesaw mechanisms and found the constrains on the relevant parameters from the present and future experiments.
- In type I seesaw, in case of IH, all of the LFV observables, can be strongly suppressed if $\sin_{\theta_{13}}$, δ and α_{21} satisfy a relation and $0 < \delta < 0.7$. The rate of $\mu - e$ conversion in Al, Ti and Au can also be strongly suppressed, however it can be efficient in case of Ti and Al or in Au only.
- All of the observable rates in type II seesaw are proportional to M_{Δ}^{-4}, it is impossible to detect any of the LFV processes if M_{Δ} is too large. For M_{Δ} is in the under discussion range, the $\mu - e$ conversion, and $\mu \to 3e$ decay rates depend strongly on CPV phases. It can be used to determined the Dirac and Majorana phases.