Yukawa hierarchies from spontaneous flavor symmetry breaking

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[E. Nardi, PRD**84**, 036008 (2011)]

[J.R. Espinosa, CSF, E. Nardi, To appear]

Outline

1. Motivation: The origin of the Yukawa hierarchies ?
2. Yukawa hierarchies from SSB
   - First attempt: Via the one-loop effective potential
   - Second attempt: Via reducible representations
3. Conclusions and on-going work

Motivation: The origin of the Yukawa hierarchies ?

In the *Standard Model* (SM), the $q$uark and ℓepton gauge invariant kinetic terms possess the global symmetry [Chivukula, Georgi (1987)]

$$\begin{aligned}\mathcal{G} &= \mathcal{G}^q \times \mathcal{G}^\ell \\ \mathcal{G}^q &= U(3)_Q \times U(3)_u \times U(3)_d \\ \mathcal{G}^\ell &= U(3)_\ell \times U(3)_e\end{aligned}$$

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Yukawa couplings break explicitly the $\mathcal{G} \to U(1)_B \times U(1)_Y^2$.

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- NO. They are different at the fundamental level but the replication is just an illusion of the low energy theory e.g. Froggatt-Nielsen mechanism [Froggatt, Nielsen (1979)], Randall-Sundrum warped extra-dimensional model (1999)

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- YES. They are exact replica at the fundamental level, but the flavor symmetry is broken spontaneously (SSB) at low energy.

  [Koide (2008), (2009)], [Koide, Nishiura (2012)], [Feldmann, Jung, Mannel (2009)], [Albrecht, Feldmann, Mannel (2010)], [Grinstein, Redi, Villadoro (2010)], [Alonso, Gavela, Merlo, Rigolin (2011)], [Nardi (2011)]

Yukawa hierarchies from SSB: The $T, A, \mathcal{D}$ invariants

$$\mathcal{G}_{\mathcal{F}} = SU(3)_L \times SU(3)_R$$

$$-\mathcal{L}_Y = \frac{1}{\Lambda}\overline{\psi_L} Y \psi_R H$$

$$\psi_L = (3,1), \qquad \psi_R = (1,3), \qquad Y = (3,\bar{3})$$

We have $T, A, \mathcal{D}$ Invariants [Alonso et. al. (2011)], [Nardi (2011)]

$$\det(\xi I_{3\times 3} - YY^\dagger) = \xi^3 - T\xi^2 + A\xi - D^2 = 0$$

$$\begin{aligned} T &= \mathrm{Tr}(YY^\dagger), \\ A &= \mathrm{Tr}[\mathrm{Adj}(YY^\dagger)] = \frac{1}{2}\left[T^2 - \mathrm{Tr}(YY^\dagger YY^\dagger)\right], \\ \mathcal{D} &= \det(Y) = e^{i\delta} D \end{aligned}$$

Hierarchy $\implies \langle D\rangle^{1/3} \ll \langle A\rangle^{1/4} \ll \langle T\rangle^{1/2}$

Yukawa hierarchies from SSB: The scalar potential

The most general renormalizable potential for Y invariant under $SU(3)_L \times SU(3)_R$

$$
\begin{aligned}
\frac{\hat{V}_0}{\Lambda^4} &= V_0 = V_T + V_A + V_D, \\
V_T &= \lambda \left(T - \frac{m^2}{2\lambda} \right)^2 = \lambda \left(T - v^2 \right)^2, \\
V_A &= \lambda' A, \\
V_{\mathcal{D}} &= \tilde{\mu} \mathcal{D} + \tilde{\mu}^* \mathcal{D}^* \equiv 2\mu D \cos \phi_{\mathcal{D}}
\end{aligned}
$$

The most general background field

$$
\langle Y \rangle = \frac{1}{\sqrt{2}} \mathrm{diag}(R_{11}, R_{22}, R_{33} + iJ_{33})
$$

At the minimum

$$
\begin{aligned}
V_{\mathcal{D}}^{\min} &= V_D = -2\mu D, \\
\langle Y \rangle &= \frac{1}{\sqrt{2}} \mathrm{diag}(R_{11}, R_{22}, R_{33})
\end{aligned}
$$

Yukawa hierarchies from SSB: The tree-level vacua

[Alonso et. al. (2011)], [Nardi (2011)]

$$T = \frac{1}{2}\left(R_{11}^2 + R_{22}^2 + R_{33}^2\right), \qquad A = \frac{1}{4}\left(R_{11}^2 R_{22}^2 + R_{11}^2 R_{33}^2 + R_{22}^2 R_{33}^2\right), \qquad D = \frac{1}{\sqrt{2}} R_{11} R_{22} R_{33}.$$

At tree-level

(i) For $\lambda' < 0$, we have $\langle Y \rangle^s = \frac{1}{\sqrt{3}} v\, \mathrm{diag}(1,1,1)$
$\implies \langle D \rangle^{1/3} \approx \langle A \rangle^{1/4} \approx \langle T \rangle^{1/2}$

$$\mathcal{G}_{\mathcal{F}} \to H_s = SU(3)_{L+R}$$

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(ii) For $\lambda' > 0$, we have $\langle Y \rangle^h = v\, \text{diag}(0,0,1)$ as long as [Nardi (2011)]

$$V_0(\langle Y \rangle^s) > 0 \implies \frac{\mu^2}{m^2} < 2\lambda \left[\left(4 + \frac{\lambda'}{\lambda}\right)^{3/2} - \left(8 + 3\frac{\lambda'}{\lambda}\right)\right]$$

Then $\langle T \rangle \approx 1$ and $\langle D \rangle = \langle A \rangle = 0$.

$$\mathcal{G}_\mathcal{F} \to H_h = SU(2)_L \times SU(2)_R \times U(1)_{L+R}$$

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The question: Is it possible to lift the zeros? $\langle Y \rangle^h = v \,\mathrm{diag}(0,0,1) \longrightarrow v \,\mathrm{diag}(\epsilon', \epsilon, 1)$

First attempt: Via the one-loop effective potential

It was hypothesized that loop corrections to the effective potential could yield a structure $\langle Y \rangle = v\,\mathrm{diag}(\epsilon', \epsilon, 1)$ i.e. $\mathcal{G}_{\mathcal{F}} \to U(1)^2_{L+R}$ [Nardi (2011)]

$\implies$ Michel's conjecture [Michel (1979)] states that the maximal little groups H_s and H_h are the maximal stability groups of the most general 4-th order function of the invariants. Is this true also at the loop-level?

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Including the $SU(3)_L \times SU(3)_R$ invariant one-loop Coleman Weinberg effective potential [Coleman, Weinberg (1973)], [Jackiw (1974)] we have

$$\begin{aligned} V_{\text{eff}} &= V_0 + V_1, \\ V_1 &= \frac{1}{64\pi^2}\sum_i M_i^4(Y)\left[\log\frac{M_i^2(Y)}{\Lambda^2} - \frac{3}{2}\right] \end{aligned}$$

with $M_i^2(Y)$ the eigenvalues of

$$[\mathcal{M}]_{ij,kl} = \left.\frac{\partial^2 V_0}{\partial\mathcal{Y}_{ij}\partial\mathcal{Y}_{kl}}\right|_{\langle Y\rangle}, \qquad \mathcal{Y}_{ij} = \{\mathrm{Re}(Y_{ij}), \mathrm{Im}(Y_{ij})\}$$

First attempt: Via the one-loop effective potential

(A) Brute force verification:
We determined the analytical expressions for the eigenvalues
(18th-order polynomial equation! But we somehow managed ...)

$$\det(M^2 \cdot I_{18\times 18} - \mathcal{M}^2) = P^{(6)}(M^2) \times \Pi_{i=1}^{3}(M^2 - M_{i+}^2)^2(M^2 - M_{i-}^2)^2 = 0$$

Numerically minimized the effective potential and found that vacuum structure remains separated into two: $\langle Y\rangle^s \sim \mathrm{diag}(1,1,1)$ and $\langle Y\rangle^h \sim \mathrm{diag}(0,0,1)$ [Epsinosa, CSF, Nardi, To appear]

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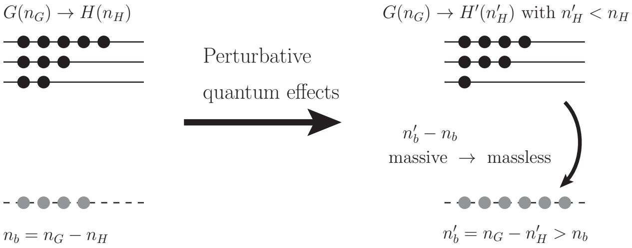
(B) A heuristic argument:



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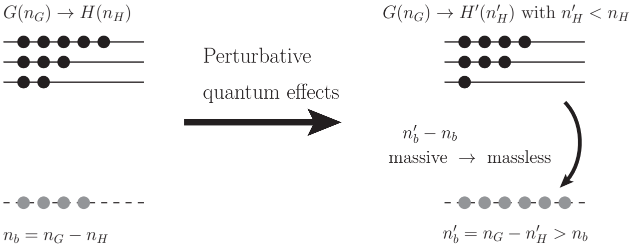

First proof given in app. of [Georgi, Glashow (1972)]. More rigorous proof in [Georgi, Pais (1977)].
$\Longrightarrow$ **cannot** change the tree-level vacuum through perturbative quantum effects
$\Longrightarrow$ in agreement with Michel's conjecture.

Second attempt: Via reducible representations

We need to break $\mathcal{G}_{\mathcal{F}} \to U(1)^2_{L+R}$ already at tree-level. A minimal enlargement of the scalar sector by introducing

$$Z_L = (3,1), \qquad Z_R = (1,3)$$

The most general renormalizable $SU(3)_L \times SU(3)_R$ invariant scalar potential

$$V = \lambda' A + \tilde{\mu}\mathcal{D} + \tilde{\mu}^* \mathcal{D}^* + V_l + V_m + V_\nu,$$

where

$$\begin{aligned} V_l \quad = \quad & \lambda\left(T - v^2\right)^2 + \lambda_L\left(|Z_L|^2 - v_L^2\right)^2 + \lambda_R\left(|Z_R|^2 - v_R^2\right)^2 \\ & + g\left[\left(T - v^2\right) + \frac{g_{1L}}{g}\left(|Z_L|^2 - v_L^2\right) + \frac{g_{1R}}{g}\left(|Z_R|^2 - v_R^2\right)\right]^2, \end{aligned}$$

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$$V_m = g_{2L}\, Z_L^\dagger YY^\dagger Z_L + g_{2R}\, Z_R^\dagger Y^\dagger Y Z_R$$

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$$V_{\tilde{\nu}} = \tilde{\nu}\, Z_L^\dagger Y Z_R + \tilde{\nu}^*\, Z_R^\dagger Y^\dagger Z_L \equiv 2\nu\left|Z_L^\dagger Y Z_R\right| \cos\phi_{LR}$$

Second attempt: Via reducible representations

Consider only the case $g_{2L}, g_{2R}, \lambda' > 0$.
Since V_l fixes the 'length', we only have to consider

$$V_\epsilon = \lambda' A - 2\mu D + g_{2L} Z_L^\dagger Y Y^\dagger Z_L + g_{2R} Z_R^\dagger Y^\dagger Y Z_R - 2\nu \left| Z_L^\dagger Y Z_R \right|$$

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Let us take the new vacua to be

$$\begin{aligned}
\langle Y \rangle &= v\,\mathrm{diag}(\epsilon', \epsilon, y) \quad \text{with } \epsilon', \epsilon \ll y, \quad \epsilon'^2 + \epsilon^2 + y^2 = 1 \\
\langle Z_L \rangle &= v_L\,(z_L, \epsilon_L, \epsilon'_R) \quad \text{with } \epsilon'_L, \epsilon_L \ll z_L, \quad \epsilon_L'^2 + \epsilon_L^2 + z_L^2 = 1 \\
\langle Z_R \rangle &= v_R\,(z_R, \epsilon_R, \epsilon'_R) \quad \text{with } \epsilon'_R, \epsilon_R \ll z_R, \quad \epsilon_R'^2 + \epsilon_R^2 + z_R^2 = 1
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For simplicity setting $v = v_L = v_R = 1$ and $g_{2L} = g_{2R} = \lambda'$, then solving $\frac{\partial V_\epsilon}{\partial \epsilon} = ... = 0$ and by truncating to terms $\mathcal{O}(\epsilon^2)$ we obtain a unique global minimum
[Epsinosa, CSF, Nardi, To appear]

$$\epsilon' = \frac{\lambda' \nu}{3\lambda'^2 - 2\mu^2}, \qquad \epsilon = \sqrt{2}\frac{\mu}{\lambda'}\epsilon', \qquad \epsilon_{L,R} = \epsilon'_{L,R} = 0$$

Second attempt: Via reducible representations

The new vacua

$$\langle Y \rangle \simeq v\,\mathrm{diag}(\epsilon', \epsilon, 1)$$
$$\langle Z_L \rangle \simeq v_L\,(1,0,0)$$
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$$-\mathcal{L}_Y = \frac{1}{\Lambda}\overline{\psi}_L Y \psi_R H + \frac{1}{\Lambda^2}\overline{\psi}_L Z_L Z_R^\dagger \psi_R H$$

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For example for $\nu \sim \mu \sim 10^{-2}\lambda'$, we have $\epsilon' \sim 10^{-2}$ and $\epsilon \sim 10^{-4}$.

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The symmetry of the new vacua is

$$\begin{aligned} H_{LR} &= SU(2)_L \times SU(2)_R, \\ H_Y &= U(1)^2_{L+R}, \\ H' &= H_{LR} \cap H_Y = U(1)_{L+R} \end{aligned}$$

Conclusions and on-going work

Summary:

- Consider the most general $SU(3)_L \times SU(3)_R$ invariant scalar potential of Y, we have two possible tree-level vacua: $\langle Y \rangle^s \sim \mathrm{diag}(1,1,1)$ and $\langle Y \rangle^h \sim \mathrm{diag}(0,0,1)$
- Yukawa hierarchies $\langle Y \rangle \sim \mathrm{diag}(\epsilon', \epsilon, 1)$ cannot be obtained through quantum corrections to V_0 but the flavour symmetry has to be broken at tree-level via reducible representations with $Y = (3, \bar{3}),\ Z_L = (3,1),\ Z_R = (1,3)$: $SU(3)_L \times SU(3)_R \to U(1)_{L+R}$

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Other considerations:

- Other interesting possibility: consider a scalar potential with additional Nambu-Goldstone massless fields in the tree-level vacuum from accidental symmetry e.g. $V_0(\lambda', \mu \to 0) = \lambda(T - v^2)^2$ (i.e. $O(18) \longrightarrow O(17)$). Can additional interactions e.g. gauge be able to induce nonzero $\langle D \rangle, \langle A \rangle \neq 0$ at the loop-level?

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- Couple up and down sectors: CKM mixing obtainable? [Alonso et. al. (2011)], [Nardi (2011)]
- Consider lepton sector with 3 right-handed neutrinos: PMNS mixing obtainable?

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- Yukawa hierarchies $\langle Y \rangle \sim \mathrm{diag}(\epsilon', \epsilon, 1)$ cannot be obtained through quantum corrections to V_0 but the flavour symmetry has to be broken at tree-level via reducible representations with $Y = (3, \bar{3}),\ Z_L = (3,1),\ Z_R = (1,3)$: $SU(3)_L \times SU(3)_R \to U(1)_{L+R}$

Other considerations:

- Other interesting possibility: consider a scalar potential with additional Nambu-Goldstone massless fields in the tree-level vacuum from accidental symmetry e.g. $V_0(\lambda', \mu \to 0) = \lambda(T - v^2)^2$ (i.e. $O(18) \longrightarrow O(17)$). Can additional interactions e.g. gauge be able to induce nonzero $\langle D \rangle, \langle A \rangle \neq 0$ at the loop-level?
- Couple up and down sectors: CKM mixing obtainable? [Alonso et. al. (2011)], [Nardi (2011)]
- Consider lepton sector with 3 right-handed neutrinos: PMNS mixing obtainable?
- Gauging the flavour symmetry to get rid of the massless Nambu-Goldstone bosons [Albrecht et. al. (2010)], [Grinstein et. al. (2010)]

Thank you for your attention!

Questions/comments?

Yukawa hierarchies as a function of ν/λ'

$R = \{\epsilon', \epsilon, y\}$ and $\mu = 0.05\lambda'$