On the factorization of the scattering of W bosons

arXiv:1202.1904 **(EWA: cross-section level)**
arXiv:12mm.xxxx **(gEWA: amplitude level (this talk))**

a work in progress
with
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Outline

Motivation

- Factorization in QFT
- EWSB and the scattering of W bosons

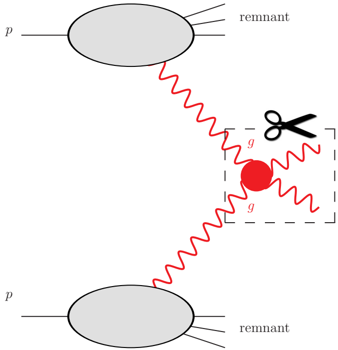
Results: $WW \to WW$ scattering from the process $qq \to qqWW$

- High energy behavior of the WW scattering amplitudes
- corrections to the EWA at the amplitude-level
- EWA and the exact amplitude

Conclusions

- Outlook on WW scattering

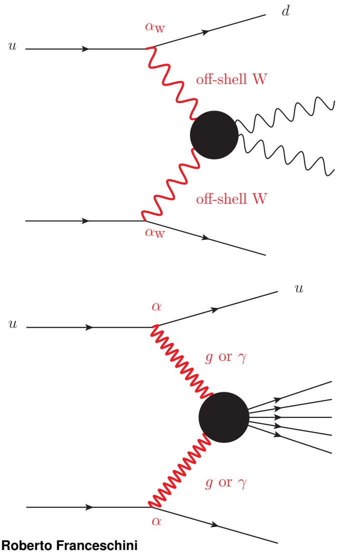
Factorization



Field theory question

How this generalizes to the massive case?

Analogous, but quite different



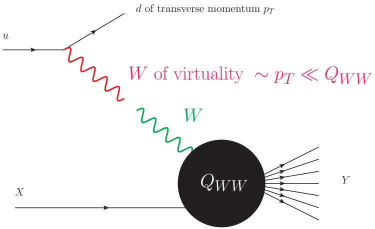
Qualitively different

- the W is never on-shell $p_W^2 = (p_u - p_d)^2 < 0 < m_W^2$
- a third polarization mode, $\epsilon_L \sim \frac{E}{m}$
- the new mass scale m_W

- The energy of LHC is finite
- $\sigma(pp \to WWjj)$ only few fb

... you are asking for a beam of W bosons(!)

- our source of W is $f \to f' W^*$
- $ff \to f' f' W^* W^* \to X_{WW} f' f'$



Effective W Approximation: (Fermi '24, Weizsäcker, Williams '34, Cahn, Chanowitz, Dawson, Gaillard, Kane, Repko, Rolnick '84-'85)

- each W^* has virtuality

$$V = \sqrt{m_W^2 - (p_f - p_{f'})^2} \sim \sqrt{p_T^2 + m_W^2}$$

- $W^* W^* \to X_{WW}$ of virtuality $Q_{WW} \sim E$

$t_{hard} \sim \frac{1}{Q_{WW}} \ll \Delta t_W \sim \frac{1}{\Delta E} \sim \frac{E}{V^2}$

- $V \ll Q_{WW}$

for $ff \to ffWW$

- $p_{T,f'} \ll p_{T,W^{out}}$ and $m_W \ll p_{T,W^{out}}$

that's pure kinematics!

- factorization of a hard (fast) process and a soft (slow) process
- expansion in $V/Q_{WW} \simeq p_{T,jet}/p_{T,W_{out}}$

Why I care so much about processes initiated by W bosons?

Goldstone scattering: is weak or strong?

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma)$$

$$\mathcal{A}(\pi\pi \to \pi\pi) \sim \frac{s}{v^2}$$

- a weakly coupled moderator of the growth of the amplitude at high energy must appear
- the Goldstone bosons are strongly coupled

$WW \to WW$ is a direct probe of the Goldstone sector

- Do the Goldstones experience a strong or a weak force?
- $WW \to WW$ scattering rather than a complicated $qq \to qqWW$ process (in QCD you don't want to go back to the proton!)
- concentrate all our knowledge of the EWSB sector in the form of a detailed measurement of the $WW \to WW$ cross-section

Why do I want to know about the details of this factorization?

Factorization in massive gauge theories

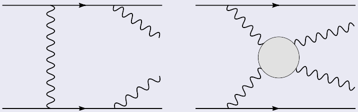
- The same story of the massless case?

Simplicity of understanding the EWSB sector: $|\mathcal{A}_{WW\to WW}(s,t)|^2$ is all that you want

- Ideally our knowledge of the EWSB can be encoded in the behavior of a $2 \to 2$ scattering process $WW \to WW$

Effectiveness and robustness of LHC data analysis

- Where the factorization works best is where the EWSB is more at display, there you can see $WW \to WW$ and nothing else.



Status of the EWA: at cross-section level $\sigma = \int \frac{d\sigma}{d\phi_{jet}} d\phi_{jet}$

surprisingly no complete and clear statement

- $ff \to ffWW$ only for heavy Higgs boson or Higgless (Kunstz,Soper '88)
- $ff \to ff\, h$

that's pure kinematics!

- factorization of a hard (fast) process and a soft (slow) process
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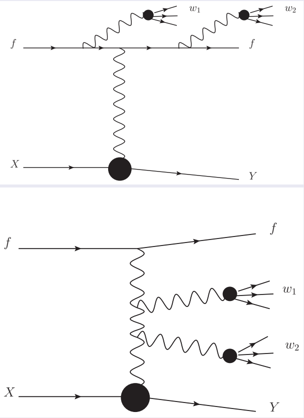
The EWA from the expansion of the exact amplitude

$f\,X \to f\,Y\,WW$: Enhanced diagrams (from dimensional analysis)

scattering ⇔ non-scattering



- reattaching W lines a non-scattering diagram becomes a scattering with the same couplings
- different numbers of fermionic and W propagators, and of g_{WWW} and g_{qqW}

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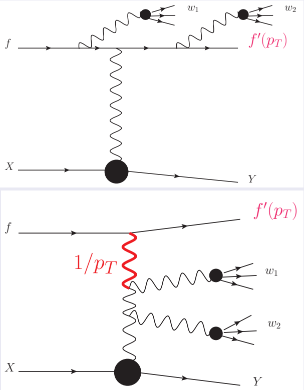

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away from singular regions

- $\mathcal{A}_{\text{non-scattering}} \sim g^v \left(\frac{1}{E}\right)^k$
- $\mathcal{A}_{\text{scattering}} \sim g^v \frac{1}{p_{T,f}} \left(\frac{1}{E}\right)^{k-1} + \ldots$
- gauge invariant kinematical enhancement
- irrespectively of the nature of h and of m_h

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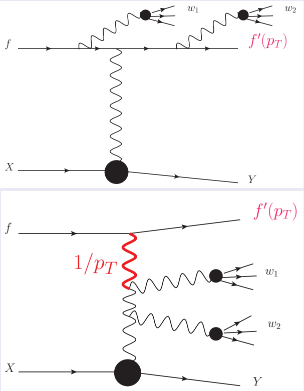

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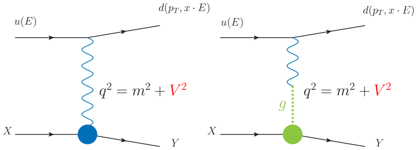
in the EWA region: $p_T \ll Q_{WW} \sim E$

- $\mathcal{A}_{\text{exact}} = \mathcal{A}_{\text{scattering}}(1 + \mathcal{O}(\frac{p_T}{Q_{WW}}))$

subleading terms are expected

- $\mathcal{A}_{\text{scattering}} \supset \frac{\mathcal{A}_{\text{contact-scattering}}}{E}$

Anatomy of a scattering amplitude



$$\mathcal{A}_{\text{scattering}} = \frac{\imath}{V^2}\Bigg(J^\mu \epsilon^*_{T,\mu}\epsilon_{T,\nu}\mathcal{A}^\nu_{Txy}$$

$$+ \quad J^\mu \epsilon^*_{0,\mu}\epsilon_{0,\nu}\mathcal{A}^\nu_{0xy}\frac{1+\frac{q_L^2}{m^2}}{1+\frac{q_L^2}{q^2}}$$

$$+ \quad J^\mu \epsilon^*_{0,\mu}\epsilon_g\mathcal{A}_{gxy}\Bigg)$$

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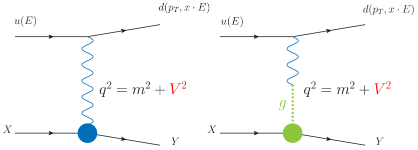

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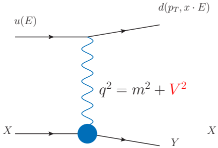

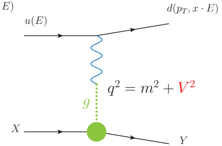

$$\begin{aligned}
\mathcal{A}_{\text{scattering}} &= \frac{\imath}{V^2}\Big(J^\mu \epsilon^*_{T,\mu} \epsilon_{T,\nu} \mathcal{A}^\nu_{Txy} \\
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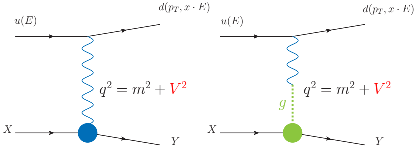
&+ J^\mu \epsilon^*_{0,\mu} \epsilon_g \mathcal{A}_{gxy} \Big) \\
&= \frac{1}{V^2} \left(\mathcal{A}_{\text{scattering-diag}} + \mathcal{A}_{\text{scattering-mix}} \right) \\
&+ \mathcal{A}_{\text{contact-scattering}}
\end{aligned}$$

$\mathcal{A}_{\text{contact-scattering}}$

- it is representative of the size of the non-scattering diagrams
- it is a correction to $\mathcal{A}_{Txy}$ not to $\mathcal{A}_{gxy}$

Surgery on a scattering amplitude



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to make contact with on-shell

$$q_\mu = \left(\sqrt{q^2+|\bar{q}|^2}, \bar{q}\right) \to$$
$$q_\mu = \left(\sqrt{m^2+|\bar{q}|^2}, \bar{q}\right)$$

kinematical corrections

- $\frac{\delta q_0}{q_0} \simeq \frac{V^2}{|\bar{q}|^2} \equiv \kappa^2$
- $\frac{\delta \epsilon_\mu}{\epsilon_\mu}, \frac{\delta J\cdot\epsilon}{J\cdot\epsilon}$ and $\frac{\delta \mathcal{A}}{\mathcal{A}} \sim \kappa^2$

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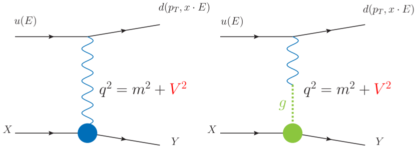

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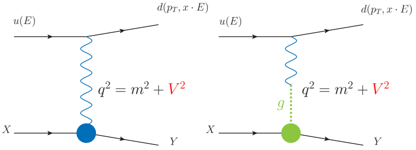

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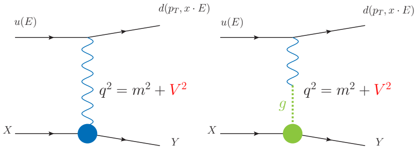

$$\mathcal{A}_{\text{scattering}} = \frac{p_T}{V^2} e^{\pm \imath\phi} g_\pm(x)\epsilon_\pm \cdot \mathcal{A}^{\text{on}}_{\pm xy} + \frac{m}{V^2} g_0(x)\left(\epsilon_g \cdot \mathcal{A}^{\text{on}}_{gxy} + \frac{m}{q_L}\tilde{\epsilon}_0 \cdot \mathcal{A}^{\text{on}}_{0xy}\right) + \frac{1}{q_L} g_0(x)\tilde{\epsilon}_0 \cdot \mathcal{A}^{\text{on}}_{0xy} + \mathcal{O}(\kappa^2)$$

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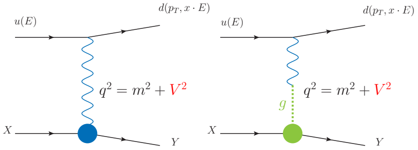

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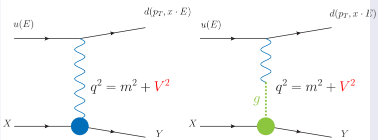
non-scattering corrections

- $\Delta_T \equiv \frac{V}{q_L} \sim \kappa$
- w.r.t $\mathcal{A}_{\text{scattering-diag}}$

Two lessons from the explicit computation

Goldstone bosons and transverse vectors have parametrically different amplitudes (different Feynman rules)



Two sources of error:

- Kinematical (put the internal W on-shell) $\mathcal{O}(\frac{p^2_{T,jet}}{p^2_{T,W_{out}}})$
- Diagrammatical (from the strahlung-like diagrams) $\mathcal{O}(\frac{p_{T,jet}}{p_{T,W_{out}}})$ w.r.t the Transverse scattering

The approximated amplitude

$\mathcal{A}_{EWA} = f_{\pm}(p_T, m, x)\mathcal{A}^{\pm}_{\text{scattering-diag}} + f_0(p_T, m, x)\mathcal{A}_{\text{scattering-mix}}$

- $\mathcal{A}_{exact} = \frac{1}{V^2}\mathcal{A}_{\text{EWA}}\left(1 + \mathcal{O}(\Delta_T) + \mathcal{O}(\kappa^2)\right) + \mathcal{A}_{\text{non-scattering}}$
- $\mathcal{A}_{\text{non-scattering}}$ is comparable to the $\mathcal{O}(\Delta_T)$ correction to $\mathcal{A}_{\text{scattering-diag}}$

$\frac{\mathcal{A}_{\text{scattering-mix}}}{\mathcal{A}_{\text{scattering-diag}}} \equiv \rho$

- ρ depends on the model and on the external states
- in typical cases $\rho \simeq \kappa^{\pm 1}$

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e.g. in the Higgs model: $\Phi = \begin{pmatrix} \pi^\pm \\ v + \frac{h + \imath\pi}{\sqrt{2}} \end{pmatrix}$

- $v \to -v,\ h \to -h,\ \pi \to -\pi,\ \pi^\pm \to -\pi^\pm$ is a symmetry
- $\mathcal{A}(\pi_1^a \ldots \pi_{2k}^b \ldots) \sim v^{2n} \Rightarrow \mathcal{A}(LL \to LL) \sim v^{2k}$
- $\mathcal{A}(\pi_1^a \ldots \pi_{2k+1}^b \ldots) \sim v^{2n+1} \Rightarrow \mathcal{A}(LT \to LL) \sim v^{2k+1}$

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When the exchange of transverse bosons dominates the scattering

- $\mathcal{A}_{exact} = \mathcal{A}_{EWA} + \mathcal{O}(p_{T,jet}/p_{T,W_{out}})$

When the exchange of Goldstone bosons dominates the scattering

- $\mathcal{A}_{exact} = \mathcal{A}_{EWA} + \mathcal{O}(p^2_{T,jet}/p^2_{T,W_{out}})$

Factorization of the amplitude

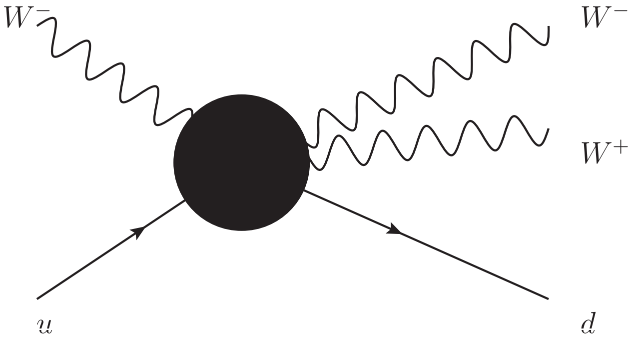
- irrespective of the mass of the Higgs in the limit of a *soft jet emission* compared to the *hard scattering* factorization holds for the **amplitude** (previous statements were about the cross-section $\sigma = \int \frac{d\sigma}{d\phi_{jet}} d\phi_{jet}$)
- $\frac{d\sigma}{d\phi_{jet}}$ now predictable with gEWA
- several sources of corrections have been identified (κ, Δ, ...)

Quantitatively we check the validity of the approximation:

- evaluating the (integratal of) the exact amplitude and the EWA amplitude in fixed points of the phase space to study the behavior of the corrections
- using the approximated $\mathcal{A}_{exact} \simeq \frac{1}{V^2}\mathcal{A}_{EWA}$ to generate LHE events with a parton level MC (http://code.google.com/p/ewangelion) and comparing kinematical distributions to those from the exact amplitude (MadGraph)

Numerical Results

$uW^+_{h_1} \to dW^+_{h_2} W^-_{h_3}$: EWA Amplitude vs. Exact Amplitude (FeynArts+FormCalc)



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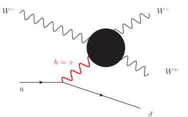
$$A^{(h_1 h_2 h_3)}_{\text{exact}} = f_0 A^{(0)}_{(h_1 h_2 h_3)} + f_+ A^{(+)}_{(h_1 h_2 h_3)} + f_- A^{(-)}_{(h_1 h_2 h_3)} + \textbf{corrections}$$



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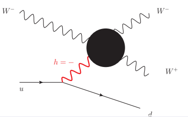

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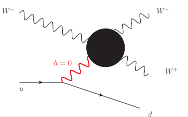

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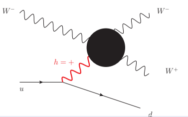

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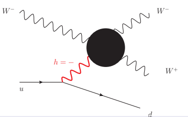

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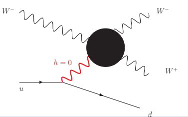
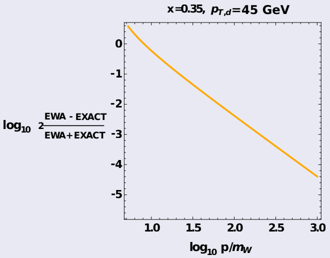


$h_1 = 0, h_2 = 0, h_3 = 0$: 3 longitudinal external states

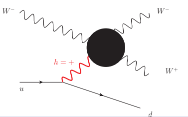
- $f_\pm = \frac{p_T e^{\pm \imath \phi}}{V^2} g_\pm(x)$
- $f_0 = \frac{m}{V^2} g_0(x)$
- $A^{(0)}_{000} = \mathcal{O}(1) + \ldots$
- $A^{(\pm)}_{000} = \frac{v}{E} \mathcal{O}(1) + \ldots$



Agreement in the amplitude at $\mathcal{O}(p_T^2/E^2)$

$uW^+_{h_1} \to dW^+_{h_2} W^-_{h_3}$: EWA Amplitude vs. Exact Amplitude (FeynArts+FormCalc)

$$A^{(h_1 h_2 h_3)}_{\textbf{exact}} = f_0 A^{(0)}_{(h_1 h_2 h_3)} + f_+ A^{(+)}_{(h_1 h_2 h_3)} + f_- A^{(-)}_{(h_1 h_2 h_3)} + \textbf{corrections}$$



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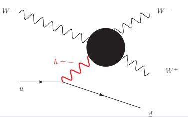

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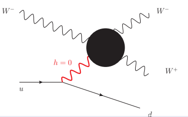
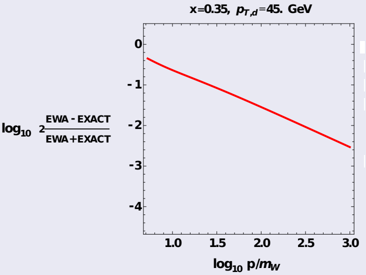


$h_1 = +, h_2 = +, h_3 = +$: 3 transverse external states

- $f_\pm = \frac{p_T e^{\pm \imath \phi}}{V^2} g_\pm(x)$
- $f_0 = \frac{m}{V^2} g_0(x)$
- $A^{(+)}_{000} = \mathcal{O}(1) + \ldots$
- $A^{(0)}_{000} = \frac{v}{E}\mathcal{O}(1) + \ldots$

Agreement in the amplitude at $\mathcal{O}(p_T/E)$



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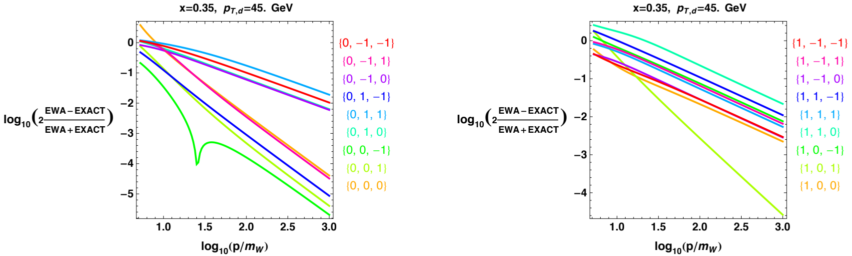

$p_T \ll m$ behavior

effects of the massive propagator

$$\mathcal{A} \sim f_{\pm,0} \frac{1}{p_T^2 + m^2} \mathcal{A}_{hard,\pm 0}$$

$$V = \sqrt{m_W^2 - (p_u - p_d)^2} = \sqrt{m_W^2 + p_T^2/x}$$

$$\begin{aligned} \mathcal{A}_{exact} &= \frac{p_T}{V^2} e^{\pm \imath \phi} g_\pm(x) \epsilon_\pm \cdot \mathcal{A}^{\text{on}}_{\pm xy} \\ &+ \frac{m}{V^2} g_0(x) \left(\epsilon_g \cdot \mathcal{A}^{\text{on}}_{gxy} + \frac{m}{q_L} \tilde{\epsilon}_0 \cdot \mathcal{A}^{\text{on}}_{0xy} \right) \\ &+ \frac{1}{q_L} g_0(x) \tilde{\epsilon}_0 \cdot \mathcal{A}^{\text{on}}_{0xy} + \mathcal{O}(\kappa^2) \end{aligned}$$

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\end{aligned}$$

corrections at $p_{T,d} \ll m$

- if T dominates: $\mathcal{O}(\frac{V^2}{p_T q_L}) \sim \mathcal{O}(\frac{m^2}{p_T q_L})$
- if L dominates: $\mathcal{O}(\frac{\kappa V^2}{m q_L}) \sim \mathcal{O}(\frac{\kappa m}{q_L})$

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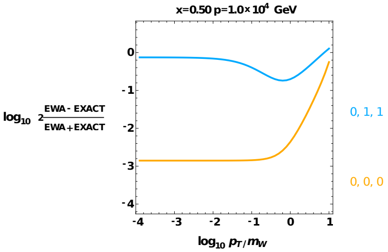

$p_T \ll m$ behavior

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- if T dominates: $\mathcal{O}(\frac{V^2}{p_T q_L}) \sim \mathcal{O}(\frac{m^2}{p_T q_L})$
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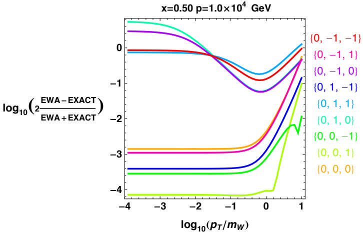
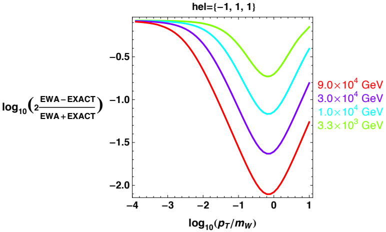


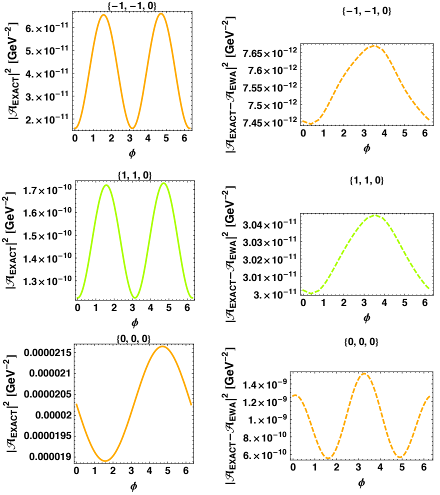


$\frac{d\sigma}{d\phi_{jet}}$ from $A^{(h_1 h_2 h_3)}_{\mathrm{exact}} = f_0 A^{(0)}_{(h_1 h_2 h_3)} + f_+ A^{(+)}_{(h_1 h_2 h_3)} + f_- A^{(-)}_{(h_1 h_2 h_3)} + \text{corrections}$ (PRELIMINARY)

Amplitude means interference

- $f_0 = \frac{m}{V^2} g_0(x)$
- $f_\pm = \frac{p_T e^{\pm \imath \phi}}{V^2} g_\pm(x)$

$\mathcal{A}_{h\lambda(W^-_{in})\to\lambda(W^+)\lambda(W^-)}$			$\lambda(W^-_{in})$	$\lambda(W^+)$	$\lambda(W^-)$	$d\sigma/d\phi$
$h=0$	$h=-1$	$h=1$				
1	ϵ	ϵ	0	0	0	$1+\epsilon\,[\sin\phi+\Delta\cdot f(\phi)]$
ϵ	ϵ^2	ϵ^2	0	0	1	$1+\epsilon\,[\sin\phi+\Delta\cdot f(\phi)]$
ϵ	ϵ^2	ϵ^2	0	0	-1	$1+\epsilon\,[\sin\phi+\Delta\cdot f(\phi)]$
ϵ	1	ϵ^2	0	1	0	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ^2	ϵ	ϵ^3	0	1	1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
1	ϵ	ϵ	0	1	-1	$1+\epsilon\,[\sin\phi+\Delta\cdot f(\phi)]$
ϵ	ϵ^2	1	0	-1	0	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
1	ϵ	ϵ	0	-1	1	$1+\epsilon\,[\sin\phi+\Delta\cdot f(\phi)]$
ϵ^2	ϵ^3	ϵ	0	-1	-1	$1+\sin\phi+\Delta\cdot f(\phi)$
ϵ	ϵ^2	1	1	0	0	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
1	ϵ	ϵ	1	0	1	$1+\epsilon\,[\sin\phi+\Delta\cdot f(\phi)]$
ϵ^2	ϵ^3	ϵ	1	0	-1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ^2	ϵ	ϵ	1	1	0	$1+\sin 2\phi+\Delta f(\phi)+\epsilon\sin\phi$
ϵ	1	ϵ^2	1	1	1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ	ϵ^2	1	1	1	-1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ^2	ϵ^3	ϵ	1	-1	0	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ	ϵ^2	1	1	-1	1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ^3	ϵ^4	ϵ^2	1	-1	-1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ	1	ϵ^2	-1	0	0	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ^2	ϵ	ϵ^3	-1	0	1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
1	ϵ	ϵ	-1	0	-1	$1+\epsilon\,[\sin\phi+\Delta\cdot f(\phi)]$
ϵ^2	ϵ	ϵ^3	-1	1	0	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ^3	ϵ^2	ϵ^4	-1	1	1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ	1	ϵ^2	-1	1	-1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ^2	ϵ	ϵ	-1	-1	0	$1+\sin 2\phi+\Delta f(\phi)+\epsilon\sin\phi$
ϵ	1	ϵ^2	-1	-1	1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$
ϵ	ϵ^2	1	-1	-1	-1	$1+\epsilon\sin\phi+\Delta\cdot f(\phi)$

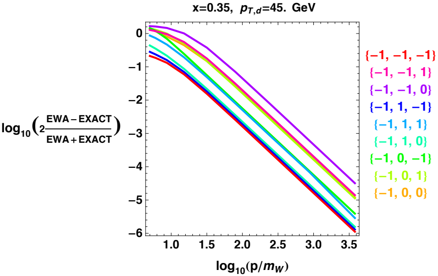


$uW^+ \to dW^+W^-$: $\int d\phi |\mathcal{A}_{EWA}|^2$ vs. $\int d\phi |\mathcal{A}_{exact}|^2$

so far $\bar{p}_{W,virtual} = \bar{p}_u - \bar{p}_d$

$$\begin{aligned}\mathcal{A}_{exact} &= \frac{p_T}{V^2} e^{\pm\imath\phi} g_\pm(x)\epsilon_\pm \cdot \mathcal{A}^{on}_{\pm xy} \\ &+ \frac{m}{V^2} g_0(x)\left(\epsilon_g \cdot \mathcal{A}^{on}_{gxy} + \frac{m}{q_L}\tilde{\epsilon}_0 \cdot \mathcal{A}^{on}_{0xy}\right) \\ &+ \frac{1}{q_L} g_0(x)\tilde{\epsilon}_0 \cdot \mathcal{A}^{on}_{0xy} + \mathcal{O}(\kappa^2)\end{aligned}$$

- $\mathcal{A} \sim p_T e^{\pm\imath\phi}\mathcal{A}_\pm + \mathcal{A}_0$
- $\int d\phi \; e^{\pm\imath\phi}\mathcal{A}_\pm\mathcal{A}_0^* + h.c. = 0$
- $|\mathcal{A}|^2 = |\mathcal{A}_\pm|^2 + |\mathcal{A}_0|^2 = |\mathcal{A}_{EWA}|^2 + \mathcal{O}(\kappa^2)$



$uW^+ \to dW^+W^-$: $\int d\phi |\mathcal{A}_{EWA}|^2$ vs. $\int d\phi |\mathcal{A}_{exact}|^2$

$p_{T,W,virtual} = 0$

$$
\begin{aligned}
\mathcal{A}_{exact} &= \frac{p_T}{V^2} e^{\pm\imath\phi} g_\pm(x)\epsilon_\pm \cdot \mathcal{A}^{\rm on}_{\pm xy} \\
&+ \frac{m}{V^2} g_0(x)\left(\epsilon_g \cdot \mathcal{A}^{\rm on}_{gxy} + \frac{m}{q_L}\tilde{\epsilon}_0 \cdot \mathcal{A}^{\rm on}_{0xy}\right) \\
&+ \frac{1}{q_L} g_0(x)\tilde{\epsilon}_0 \cdot \mathcal{A}^{\rm on}_{0xy} + \mathcal{O}(\kappa)
\end{aligned}
$$

- $\mathcal{A} \sim p_T e^{\pm\imath\phi}\mathcal{A}_\pm + \mathcal{A}_0$
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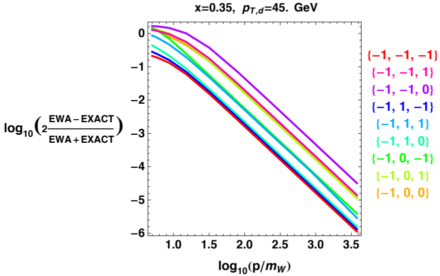
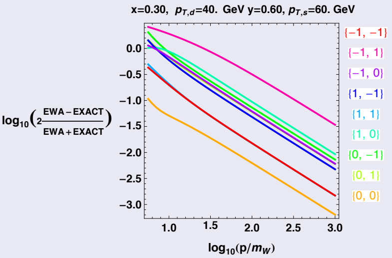


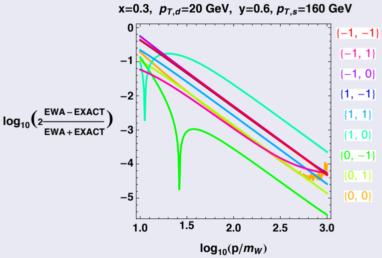
$u\bar{c} \to d\bar{s}W^+W^-$

The process studied so far, $uW^+ \to dW^+W^-$, is only a toy, but displays all the interesting physics (even more indeed), of the "interesting" process $qq \to qqWW$.

fixed jets angles



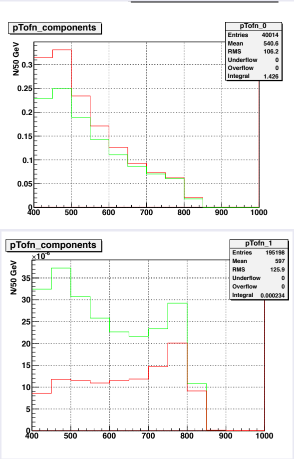
int. cross-section $\sigma = \int \frac{d\sigma}{d\phi_{jet}} d\phi_{jet}$



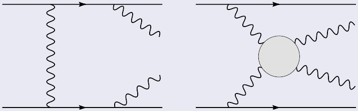
EWA vs. MadGraph: $d\sigma/dp_{T,W^+}$ for $uW^- \to dW^+W^-$ at $\sqrt{\hat{s}}$=2 TeV (PRELIMINARY)

in the $SU(2)$ Higgs model (m_h=160 GeV) in the region 30 GeV $< p_{T,d} <$ 60 GeV, $0.3 < x < 0.4$, $m_{WW} >$ 400 GeV

$d\sigma/dp_{T,W}$ EWA versus Exact



- significant variance of the accuracy at fixed cuts
- useful to check the sensitivity of the WW scattering analysis to the physics of EWSB (not W-strahlung!)



- “all purpose” method for the assessment of the irreducible background to *WW* scattering

Conclusions

on-shell WW scattering:

- direct probe of the EWSB sector
- simple to understand (compared to $qq \to qqWW$)

EWA:

- (re)-established the EWA as an expansion in $p_{T,jet}/p_{T,W_{out}}$
- assessed the origin and predicted the size of the corrections to EWA
  - $\mathcal{A}$ correct up to $\mathcal{O}(p_T^2/E^2)$ when W_L dominate
  - $\mathcal{A}$ correct up to $\mathcal{O}(p_T/E)$ when W_T dominate
  - $\int d\phi |\mathcal{A}|^2$ up to $\mathcal{O}(p_T^2/E^2)$ in all cases
- $p_T \ll$ m has been investigated (different from the massless case)
- numerical checks of our prediction with analytic amplitudes

- prediction of $\frac{d\sigma}{d\phi_{jet}}$ (sensitive to the polarization of the W)
- EWA generator for partonic collisions

The example of the scalar as moderator (Higgs-like model)

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma)$$

$$\mathcal{A}(\pi\pi \to \pi\pi) \sim \frac{s}{v^2}$$

The example of the scalar as moderator (Higgs-like model)

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma)\left(1 + a\frac{h}{v}\right)$$

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$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma)\left(1 + a\frac{h}{v} + b\frac{h^2}{v^2}\right)$$

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an interpolator

- $a = b = 0$ corresponds to the strongly coupled Goldstones
- $a = b = 1$ corresponds to weakly coupled Goldstones, i.e. the SM

$$\mathcal{L} = |D_\mu \Phi|^2 \text{ with } \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \pi_1 + \imath\pi_2 \\ v + h + \imath\pi_3 \end{pmatrix}$$

The example of the scalar as moderator (Higgs-like model)

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma)\left(1 + a\frac{h}{v} + b\frac{h^2}{v^2}\right) + m_f \bar{\psi}_L \psi_R \left(1 + c\frac{h}{v}\right)$$

$$\mathcal{A}(\pi\pi \to \pi\pi) \sim \frac{s}{v^2}\left(1 - a^2\right)$$

$$\mathcal{A}(\pi\pi \to hh) \sim \frac{s}{v^2}\left(a^2 - b\right)$$

$$\mathcal{A}(\pi\pi \to \psi\psi) \sim \frac{\sqrt{s} m_f}{v^2}\left(1 - ac\right)$$

an interpolator

- $a = b = 0$ corresponds to the strongly coupled Goldstones
- $c = a = b = 1$ corresponds to weakly coupled Goldstones, i.e. the SM

$$\mathcal{L} = |D_\mu \Phi|^2 \text{ with } \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \pi_1 + \imath\pi_2 \\ v + h + \imath\pi_3 \end{pmatrix}$$

on-shell WW scattering: a SM process that knows BSM

$W_L W_L \to W_L W_L$

- W_L described by the Goldstone's bosons
- $\Sigma \equiv e^{\imath\pi^a\sigma^a/v}$
- a scalar h coupled to the Goldstones

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}(\partial h)^2 - V(h) \\
&+ \frac{v^2}{4} Tr\left(D_\mu \Sigma D^\mu \Sigma\right)\left(1 + a\frac{h}{v} + b\frac{h^2}{v^2} + \dots\right) \\
&+ m\bar{\psi}_R \Sigma \psi_L \left(1 + c\frac{h}{v}\right) + h.c.
\end{aligned}
$$

$\mathcal{A}$ grows with the energy

$$\mathcal{A}(\pi\pi \to \pi\pi) = (1 - a^2)\frac{s}{v^2} + \dots$$

Strong or Weak coupling

- a,b,c are in principle free parameters
- a: $W_L W_L \to W_L W_L$
- b: $W_L W_L \to hh$
- c: $W_L W_L \to f\bar{f}$
- Strong if a=0 or b=0 or c=0
- SM is a=b=c=1

- The Higgs is part of new physics

whatever breaks the EW symmetry

- measuring a,b,c tells about EWSB (and tells what is h)

Status of the EWA: at cross-section level $\sigma = \int \frac{d\sigma}{d\phi_{jet}} d\phi_{jet}$

surprisingly no complete and clear statement

- $ff \to ffWW$ only for heavy Higgs boson or Higgless (Kunstz,Soper '88)
- $ff \to ff\, h$

Checks of the EWA: at cross-section level $\sigma = \int \frac{d\sigma}{d\phi_{jet}} d\phi_{jet}$

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Higgs:

- total rate: $ff \to ff\,h$ in agreement up to $\mathcal{O}(10\%)$ (Cahn '85, Altarelli et al. '87)

WW:

- $d\sigma/dm_{WW}$ easily off by a factor $\mathcal{O}(1)$ (Gunion et al. '86, Accomando et al. '06)
- $d\sigma/dp_{T,jet}dp_{T,jet}$ easily off by a factor $\mathcal{O}(1)$ (Accomando et al. '06)

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validity of the EWA has been questioned

- the goal is not to compute the rate
- most of the attention was on the total cross-section
- cuts were not selecting the region $V \ll Q_{WW}$