# Discrete Flavour Symmetries after Daya Bay and RENO

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based on

AFMS = G. Altarelli, F.F., L. Merlo and E. Stamou hep-ph/1205.4670
AFM = G. Altarelli, F.F. and L. Merlo hep-ph/1205.5133
FHT1=F.F., C. Hagedorn, R. de A.Toroop hep-ph/1107.3486
FHT2=F.F., Hagedorn, R. de A. Toroop hep-ph/1112.1340

## lepton mixing matrix $U_{PMNS}$

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{l}_L \gamma^\mu U_{PMNS} \nu_l$$

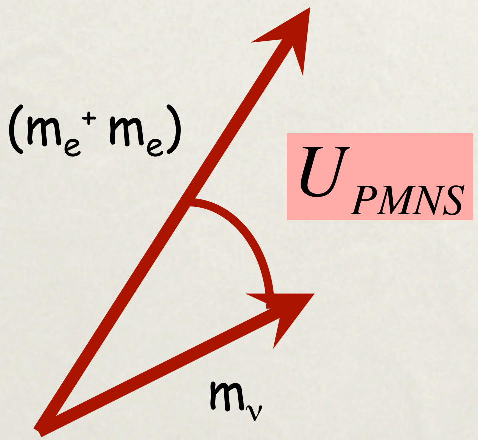
$$U_e^+ (m_e^+ m_e) U_e = (m_e^+ m_e)_{diag}$$

$$U_\nu^T m_\nu U_\nu = m_{\nu\, diag}$$

$\Rightarrow$

$$U_{PMNS} = U_e^+ U_\nu$$

$(m_e^+ m_e)$ and $m_\nu$ misaligned in flavour space




what matters is the relative orientation

$U_{PMNS}$ parametrized in terms of 3 mixing angles and 1 phase
(2 more phases are not measurable in neutrino oscillations)

$\vartheta_{12}$ $\vartheta_{23}$ $\vartheta_{13}$ $\delta_{CP}$

# 2011/2012 breakthrough

- from LBL experiments searching for $\nu_\mu \to \nu_e$ conversion

T2K: muon neutrino beam produced at JPARC [Tokai] E=0.6 GeV and sent to SK 295 Km apart [1106.2822]

MINOS: muon neutrino beam produced at Fermilab [E=3 GeV] sent to Soudan Lab 735 Km apart [1108.0015]

$$P(\nu_\mu \to \nu_e) = \sin^2\vartheta_{23} \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m^2_{32} L}{4E} + \ldots$$

both experiments favor $\sin^2\vartheta_{13} \sim$ few %

- from SBL reactor experiments searching for anti-$\nu_e$ disappearance

Double Chooz (far detector): $\sin^2\vartheta_{13} = 0.022 \pm 0.013$

Daya Bay (near + far detectors): $\sin^2\vartheta_{13} = 0.024 \pm 0.004$

RENO (near + far detectors): $\sin^2\vartheta_{13} = 0.029 \pm 0.006$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m^2_{32} L}{4E} + \ldots$$

- SBL reactors are sensitive to $\vartheta_{13}$ only
  LBL experiments anti-correlate $\sin^2 2\vartheta_{13}$ and $\sin^2\vartheta_{23}$
  also breaking the octant degeneracy $\vartheta_{23} \leftrightarrow (\pi-\vartheta_{23})$

# updated global fit

| | Lisi [Neutel2011] [0806.22517update] | Fogli et al. [1205.5254] |
|---|---|---|
| $\sin^2\vartheta_{12}$ | $0.307^{+0.018}_{-0.016}$ | $0.307^{+0.018}_{-0.016}$ |
| $\sin^2\vartheta_{23}$ | $0.42^{+0.09}_{-0.04}$ | $0.398^{+0.030}_{-0.026}$ [NO]<br>$0.408^{+0.035}_{-0.030}$ [IO] |
| $\sin^2\vartheta_{13}$ | $0.014^{+0.009}_{-0.008}$ | $0.0245^{+0.0034}_{-0.0031}$ [NO]<br>$0.0246^{+0.0034}_{-0.0031}$ [IO] |
| $\Delta m^2_{sol}\ (eV^2)$ | $(7.54^{+0.25}_{-0.22})\times 10^{-5}$ | $(7.54^{+0.26}_{-0.22})\times 10^{-5}$ |
| $\left\lvert\Delta m^2_{atm}\ (eV^2)\right\rvert$ | $(2.36^{+0.12}_{-0.10})\times 10^{-3}$ | $(2.43^{+0.07}_{-0.09})\times 10^{-3}$ [NO]<br>$(2.42^{+0.07}_{-0.10})\times 10^{-3}$ [IO] |

hint for non maximal $\vartheta_{23}$ ?

$7\sigma$ away from 0

$$\vartheta_{13} = (9.0 \pm 0.6)^0$$

open questions
- is L violated or not?
- mass ordering: Normal or Inverted?
- is $\vartheta_{23}$maximal or not?
- $\delta_{CP}$

# on the theory side

now data seem sufficiently precise to allow for a strong selection among the existing models/ideas

does a coherent and unique theoretical picture emerge from the data?

how should we read the data?

$$\left.\begin{array}{l}\vartheta_{13} << \vartheta_{12}, \vartheta_{23} \\ \vartheta_{23} \approx \text{maximal}\end{array}\right\}$$ less sharp after the 2012 data

$$\vartheta_{12} + O(\lambda_C) \approx \pi/4$$

$$\Delta m^2_{sol} << \left|\Delta m^2_{atm}\right|$$

accidental features
mixing angles and mass ratios are O(1)
no special pattern beyond the data:
Anarchy [Hall, Murayama, Weiner 1999]

"Evidence" for some property of the fundamental theory

the new data have strengthened the case for Anarchy

this talk:
there is a limit of the theory where lepton mixing angles become simple
[like $V_{CKM}=1 + O(\lambda_C)$]

$$U_{PMNS} = U^0_{PMNS} + \text{corrections}$$

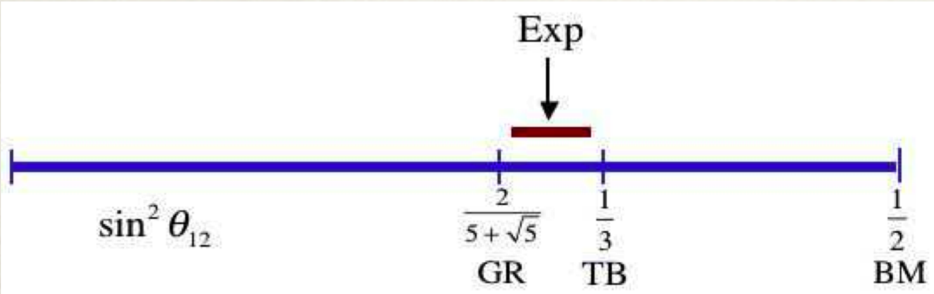
# Mixing patterns $U^0_{PMNS}$ (an incomplete list)

| | $U^0_{PMNS}$ | $\sin^2\vartheta^0_{23}$ | $\sin^2\vartheta^0_{13}$ | $\sin^2\vartheta^0_{12}$ |
|---|---|---|---|---|
| TB | $\begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$ | $1/2$ | $0$ | $1/3$ |
| GR | $\begin{pmatrix} c & s & 0 \\ -s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ -s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ $s/c = 1/\varphi$ | $1/2$ | $0$ | $1/(\sqrt{5}\varphi)$ $\approx 0.276$ |
| BM | $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}$ | $1/2$ | $0$ | $1/2$ |
| | $3\sigma$ range [NO] | $(0.330 \div 0.638)$ | $(0.0149 \div 0.0344)$ | $(0.259 \div 0.359)$ |

[TB <->Harrison, Perkins and Scott]

[GR<-> Kajiyama, Raidal, Strumia 2007]

$\varphi = \frac{(1+\sqrt{5})}{2}$ Golden Ratio



# Mixing patterns $U^0_{PMNS}$ from discrete symmetries

[in some limit: u->0]

[He, Keum, Volkas 0601001
Lam 0708.3665 + 0804.2622]

$$G_e \leftarrow G_f \rightarrow G_\nu$$

residual symmetry of $(m_e^+ m_e)$ — $G_e$

$G_\nu$ — residual symmetry of $m_\nu$

$(m_e^+ m_e)$ and $m_\nu$ misaligned because $G_e$ and $G_\nu$ do not commute

[non degenerate mass spectrum: $G_e$ and $G_\nu$ abelian]

assign l to a 3-dim irrep $\rho(g)$ of $G_f$

$$U_\nu^+ \rho(g_\nu) U_\nu = \rho(g_\nu)_{diag}$$
$$U_e^+ \rho(g_e) U_e = \rho(g_e)_{diag}$$

$$\Rightarrow \quad U_{PMNS} = U_e^+ U_\nu$$

LO result gets corrected in the full theory $\quad \vartheta_{ij} = \vartheta_{ij}^0 + O(u)$

- the most general group leaving $\nu^T m_\nu \nu$ invariant, and $m_i$ unconstrained $\quad G_\nu = Z_2 \times Z_2$ $\quad$ Majorana neutrinos imply $G_\nu$ discrete!

- $G_e$ can be continuous but the simplest choice is $G_e$ discrete

$$G_e = \begin{cases} Z_2 \times Z_2 & \\ Z_n & n \geq 3 \end{cases}$$

# empirical mixing patterns arise from small groups

| $G_f$ | $G_e$ | $U^0_{PMNS}$ |
|---|---|---|
| $A_4$ | $Z_3$ | $U_{TB}$ |
| $S_4$ | $Z_3$<br>$Z_4$<br>$Z_2 \times Z_2$ | $U_{TB}$<br>$U_{BM}$ |
| $A_5$ | $Z_5$ | $U_{GR}$ |

generators

$G_\nu = Z_2 \times Z_2$ (S,S′)

[although S′ does not belong to $A_4$, it can arise as an accidental symmetry]

$G_e = Z_n$ T

$G_e = Z_2 \times Z_2$ (T,T′)

- $\vartheta_{13}$=0 and $\vartheta_{23}$=π/4 originate from the generator S′ of $G_\nu$

in the basis where the elements of $G_e$ are diagonal

$$S' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

μ-τ or 2-3 exchange symmetry

- invariance under a single $Z_2$ parity in $G_\nu = Z_2 \times Z_2$ determines two (combinations of) mixing angles: $\vartheta_{13}$=0 and $\vartheta_{23}$=π/4 in case of S′
- the second $Z_2$ parity determines the third angle and a phase
- neutrino masses unconstrained: fitted, not predicted

general feature

$$U_{PMNS} = U^0_{PMNS} + O(u) \quad u \equiv \frac{\langle \varphi \rangle}{\Lambda} < 1$$

$$\vartheta_{ij} = \vartheta^0_{ij} + O(u)$$

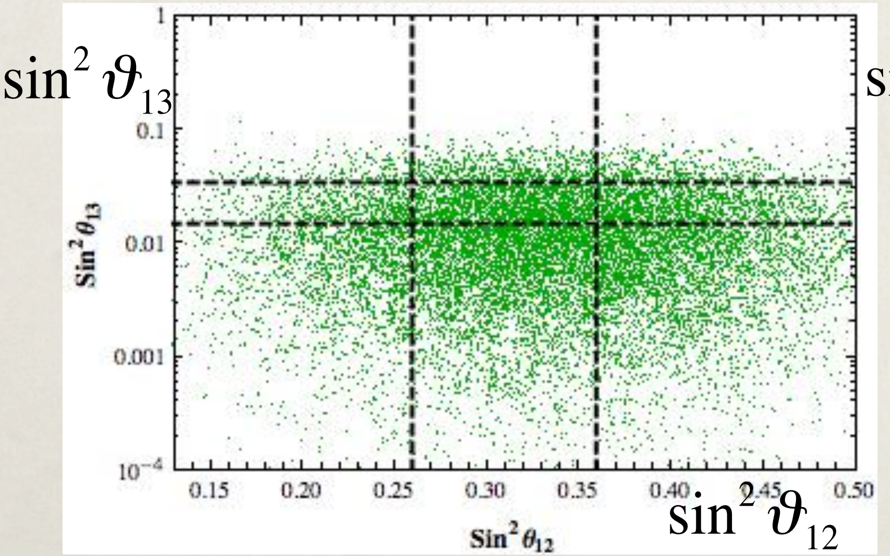
LO result gets corrected in the full theory

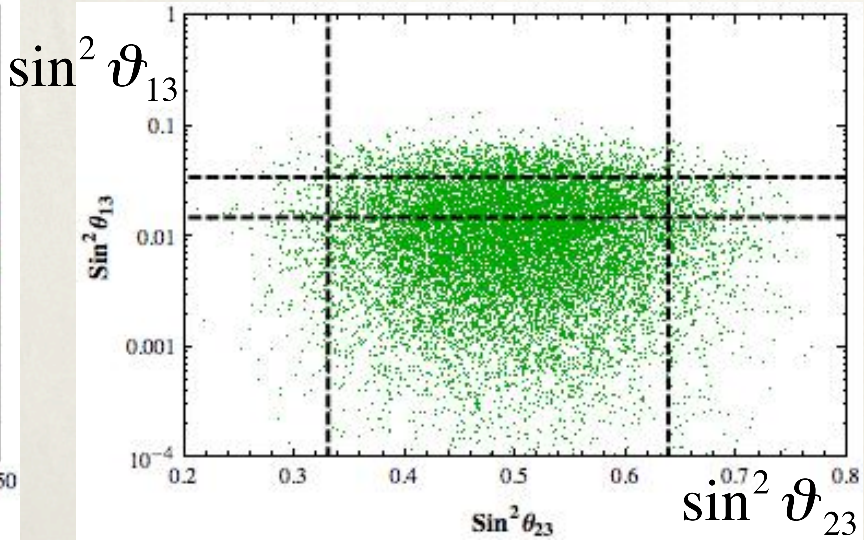
[depending on $U^0$ we might need u small or very small]

when $U^0$ is TM (or GR), we expect $\vartheta_{13}$ and $(\vartheta_{23}-\pi/4) \approx$ few 0.01
[not to spoil the agreement with $\vartheta_{12}$]

a challenge for models such as $A_4$ leading to $U^0 = U_{TB}$
is to generate $\vartheta_{13} \approx 0.1$ while keeping $\vartheta_{12}$ almost unchanged

$A_4$ model with typical O(0.1) corrections
[size of the corrections – 0.08 - optimized to maximize the success rate]




lack of predictability: $\sin^2\vartheta_{12}$ ranges from 0.2 up to 0.45 now
success rate (about 13%) indicates the need of tuning

## $A_4$ models with special corrections

group theoretical origin of TB mixing suggests how to modify $\vartheta_{13} \approx 0.1$ while keeping $\vartheta_{12}$ almost unchanged

assume $G_e = Z_3$ (generated by T) and $G_\nu = Z_2$ (generated by S) i.e. remove S′ generator

-- natural in the context of $A_4$ that contains S and T, but not S′
-- explicit constructions proposed before T2K,... [Lin 2009]
-- starting from the full $G_\nu = Z_2 \times Z_2$, the parity S′ can be broken at a high scale

$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & e^{i\delta}\sin\alpha \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

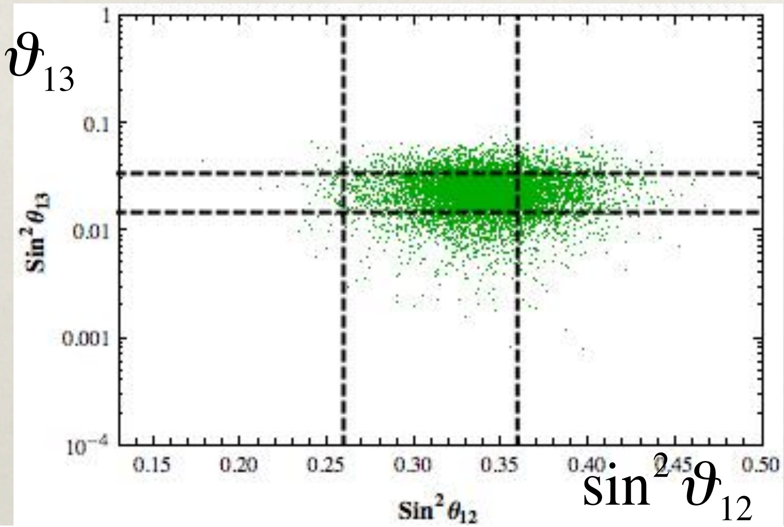
$$0 \le \alpha \le \pi/2$$
$$0 < \delta \le 2\pi$$

$$\sin\vartheta_{13} = \sqrt{2/3}\,\alpha + \ldots$$
$$\sin^2\vartheta_{12} = 1/3 + 2/9\,\alpha^2 + \ldots$$
$$\sin^2\vartheta_{23} = 1/2 + \alpha/\sqrt{3}\cos\delta + \ldots$$
$$\delta_{CP} = \delta$$

Trimaximal mixing
gives back TB when α=0

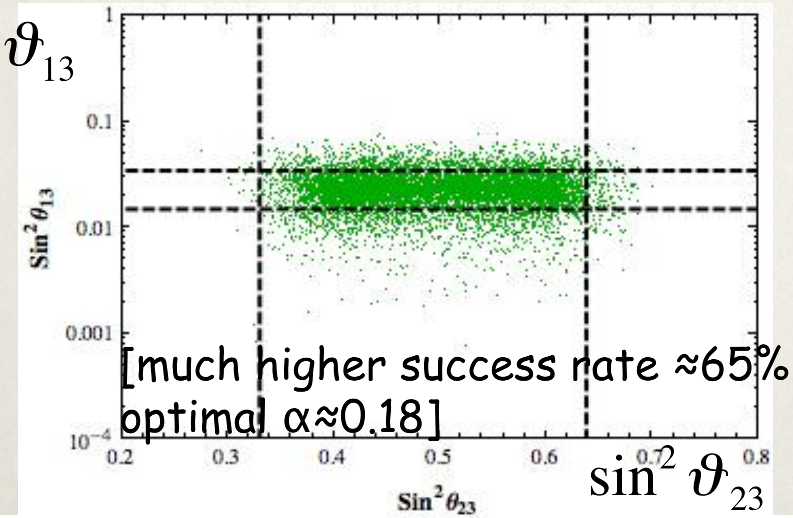
[assuming α=0.1 and expanding in powers of α]

$\sin^2\vartheta_{13}$




$\sin^2\vartheta_{12}$

$\sin^2\vartheta_{13}$

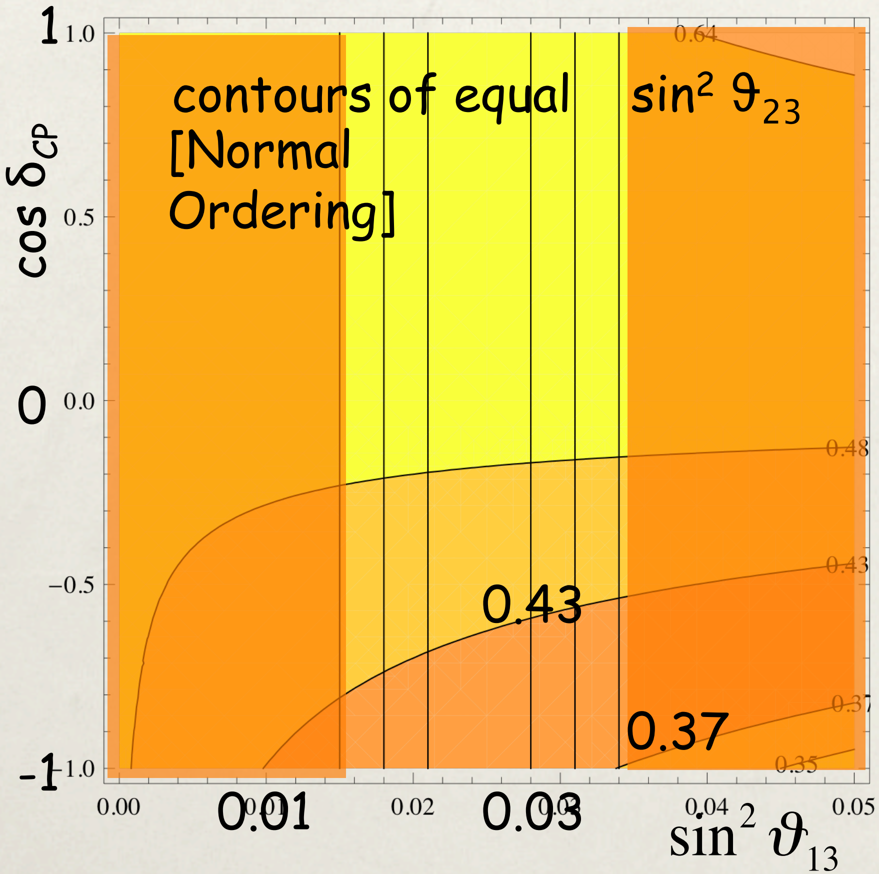



[much higher success rate ≈65%
optimal α≈0.18]

$\sin^2\vartheta_{23}$

from the previous relations

$$\sin^2\vartheta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$




indication for $\sin^2\vartheta_{23} \approx 0.4$
would favor $-1 < \cos\delta_{CP} < -0.5$

can be tested by measuring $\delta_{CP}$
and improving on $\sin^2\vartheta_{23}$

Trimaximal ansatz proposed with different motivations by many authors

[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011]

[similar tests can be realized in $S_4$ (TM) and $A_5$ (GR)
more possibilities by enforcing $G_\nu = Z_2$ generated by SxS']

## corrections to $U^0_{PMNS}=U_{BM}$ realized in $S_4$

in this case removing S' would not help since it would maintain $\vartheta_{12}$ very close to $\pi/4$, i.e. the LO BM prediction

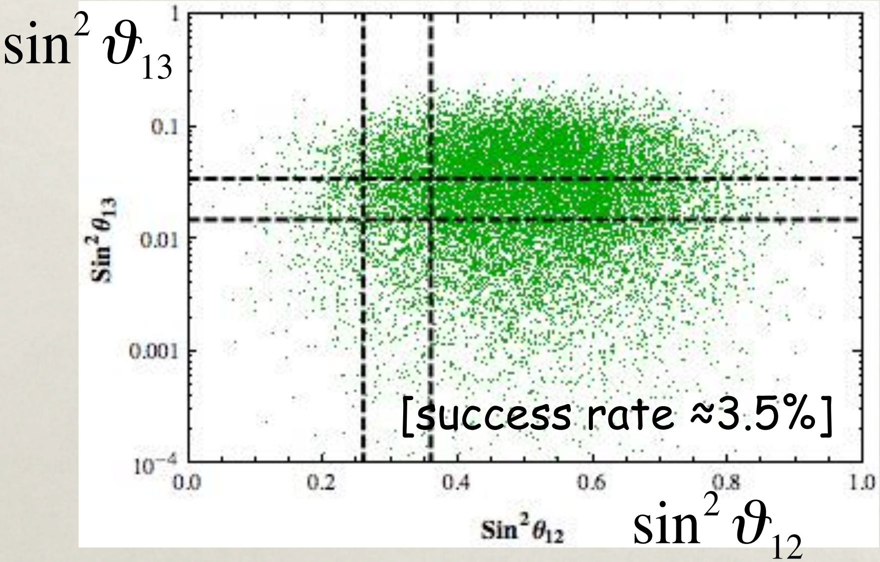
as observed long ago, the most efficient correction is of the following type

$$U_{BM} \to U_e^+ U_{BM}$$

a correction from the charged lepton sector, mainly through rotations in the 12 and 13 sectors, to preserve $\vartheta_{23}=\pi/4$

several existing models incorporate this idea, in particular in the context of $G_f=S_4$

$S_4$ model with $U^0_{PMNS}=U_{BM}$ and typical O(0.1) corrections from $U_e$
[size of the corrections – 0.17 - optimized to maximize the success rate]




-- a tuning of the parameters in $U_e$ is needed to reproduce both $\vartheta_{13}$ and $\vartheta_{12}$ otherwise $\sin^2\vartheta_{12}$ ranges from 0.2 up to 0.8

-- required tuning is worse than in $A_4$ model with typical O(0.1) corrections

## $S_4$ models with special corrections

BM mixing can also arise from $S_4$ when $G_e=Z_2\times Z_2$ (generated by T,T') and $G_\nu=Z_2\times Z_2$ (generated by S,S') [FHT2]

$$T=\begin{pmatrix}+1 & 0 & 0\\ 0 & +1 & 0\\ 0 & 0 & -1\end{pmatrix} \qquad T'=\begin{pmatrix}+1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & +1\end{pmatrix}$$

assume $G_e=Z_2$ (generated by T) and $G_\nu=Z_2\times Z_2$ (generated by S,S')
i.e. remove T' generator

-- starting from the full $G_e=Z_2\times Z_2$, the parity T' can be broken at a high scale

$$U^0=\begin{pmatrix}\cos\alpha & -e^{i\delta}\sin\alpha & 0\\ e^{-i\delta}\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}\times U_{BM}$$

$$0\le\alpha\le\pi/2$$
$$0<\delta\le 2\pi$$

$$\sin\vartheta_{13}=\alpha/\sqrt{2}+\ldots$$
$$\sin^2\vartheta_{12}=1/2+\alpha\cos\delta/\sqrt{2}+\ldots$$
$$\sin^2\vartheta_{23}=1/2-\alpha^2/4\ldots$$
$$\delta_{CP}=-\delta$$

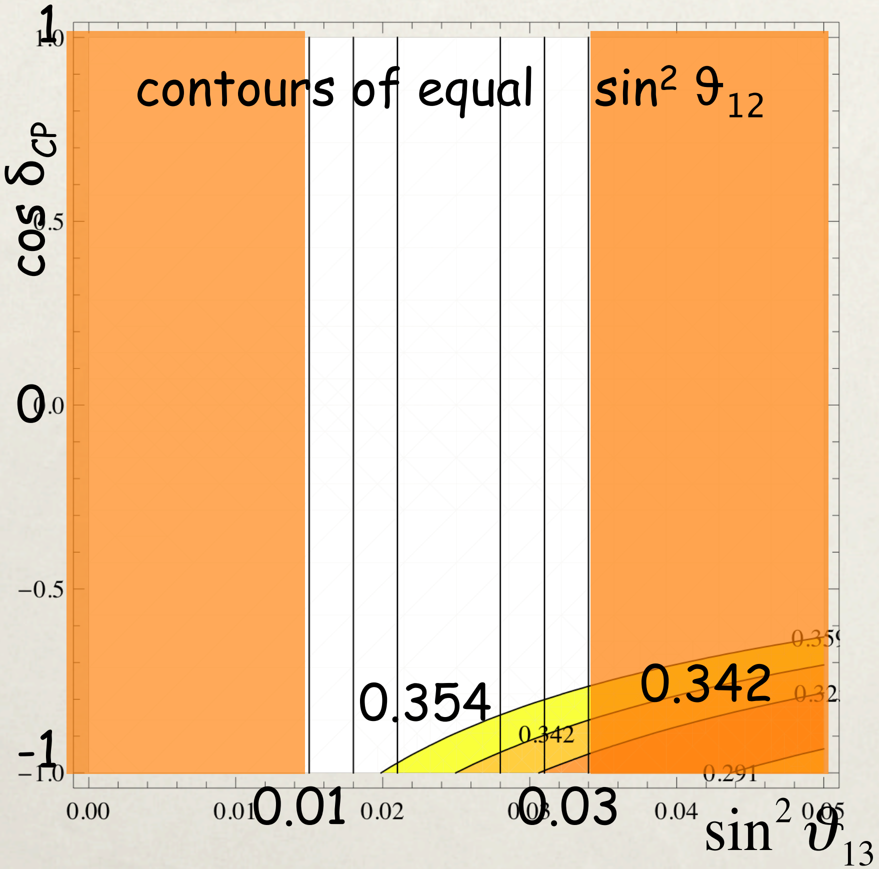
reasonable correction if charged leptons are similar to quarks, i.e. dominant mixing is in 12 sector

[assuming α=0.1 and expanding in powers of α] from the previous relations

$$\sin^2\vartheta_{12} = \frac{1}{2} + \sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$

since $\vartheta_{13}=O(\lambda_C)$ this realizes a form of QLC

[Raidal 0404046
Minakata, Smirnov 0405088]




reduced parameter space still allowed

strong preference for $\delta_{CP}=\pi$
[no CP violation in lepton sector]
and for the higher side of $\sin^2\vartheta_{12}$

testable by measuring $\delta_{CP}$

[Frampton, Petcov, Rodejohann 0401206
Altarelli, F, Masina 0402155
Romanino 0402508,
Marzocca, Petcov, Romanino, Spinrath 1108.0614]

So far $U_{PMNS} = U^0_{PMNS}$ + corrections

## $\vartheta_{13} > 0$ from any discrete symmetry, at the LO ? [FHR1,FHR2]

how to "deform" $A_4$ and/or $S_4$? no continuous parameter

abstract definition
in terms of generators
and relations

$$S^2 = (ST)^3 = T^n = 1$$

$n = 3 \quad A_4$

$n = 4 \quad S_4$

both subgroups of the (infinite) modular group Γ

$$S^2 = (ST)^3 = 1$$

we looked for other subgroups of Γ, the so-called finite modular groups $\Gamma_N$
an infinite series, but there are only six of them admitting (independent)
3-dimensional irreducible representations [Nobs, 1976]

| $N$ | 3 | 4 | 5 | 7 | 8 | 16 |
|---|---|---|---|---|---|---|
| $\Gamma_N$ | $A_4$ | $S_4$ | $A_5$ | $PSL(2,Z_7)$ | $\Gamma_8$ | $\Gamma_{16}$ |

new interesting patterns in N=8,16 choosing $G_e=Z_3$ and $G_\nu=Z_2 \times Z_2$

$$\Gamma_8 \supset \Delta(96): \qquad S^2 = (ST)^3 = T^8 = 1 \qquad \left(ST^{-1}ST\right)^3 = 1$$

$$\Gamma_{16} \supset \Delta(384): \qquad S^2 = (ST)^3 = T^{16} = 1 \qquad \left(ST^{-1}ST\right)^3 = 1$$

## new mixing patterns are special forms of Trimaximal mixing

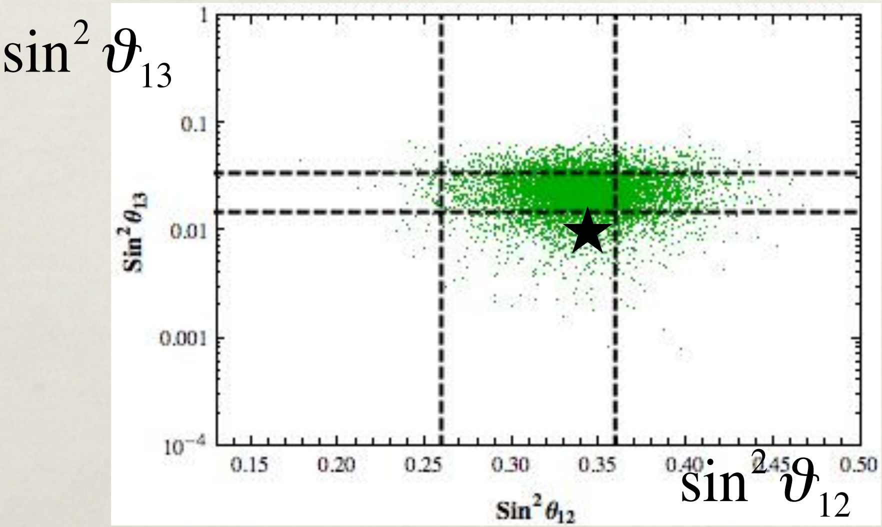
$$U^0_{PMNS} = U_{TB} U_{13}(\alpha)$$

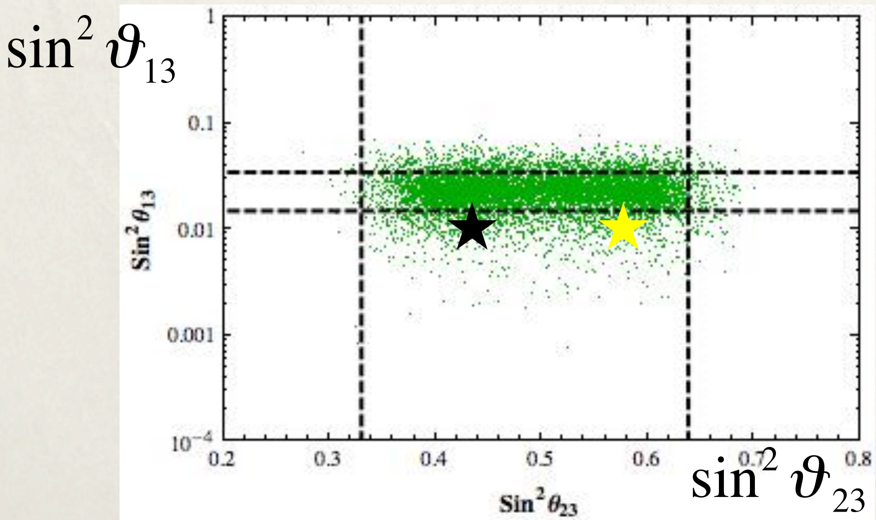
$$U_{13}(\alpha) = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

but $\delta_{CP}$ =0,π (no CP violation) and
the angle α is not a free parameter:
it is "quantized" by group theory

| $G_f$ | $\Gamma_8$ | $\Gamma_{16}$ |
|---|---|---|
| $\alpha$ | $\pm 1/12$ | $\pm 1/24$ |

patterns from $\Gamma_{16}$ (compared to $A_4$ with "special" corrections)



# conclusion

[see talks by
C. Luhn
L. Merlo
A. Meroni
G. Ross,
M. Spinrath…]

- big progress on the experimental side:
  -- precisely measured $\vartheta_{13}$: 7$\sigma$ away from zero!
  -- potentially interesting implications on $\vartheta_{23}$

- on the theory side:
  no compelling and unique picture have emerged so far
  present data can be described within widely different frameworks

- models based on "anarchy" and/or its variants - $U(1)_{FN}$ models - in good shape:
  neutrino mass ratios and mixing angles just random O(1) quantities

- -- models based on discrete symmetries and giving rise, at LO, to
  $\vartheta_{13}=0$ and $\vartheta_{23}=\pi/4$ require some tuning when generic O(0.1)
  correction are added

  -- special corrections are suggested by the group structure itself,
  leading to a good description of the data [e.g. in $A_4$]

  -- such special corrections imply restrictions on the CP violating phase $\delta_{CP}$

- there are candidate flavor symmetries for LO mixing pattern
  with non-vanishing $\theta_{13}$ and coming very close to the existing data
  [existence proof found]

back up slides

# $\Delta(384)$

$$S=\frac{1}{2}\begin{pmatrix}0 & \sqrt{2} & \sqrt{2}\\ \sqrt{2} & -1 & 1\\ \sqrt{2} & 1 & -1\end{pmatrix}$$

$$T=\begin{pmatrix}\omega_{16}^{14} & 0 & 0\\ 0 & \omega_{16}^{5} & 0\\ 0 & 0 & \omega_{16}^{13}\end{pmatrix} \qquad \omega_{16}=e^{i\frac{\pi}{8}}$$

$$G_\nu = Z_2\times Z_2$$

generated by (S,ST⁸ST⁸)

$$G_e = Z_3$$

generated by ST

$$|U_{PMNS}|=\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{4+\sqrt{2}+\sqrt{6}}/2 & 1 & \sqrt{4-\sqrt{2}-\sqrt{6}}/2\\ \sqrt{4+\sqrt{2}-\sqrt{6}}/2 & 1 & \sqrt{4-\sqrt{2}+\sqrt{6}}/2\\ \sqrt{1-1/\sqrt{2}} & 1 & \sqrt{1+1/\sqrt{2}}\end{pmatrix}$$

[by exchanging 2nd and 3rd rows in $U_{PMNS}$]

[M3]

$$\sin^2\vartheta_{13}=(4-\sqrt{2}-\sqrt{6})/12\approx 0.011$$

$$\sin^2\vartheta_{23}=\frac{(4-\sqrt{2}+\sqrt{6})}{(8+\sqrt{2}+\sqrt{6})}\approx 0.424$$

$$\sin^2\vartheta_{12}=\frac{4}{(8+\sqrt{2}+\sqrt{6})}\approx 0.337$$

$$\delta_{CP}=0$$

[M4]

$$\sin^2\vartheta_{13}=(4-\sqrt{2}-\sqrt{6})/12\approx 0.011$$

$$\sin^2\vartheta_{23}=\frac{(4+2\sqrt{2})}{(8+\sqrt{2}+\sqrt{6})}\approx 0.576$$

$$\sin^2\vartheta_{12}=\frac{4}{(8+\sqrt{2}+\sqrt{6})}\approx 0.337$$

$$\delta_{CP}=\pi$$

## $\Delta(96)$

$$S = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \end{pmatrix}$$

$$T = \begin{pmatrix} \omega_8^6 & 0 & 0 \\ 0 & \omega_8^7 & 0 \\ 0 & 0 & \omega_8^3 \end{pmatrix} \qquad \omega_8 = e^{i\frac{\pi}{4}}$$

$$G_\nu = Z_2 \times Z_2$$

generated by $(S, ST^4ST^4)$

$$G_e = Z_3$$

generated by $ST$

$$|U_{PMNS}| = \frac{1}{\sqrt{3}}\begin{pmatrix} (\sqrt{3}+1)/2 & 1 & (\sqrt{3}-1)/2 \\ (\sqrt{3}-1)/2 & 1 & (\sqrt{3}+1)/2 \\ 1 & 1 & 1 \end{pmatrix}$$

[M1]

$$\sin^2\vartheta_{13} = (2-\sqrt{3})/6 \approx 0.045$$

$$\sin^2\vartheta_{23} = (5+2\sqrt{3})/13 \approx 0.651$$

$$\sin^2\vartheta_{12} = (8-2\sqrt{3})/13 \approx 0.349$$

$$\delta_{CP} = \pi$$

[M2]

[by exchanging 2nd and 3rd rows in $U_{PMNS}$]

$$\sin^2\vartheta_{13} = (2-\sqrt{3})/6 \approx 0.045$$

$$\sin^2\vartheta_{23} = (8-2\sqrt{3})/13 \approx 0.349$$

$$\sin^2\vartheta_{12} = (8-2\sqrt{3})/13 \approx 0.349$$

$$\delta_{CP} = 0$$

# Mixing patterns $G_\nu = Z_2 \times Z_2$

[Lam 1104.0055
F., Hagedorn, Toroop in prep.]

| $G_f$ | $G_e$ | $U_{PMNS}$ | | $\sin^2\vartheta_{23}$ | $\sin\vartheta_{13}$ | $\sin^2\vartheta_{12}$ | |
|---|---|---|---|---|---|---|---|
| $A_4$ | $Z_3$ | $\frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{pmatrix}$ $\omega^3=1$ | [M] | 1/2 | $1/\sqrt{3}$ | 1/2 | ? |
| $S_4$ | $Z_3$ | $\begin{pmatrix}2/\sqrt{6} & 1/\sqrt{3} & 0\\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2}\\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2}\end{pmatrix}$ | [TB] | 1/2 | 0 | 1/3 | OK |
| | $Z_4$<br>$(Z_2 \times Z_2)'$ | $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 & 0\\ -1/\sqrt{2} & 1/\sqrt{2} & -1\\ -1/\sqrt{2} & 1/\sqrt{2} & 1\end{pmatrix}$ | [BM] | 1/2 | 0 | 1/2 | ? |
| $A_5$ | $Z_3$ | $\begin{pmatrix}c & s & 0\\ -s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2}\\ -s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}$ | [GR$_1$] $s/c=1/\varphi^2$ | 1/2 | 0 | 0.127 | ? |
| | $Z_5$ | $\begin{pmatrix}c & s & 0\\ -s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2}\\ -s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}$ | [GR$_2$] $s/c=1/\varphi$ | 1/2 | 0 | 0.276 | OK |
| | $(Z_2 \times Z_2)'$ | $\begin{pmatrix}0.81 & 0.5 & 0.31\\ 0.31 & 0.81 & 0.5\\ 0.5 & 0.31 & 0.81\end{pmatrix}$ | [GR$_2$] | 0.276 | 0.309 | 0.276 | ? |
| | | [Exp] | [3$\sigma$] | 0.39÷0.64 | <0.2 | 0.27÷0.36 | |

$\varphi = \frac{(1+\sqrt{5})}{2}$ Golden Ratio

[TB <->Harrison, Perkins and Scott]

[GR$_2$<-> Kajiyama, Raidal, Strumia 2007]

## can we test these ideas?

- none of these possibilities is supported by the quark properties!

# TESTS [I]  neutrino mass spectrum

-- LO mixing angles predicted [independently from input parameters]
-- size of NLO corrections under control, but precise values unknown

-- neutrino masses do depend on input parameters

minimal realizations of $A_4$ and $S_4$ have 2 complex parameters in neutrino sector at the LO ⇒ 1 sum rule among (complex) $m_i$

Example:

$G_f = A_4 \times Z_3 \times U(1)_{FN}$ [+ SEE-SAW]

$$\frac{1}{m_3} = \frac{1}{m_1} - \frac{2}{m_2}$$

at the LO

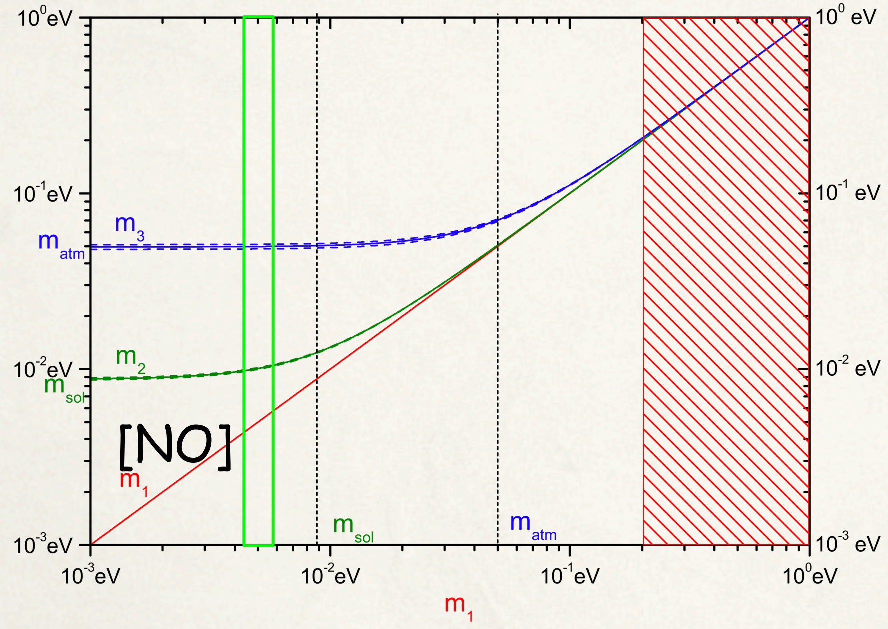
both normal [NO] and inverted [IO] orderings are allowed

[NLO corrections of order $0.005 < u < 0.05$]

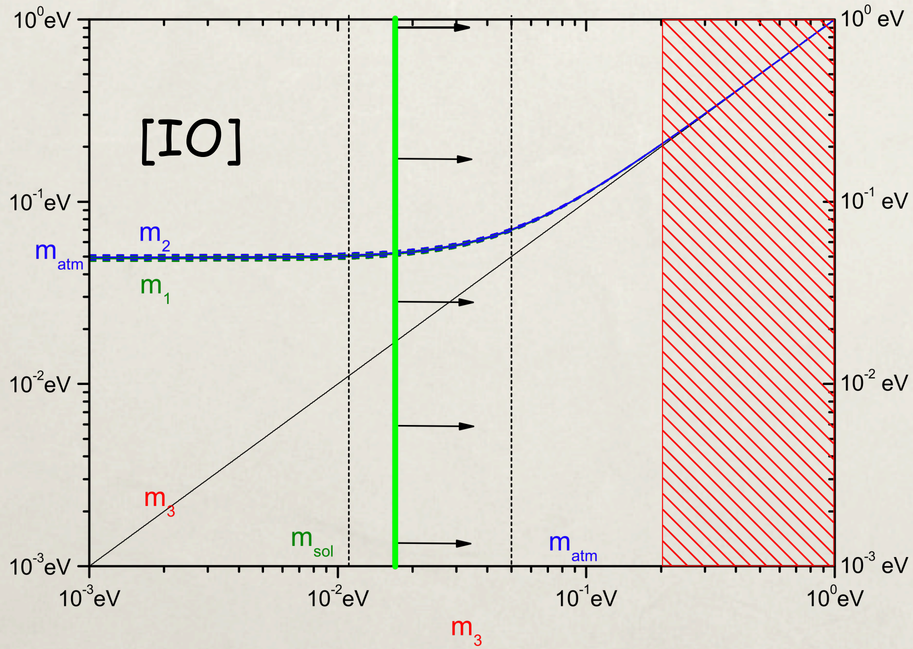
# $G_f = A_4 \times$ ... [+ SEE-SAW]

in the NO case the sum rule completely determines the spectrum

$$m_1 \approx 0.005\,eV \quad m_2 \approx 0.01\,eV \quad m_3 \approx 0.05\,eV$$
$$|m_{ee}| \approx 0.007\,eV$$



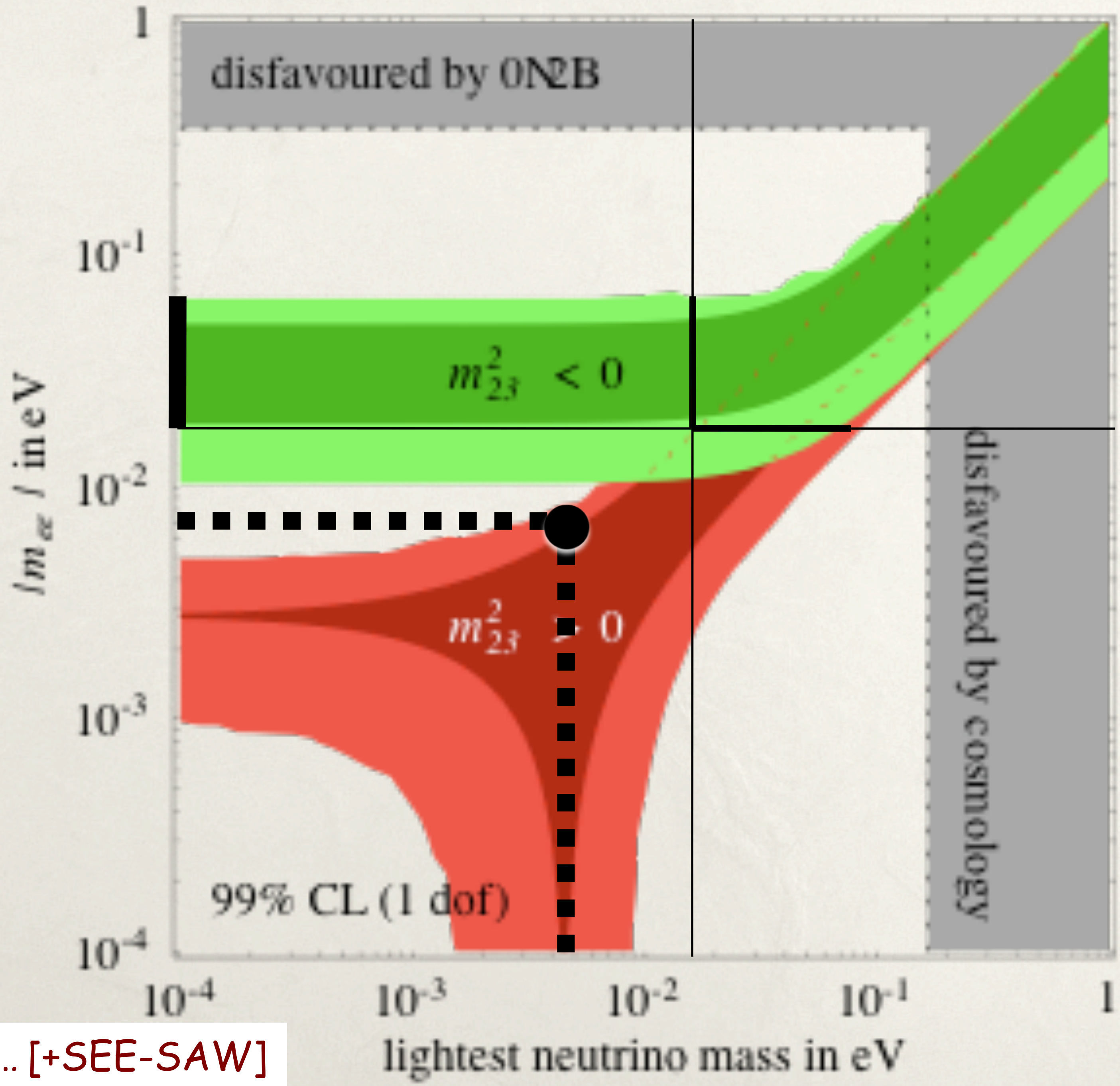

BDFN hep-ph/0908.0161




in the IO case the sum rule provides a lower bound on $m_3$

$$m_3 \geq 0.017\,eV$$
$$|m_{ee}| \geq 0.017\,eV$$

NLO corrections are negligible for NO and for IO close to the lower bound




$G_f = A_4 x$... [+SEE-SAW]

# TESTS [II] Lepton Flavour Violation

evidence for lepton flavor conversion

| | | |
|---|---|---|
| direct | $\nu_e \to \nu_\mu, \nu_\tau$ | sol |
| indirect | $\nu_\mu \to \nu_\tau$ | atm |

should show up in other processes if the scale of new physics $\Lambda_{NP} \approx 1$ TeV

1. distinctive signatures of discrete flavour symmetries

2. is BR(μ->eγ) sufficiently suppressed if $\Lambda_{NP}$=1 TeV?

$$L_{eff} = L_{SM} + i\frac{e}{\Lambda_{NP}^2} e_i^c H_d \left(\sigma^{\mu\nu} F_{\mu\nu}\right) Z_{ij}^{dip}\, l_j + \ldots$$

$Z_{ij}^{dip}$ describes lepton EDM, MDM, $l_i \to l_j \gamma$

$$BR(\mu \to e\gamma) < 1.2 \times 10^{-11} \quad \to \quad Z_{\mu e}^{dip} < 10^{-8} \times \left[\frac{\Lambda_{NP}(\mathrm{TeV})}{1\ \mathrm{TeV}}\right]^2$$

if we insist on having $\Lambda_{NP}$≈1TeV, what suppresses the rate?
[many models fail…]

flavour symmetries can generically help
what about discrete symmetries?

$$BR(\mu \to e\gamma) = O(u^p) \qquad p > 0$$

# LFV - signatures of discrete symmetries

discrete symmetries are weaker than continuous ones such as MFV, SO(3)...
and allow for $G_f$-invariant and LFV operators

in all models: l~3 of $G_f$

| | $A_4$ | $S_4$ | $A_5$ |
|---|---|---|---|
| $\frac{1}{\Lambda_{NP}^2}(\overline{\tau}\overline{\mu}ee+...)$ | Yes | Yes | Yes |
| $\frac{1}{\Lambda_{NP}^2}(\overline{\tau}\overline{e}\mu\mu+...)$ | Yes | No | No |
| $\frac{1}{\Lambda_{NP}^2}(\overline{\mu}\overline{e}\tau\tau+...)$ | Yes | No | No |

selection rule $\Delta L_e \Delta L_\mu \Delta L_\tau = 0, \pm 2$

$\tau^- \to \mu^+ e^- e^-$ in $A_4, S_4, A_5$

$\tau^- \to e^+ \mu^- \mu^-$ in $A_4$

$BR(\tau^- \to \mu^+ e^- e^-) < 2.0 \times 10^{-8}$

$BR(\tau^- \to e^+ \mu^- \mu^-) < 2.3 \times 10^{-8}$

$\Lambda_{NP} > 10$ TeV

$m_{NP} > 500$ GeV $\quad (m_{NP} = g\Lambda_{NP}/4\pi)$

- in simplest realizations of the above groups these operators are not generated at the LO

$$\frac{BR(\tau^- \to \mu^+ e^- e^-)}{BR(\tau^- \to \mu^+ \mu^- \mu^-)} = O(u^4) \qquad \frac{BR(\tau^- \to e^+ \mu^- \mu^-)}{BR(\tau^- \to \mu^+ \mu^- \mu^-)} = O(u^2 \frac{m_\mu}{m_\tau})$$

## LFV - radiative decays $l_i \to l_j\gamma$

$G_f = A_4 \times SUSY$...

from loops of SUSY particles

1. allowing for the most general slepton mass matrix compatible with pattern of flavour symmetry breaking. For instance [in super-"CKM" basis]

$$\hat{m}^2_{LL} = \begin{pmatrix} n & n_{12}u^2 & n_{13}u^2 \\ n_{12}u^2 & n & n_{23}u^2 \\ n_{13}u^2 & n_{23}u^2 & n \end{pmatrix} m^2_{SUSY} + \ldots$$

$m^2_{XY}$ (X,Y=L,R) are almost diagonal

off-diagonal terms $(\delta_{ij})_{XY}$ local in $u = \langle\varphi\rangle/\Lambda$

$$R_{ij} = \frac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j\nu_i\bar{\nu}_j)} = \begin{cases} \dfrac{6m_W^4\alpha_{em}}{\pi m^4_{SUSY}}\left|w_{ij}u\right|^2 & \text{[generic]} \\ \dfrac{6m_W^4\alpha_{em}}{\pi m^4_{SUSY}}\left[\left|w^{(1)}_{ij}u^2\right|^2 + \dfrac{m_j^2}{m_i^2}\left|w^{(2)}_{ij}u\right|^2\right] & \text{[restricted class of models]} \end{cases}$$

[$w^{(1,2)}_{ij}$ are known O(1) functions of SUSY parameters]

$$BR(\mu \to e\gamma) \approx BR(\tau \to \mu\gamma) \approx BR(\tau \to e\gamma)$$

independently from $u \approx \vartheta_{13}$

$\tau \to \mu\gamma$ and $\tau \to e\gamma$ below future experimental sensitivity

relatively light sparticle spectrum still allowed

## 2. further contributions to slepton mass matrices if ν masses come from type I see-saw [ss], through RGE running

if $G_f = A_4, S_4, A_5$

$$\left(\delta^{ss}_{\mu e}\right)_{LL} = -\frac{\left(3+a_0^2\right)y^2}{8\pi^2} U_{\mu 2} U^*_{e2} \log\frac{m_2}{m_1} + O(u)$$

$$\left(\delta^{ss}_{\tau e}\right)_{LL} = -\frac{\left(3+a_0^2\right)y^2}{8\pi^2} U_{\tau 2} U^*_{e2} \log\frac{m_2}{m_1} + O(u)$$

$$\left(\delta^{ss}_{\tau\mu}\right)_{LL} = -\frac{\left(3+a_0^2\right)y^2}{8\pi^2} \left[U_{\tau 2} U^*_{\mu 2} \log\frac{m_2}{m_1} + U_{\tau 3} U^*_{\mu 3} \log\frac{m_3}{m_1}\right] + O(u)$$

at LO they only depend on the smallest $m_i$
at variance with the general case
[18 ss parameters]

$$\left[Y_\nu^+ \log\left(\frac{M_X^2}{MM^+}\right) Y_\nu\right]_{ij}$$

Example: $A_4$ x SUSY+ see-saw [Hagedorn, Molinaro, Petcov 0911.3605]

Normal Ordering $BR(\mu \to e\gamma) \approx BR(\tau \to e\gamma) \approx O(10^{-1}) BR(\tau \to \mu\gamma)$

$\left(\delta^{ss}_{\mu e}\right)_{LL} \approx 10^{-2}$ ⇒

-- tanβ small
-- relatively heavy sparticles
-- μ-> eγ close to the present bound

Inverted Ordering $BR(\mu \to e\gamma) \approx BR(\tau \to e\gamma) << BR(\tau \to \mu\gamma)$

yet $R_{\tau\mu}$ above $10^{-9}$ practically excluded

observation of τ->μγ [$R_{\tau\mu} > 10^{-9}$ ] rules out the $A_4$ x SUSY model

# Tribimaximal Mixing

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + (\text{small corrections})$$

by unitarity

tri-bimaximal [TB] mixing pattern, completely different from the quark mixing pattern: two angles are large

$$\nu_3 = \frac{-\nu_\mu + \nu_\tau}{\sqrt{2}}$$ maximal

$$\nu_2 = \frac{\nu_e + \nu_\mu + \nu_\tau}{\sqrt{3}}$$ trimaximal

- can be a useful 1st order approximation to data, related to some limit of the underlying theory before T2K this approximation was very good

| | | | |
|---|---|---|---|
| $\sin^2\vartheta_{13}^{TB} = 0$ | $0.014^{+0.009}_{-0.008}$ | $0.010^{+0.009}_{-0.006}$ | [NO] |
| | | $0.013^{+0.009}_{-0.007}$ | [IO] |
| $\sin^2\vartheta_{23}^{TB} = \frac{1}{2}$ | $0.42^{+0.09}_{-0.04}$ | $0.51 \pm 0.06$ | [NO] |
| | | $0.52 \pm 0.06$ | [IO] |
| $\sin^2\vartheta_{12}^{TB} = \frac{1}{3}$ | $0.307^{+0.018}_{-0.016}$ | $0.312^{+0.017}_{-0.015}$ | |

experimental error on $\vartheta_{12}$ [$1\sigma$] is 0.02 rad <-> 1 degree
TB prediction for $\vartheta_{12}$ agrees within 1.5 $\sigma$
same for the other angles

# example

$$G_\nu = Z_2 \times Z_2$$

the most general group leaving $\nu^T m_\nu \nu$ invariant, if $\vartheta_{ij}$ do not depend on $m_i$

$$A = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$A^2 = B^2 = 1 \qquad [A,B] = 0$$

$$G_e = Z_3$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{i\frac{2\pi}{3}}$$

$$C^3 = 1$$

clearly $G_\nu$ and $G_e$ do not commute

$$A^T m_\nu A = m_\nu \qquad B^T m_\nu B = m_\nu \quad \rightarrow \quad U_{TB}^T m_\nu U_{TB} = m_\nu^{diag}$$

$$C^+ (m_e^+ m_e) C = (m_e^+ m_e) \quad \rightarrow \quad (m_e^+ m_e) = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}$$

$$\Rightarrow \quad U_{PMNS} = U_{TB}$$

# A,B and C generate the group $S_4$ [A and C generate the group $A_4$]

- complete models based on these symmetry groups have been constructed

[review: Guido Altarelli and F.F. hep-ph/1002.0211]

-- choice of matter representation: $l \sim 3$,...

-- symmetry breaking sector: "flavons"

$$\varphi = \begin{cases} \varphi_e & \text{couples to charged lepton sector} \\ \varphi_\nu & \text{couples to neutrinos} \end{cases}$$

at the LO

-- minimization of the energy density $V(\varphi_l,\varphi_\nu)$

$\langle \varphi_\nu \rangle$ preserving $G_\nu$

$\langle \varphi_e \rangle$ preserving $G_e$

at the LO

-- additional fields and symmetries often required to accomplish the previous step (and to explain why $m_e \ll m_\mu \ll m_\tau$)

-- predictions: constraints on neutrino masses, LFV, spectrum of SUSY particles in SUSY realizations...

- general feature

$$U_{PMNS} = U^0_{PMNS} + O(u) \qquad u \equiv \frac{\langle \varphi \rangle}{\Lambda} < 1$$

LO result gets corrected in the full theory

$$\vartheta_{ij} = \vartheta^0_{ij} + O(u)$$

- we expect $\vartheta_{13}$ to be of order u < few percent [not to spoil the agreement with $\vartheta_{12}$]

# T2K [see talk by Justyna Łogoda]

this year [1106.2822]

muon neutrino beam produced
at JPARC [Tokai]
E=0.6 GeV and sent to
SK 295 Km apart [T2K]

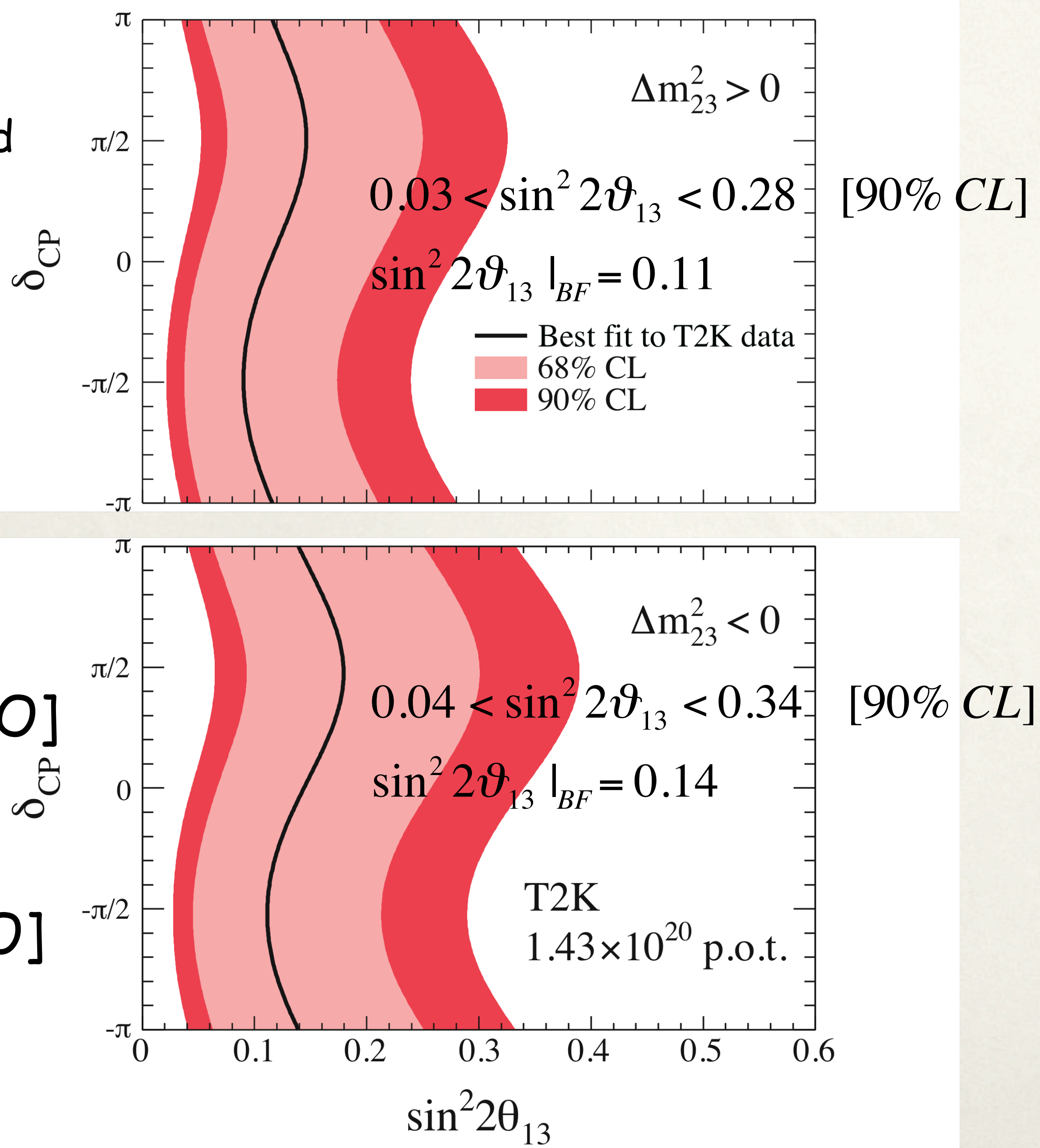
6 electron neutrino events
seen [1.5 expected]
2.5 sigma away from $\theta_{13}$=0

$\vartheta_{13}\big|_{BF} \approx 0.17$

$\vartheta_{13} > 0.09 \ \ [90\% CL]$ [NO]

$\vartheta_{13}\big|_{BF} \approx 0.19$

$\vartheta_{13} > 0.10 \ \ [90\% CL]$ [IO]



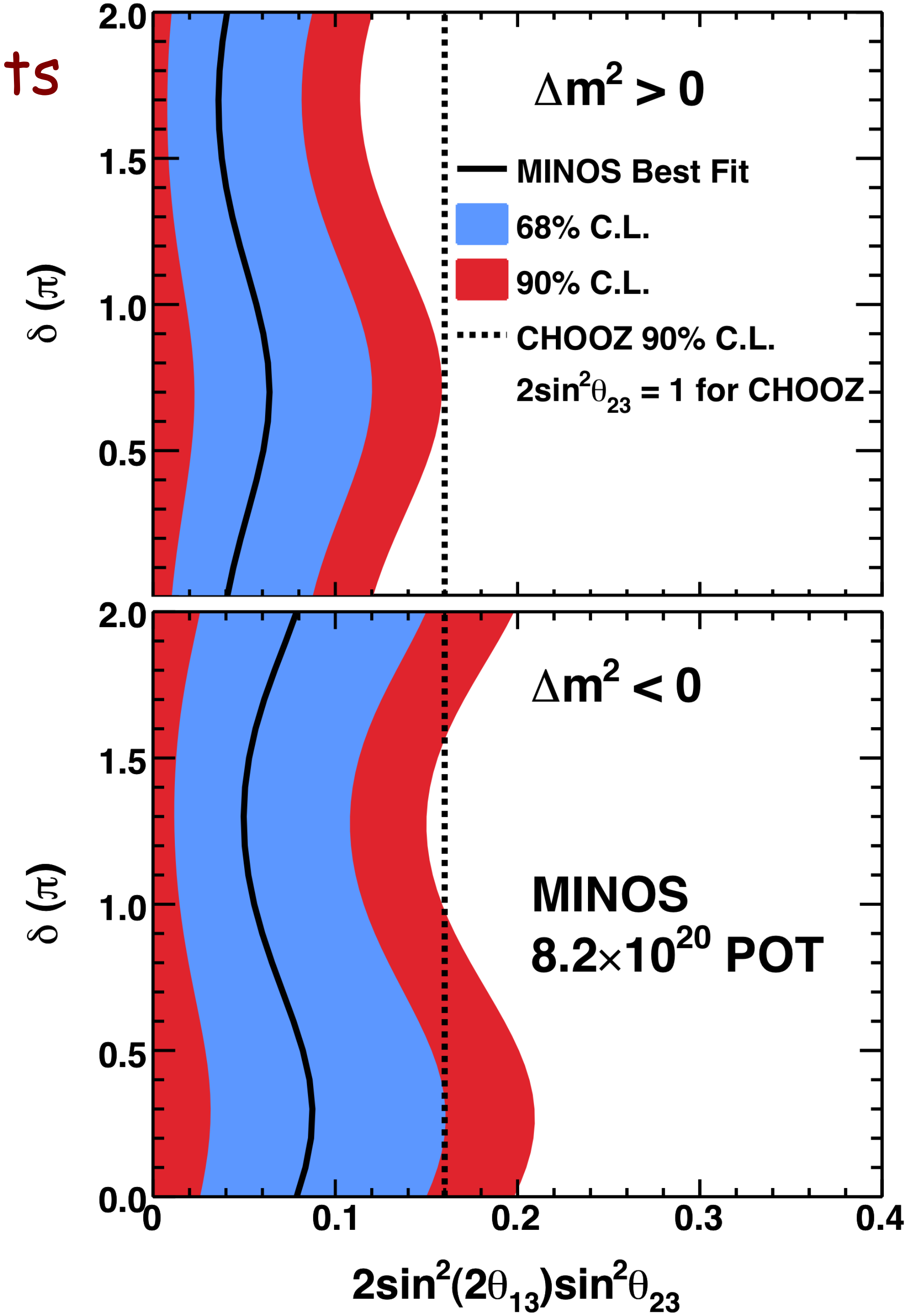
# compatible with MINOS results

[1108.0015]

muon neutrino beam produced at Fermilab [E=3 GeV] sent to Soudan Lab 735 Km apart

$\theta_{13} > 0$ at 89% CL

$$P(\nu_\mu \to \nu_e) \approx \sin^2\vartheta_{23} \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m^2_{32} L}{4E}$$



## impact on model building

- TB mixing still a good 1st order approximation, corrected by some rotation ~ 0.1 rad, coming from the neutrino sector or the charged lepton sector and leaving one row or one column of the mixing matrix unchanged
[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Morisi, Patel, Peinado 2011, Antusch, King, Luhn, Spinrath 2011]

- in previous example ($A_4$) leading to TB, the symmetry related to the B matrix receives a sizeable correction [Ma, Wegman 1106.4269, King, Luhn 2011]

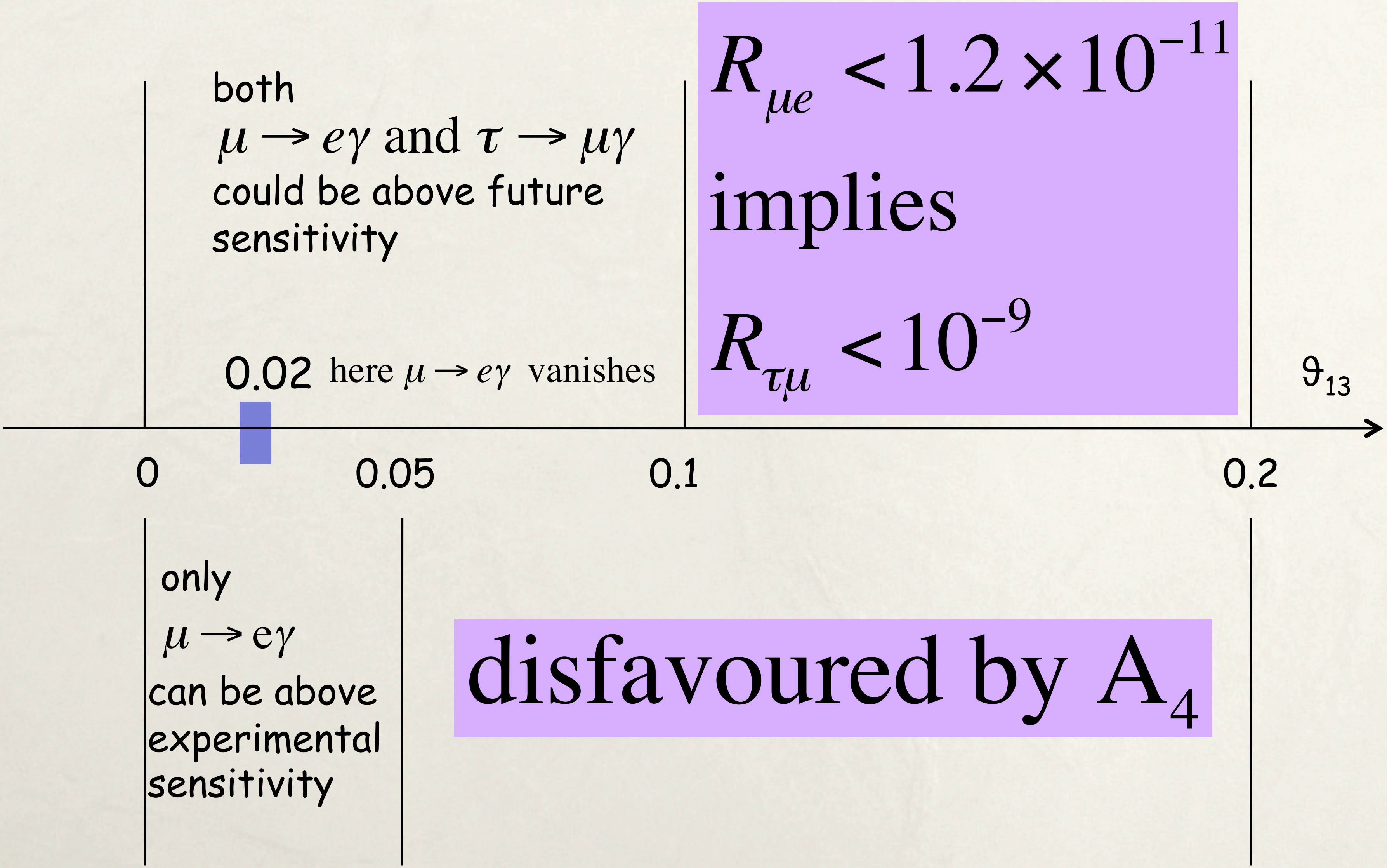
- different symmetry breaking parameters for the charged sector and for the neutrino sector [Lin 2009]

- different LO approximation, for instance a bimaximal mixing where the solar angle is $\pi/4$ and $\theta_{13}=0$, at the leading order. Corrections from the charged lepton sector bring the solar angle into agreement and generate a non-vanishing $\theta_{13}$ [Altarelli, F, Merlo 2009, Bazzocchi 2011, Meloni 2011]

- discrete (and perhaps any) flavour symmetries simply not relevant angles are random variables: anarchy [Hall, Murayama, Weiner 1999]

## MFV

[D'Ambrosio, Giudice, Isidori, Strumia 2002
Cirigliano, Grinstein, Isidori, Wise 2005]




## SUSY×$A_4$

# TESTS [III] slepton mass spectrum

in most of these constructions

$l \sim 3 \qquad e^c, \mu^c, \tau^c \sim$ singlets $\qquad$ of $G_f$

$$\left(m_{\tilde{l}}^2\right)_{LL} = \text{diag}(n,n,n)m_0^2 + O(u)$$
$$\left(m_{\tilde{l}}^2\right)_{RR} = \text{diag}(n_1^c,n_2^c,n_3^c)m_0^2 + O(u)$$

[n,$n_i^c$ are O(1) numbers]

endpoint of dilepton invariant mass distribution in $\chi_2^0 \to \chi_1^0 l^+ l^-$
can be measured at LHC with a precision <O($10^{-2}$) for l=e,μ

if $m_{\chi_2^0} > m_{\tilde{l}} > m_{\chi_1^0}$

$$\chi_2^0 \to \tilde{l} l \to \chi_1^0 l^+ l^-$$

endpoint $m_{ll}^2 = \dfrac{\left(m_{\chi_2^0}^2 - m_{\tilde{l}}^2\right)\left(m_{\tilde{l}}^2 - m_{\chi_2^0}^2\right)}{m_{\tilde{l}}^2}$

$$\frac{\left(m_{\mu\mu}^2 - m_{ee}^2\right)}{m_{\mu\mu}^2} = C\frac{\left(m_{\tilde{\mu}}^2 - m_{\tilde{e}}^2\right)}{m_{\tilde{e}}^2} = \begin{cases} O(u) & \text{[Left } \tilde{l}\text{]} \\ O(1) & \text{[Right } \tilde{l}\text{]} \end{cases}$$

| | Lisi [Neutel2011] [0806.22517update] | Schwetz et al. [1103.0734] |
|---|---|---|
| $\sin^2\vartheta_{12}$ | $0.307^{+0.018}_{-0.016}$ | $0.312^{+0.017}_{-0.015}$ |
| $\sin^2\vartheta_{23}$ | $0.42^{+0.09}_{-0.04}$ | $0.51\pm0.06$ [NO]<br>$0.52\pm0.06$ [IO] |
| $\sin^2\vartheta_{13}$ | $0.014^{+0.009}_{-0.008}$ | $0.010^{+0.009}_{-0.006}$ [NO]<br>$0.013^{+0.009}_{-0.007}$ [IO] |
| $\Delta m^2_{21}\ (eV^2)$ | $(7.54^{+0.25}_{-0.22})\times10^{-5}$ | $(7.59^{+0.20}_{-0.18})\times10^{-5}$ |
| $\left\|\Delta m^2_{31}\ (eV^2)\right\|$ | $(2.36^{+0.12}_{-0.10})\times10^{-3}$ | $(2.45\pm0.09)\times10^{-3}$ [NO]<br>$(2.34^{+0.10}_{-0.09})\times10^{-3}$ [IO] |

- many explanations of the observed mixing angles

[Hall, Murayama, Weiner 1999]

1. no special pattern behind the data, just structure-less O(1) parameters
2. accidental enhancement (cancellation) of $O(\lambda_C)$ contributions
3. accidental enhancement from RGE evolution
4. fit to (a restricted # of) parameters in a SO(10) GUT theory
5. strong or weak Quark-Lepton Complementarity from some dynamical principle
6. ...

[Smirnov;Raidal;Minakata and Smirnov 2004]

- each of them depends on what is considered relevant

Example: $\vartheta_{23} = \frac{\pi}{4} + \text{small corrections}$ not relevant for 1, 2,3, 4

- this talk: there is a limit of the underlying theory where lepton mixing angles become simple [e.g.$V_{CKM}$=1 when $\lambda_C$ is sent to zero]

## Majorana neutrinos ➡ $G_\nu$ discrete

- the most general group leaving $\nu^T m_\nu \nu$ invariant, if $\vartheta_{ij}$ do not depend on $m_i$

$$G_\nu = Z_2 \times Z_2$$

[go to the basis where $m_\nu$ is diagonal: neutrinos can only change by a sign]

- $G_e$ can be continuous but the simplest choice is $G_e$ discrete

$$G_e = \begin{cases} Z_2 \times Z_2 \\ Z_n \quad n \geq 3 \end{cases}$$

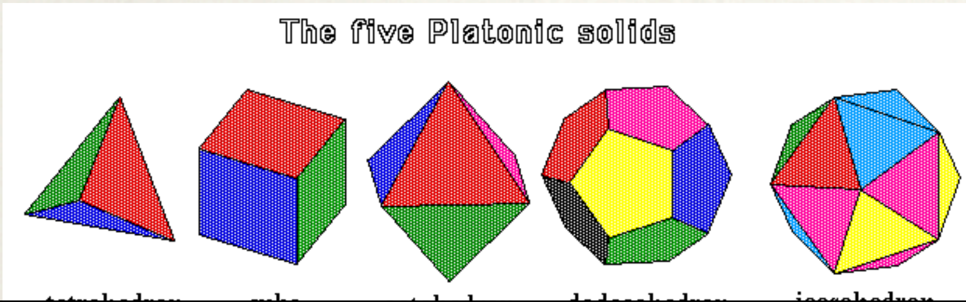
- small groups $G_f$ with 3 dimensional irreps [for l], containing $Z_2$x$Z_2$ and $Z_n$ subgroups. Consider the series defined by

$$S^2 = (ST)^3 = T^n = 1$$

[S and T are the generators of $G_f$]

n=0 -> modular group, infinite
n=1,2 -> no 3 dimensional irreps
n≥6 -> infinite groups

we are left with n=3,4,5




| duality | | group | order | n |
|---|---|---|---|---|
| tetrahedron | tetrahedron | $A_4$ | 12 | **3** |
| cube | octahedron | $S_4$ | 24 | **4** |
| dodecahedron | icosahedron | $A_5$ | 60 | **5** |