Fine-tuning revisited: status and implications for SUSY models.

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• Hierarchies of scales and fine-tuning:

• why SM EW scale $v \sim M_Z \ll M_P$, stable under quantum corrections. $\left[\frac{G_f h^2}{G_N c^2} = 1.7\times 10^{33}\right]$

$$\delta v^2 \sim \delta m_h^2 \sim f(\alpha_j)\,\Lambda^2,$$ if $\Lambda \sim M_P$: tune coupling: $1:10^{33}\,(!)$.

Hierarchy Problem $\Leftrightarrow$ fine tuning. What to do?

• A physical parameter $\rho(\mu)$ is naturally v.small if $\rho(\mu)=0$ increases the symmetry.

"Naturalness dogma": 't Hooft (1979)

Then:

$\Rightarrow$ Scale/conformal symmetry. (see talks this conference...); Bardeen 1995

$\Rightarrow$ SUSY: $\delta m_h^2 \sim m_S^2 \ln \Lambda/m_S$, $m_S \sim$TeV... no SUSY seen, $m_S \gg \mathrm{TeV} \rightarrow$ back to SM fine-tuning

.....or ignore that! $\delta m_h^2 \sim f(\alpha_j)\,\Lambda^2$, $\Lambda \ll M_{Planck}$: Λ: scale of "new physics".

.....why worry? $\underbrace{\text{worse fine tunings}}_{\text{un-related?}}$: cosmological const: $\left[\frac{\rho_v}{\rho} \approx \frac{(2.3\times 10^{-12}\mathrm{GeV})^4}{(10^{19}\,\mathrm{GeV})^4}\right]$

- Fine tuning and SUSY models:

$$V = m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 - (B_0\,\mu_0\,H_1\cdot H_2 + h.c.) + \lambda_1/2\;|H_1|^4 + \lambda_2/2\;|H_2|^4 + \lambda_3\,|H_1|^2\,|H_2|^2$$
$$+\;\lambda_4\,|H_1\cdot H_2|^2 + \Big[\lambda_5/2\;(H_1\cdot H_2)^2 + \lambda_6\;|H_1|^2\,(H_1\cdot H_2) + \lambda_7\;|H_2|^2\,(H_1\cdot H_2) + h.c.\Big]$$

$$m^2 \equiv m_1^2\cos^2\beta + m_2^2\sin^2\beta - B_0\,\mu_0\,\sin 2\beta, \quad \mathrm{UV}: m_{1,2}^2 = m_0^2 + \mu_0^2$$

$$\lambda \equiv \frac{\lambda_1}{2}\cos^4\beta + \frac{\lambda_2}{2}\sin^4\beta + \frac{\lambda_{345}}{4}\sin^2 2\beta + \sin 2\beta\left(\lambda_6\cos^2\beta + \lambda_7\sin^2\beta\right)$$

$$\textbf{2 constraints:}\;\; f_1 \equiv v^2 + \frac{m^2}{\lambda} = 0, \quad f_2 \equiv 2\lambda\frac{\partial m^2}{\partial\beta} - m^2\frac{\partial\lambda}{\partial\beta} = 0,$$

- Problem: scales vs. couplings "tension":

$$v^2 = -m^2/\lambda, \qquad v = \mathcal{O}(100\ \mathrm{GeV}), \quad \lambda < 1, \qquad \textbf{but} \quad m_{1,2}, B_0 \text{ and } m \sim \mathcal{O}(1\,\mathrm{TeV}).$$

....even worse: $m_h < m_Z$ (tree level), need large quantum corrections $\Rightarrow$ large $m_{1,2}, B_0$...

....also a problem of couplings (λ small). Solution ? increase λ by 1.- quantum corrections
2.- new physics beyond MSSM

- What is the right measure of fine tuning ?

$$\Delta_{max} \equiv \max_{\gamma} \left|\Delta_\gamma\right|, \qquad \Delta_\gamma \equiv \frac{\partial \ln v^2}{\partial \ln \gamma^2}, \quad \gamma = \{m_0, m_{1/2}, \mu_0, A_0, B_0\}, \quad v = \text{EW scale}$$

Ellis, Enqvist, Nanopoulos, Zwirner (1986)

Barbieri, Giudice (1988)

- based on physical grounds/intuition. Mathematical support?
- small Δ preferable (?); is this sufficient? what is "small" Δ ? ≤ 20? ≤ 100? ....
- many other definitions for $\Delta \Rightarrow$ different results? .... see: Anderson, Castano, hep-ph/9409419

$$\Delta_q = \Big[\sum_\gamma \Delta_\gamma^2\Big]^{1/2}, \qquad \text{"quadrature } \Delta\text{"}$$

- local in $\{\gamma\}$ space; to compare models, something global preferable $\int d\gamma$? integral measure?

- fine-tuning as "butterfly effect"‡ in dynamical systems sensitive to b.c.! [chaos theory]

classical non-linear evolution $\Leftrightarrow$ RG non-linear, quantum evolution. Edward N. Lorenz (1961)

‡ [weather forecast models]: small changes in initial conditions produce large changes in the final state!

• Fine tuning from a Bayesian approach. $p(a|b)\,p(b)=p(a\cap b)=p(b|a)\,p(a)$

Bayes theorem: [initial belief + data $\rightarrow$ updated belief].

Thomas Bayes (1761), Laplace (1812)

$$p(\gamma|\text{data}) = \frac{L(\text{data}|\gamma)\; p(\gamma)}{p(\text{data})}, \qquad p(\text{data}) = \int L(\text{data}|\gamma)\,p(\gamma)d\gamma, \qquad \gamma:\{m_0, m_{1/2}, \mu_0, A_0, B_0\}.$$

(CMSSM)

- p(data): "evidence". Models $M_{1,2}$: $p_1(\text{data})/p_2(\text{data})$
- EW constraints: $f_1(\gamma; v, \beta, y_t, y_b) = f_2(\gamma; v, \beta, y_t, y_b) = 0, \;\Rightarrow\; v(\gamma,\ldots), \tan\beta_0(\gamma\ldots)$.

$$p(\text{data}) = \int d\gamma\; p(\gamma)\, dy_t\; dy_b\; p(y_t)\; p(y_b)\;\; dv\; d(\tan\beta)\; \delta(m_Z - m_Z^0)\; \delta(m_t - m_t^0)\; \delta(m_b - m_b^0)$$
$$\times\; \delta\left(f_1(\gamma; v, \beta, y_t, y_b)\right)\; \delta\left(f_2(\gamma; v, \beta, y_t, y_b)\right)\, L(\text{data}|\gamma; \beta, v, y_t, y_b)$$

$$p(\text{data}) = \int_{f_{1,2}=0} dS_\gamma\; \frac{L(\text{data}|\gamma)}{\Delta_q'(\gamma)}\; p(\gamma)p\big(y_b(\gamma)\big)p\big(y_t(\gamma)\big), \qquad \Delta_q'(\gamma) = \Delta_q\, \frac{\partial f_2}{\partial(\tan\beta)}, \qquad \Delta_q \equiv \Big[\sum_\gamma \Delta_\gamma^2\Big]^{1/2}$$

D.G., H. M. Lee, M. Park, arXiv:1203:0569

$\Rightarrow \Delta_q \sim \Delta_q'$ (fine-tuning!) induced in p(data) by two theoretical constraints (EW min).

• Fine-tuning from a Bayesian approach.

⇒ identified mathematical support for a fine tuning measure:

$$p(\text{data}) = \int_{f_{1,2}=0} dS_\gamma \, \frac{L(\text{data}|\gamma)}{\Delta'_q(\gamma)} \times (\text{priors}), \qquad \delta(f_1)\,\delta(f_2) \to \frac{\delta(v - v_0)}{|\nabla_\gamma f_1|_{v_0}} \frac{\delta(\tan\beta - \tan\beta_0)}{|(f_2)'_{\tan\beta}|_{\beta_0}}$$

⇒ presence of Δ'_q independent of priors!

⇒ to maximize global evidence $p(\text{data})$:

small Δ (not sufficient!): max likelihood $L(\text{data}|\gamma)$ simultaneously ($\times$ priors!)

⇒ smaller Δ, good $L(\text{data}|\gamma)$ for $m_h \approx 115$ GeV.... increasing m_h ? $\Delta \sim \exp(m_h)$ dominates L?

⇒ How does Δ_q (Δ'_q) compare against Δ_{max}?

B.C. Allanach et al hep-ph/0601089, 0904.2548

J.A. Casas et al, M.E. Cabrera et al, 0812.0536

L. Roszkowski et al, C. Balazs et al, arXiv:1205.1568

• Numerical results: Models and boundary conditions: D.G., H. M. Lee, M. Park, arXiv:1203:0569

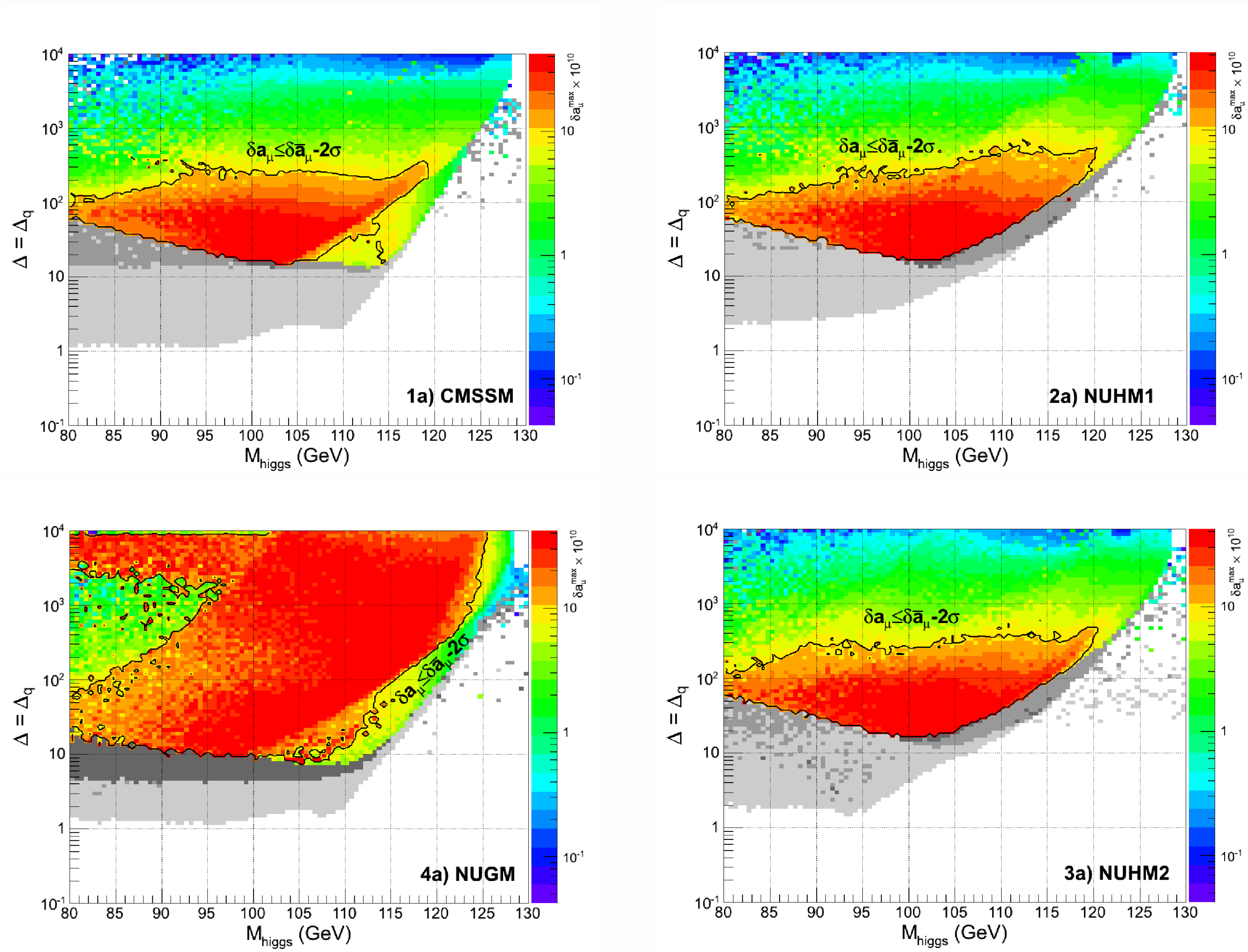
1) CMSSM (constrained MSSM) $\gamma = \{m_0, \mu_0, m_{1/2}, A_0, B_0\}$
2) NUHM1 (non universal Higgs Mass) $\gamma = \{m_0, \mu_0, m_{H_1}^{uv} = m_{H_2}^{uv}, m_{1/2}, A_0, B_0\}$
3) NUHM2 (non universal Higgs Mass) $\gamma = \{m_0, \mu_0, m_{H_1}^{uv}, m_{H_2}^{uv}, m_{1/2}, A_0, B_0\}$
4) NUGM (non universal gaugino masses) $\gamma = \{m_0, \mu_0, m_{\lambda_{1,2,3}}, A_0, B_0\}$

• Experimental Constraints:

- SUSY masses: micrOmegas 2.4.5, "MSSM/masslim.c"
- muon magnetic moment: $\delta a_\mu = (25.5 \pm 2 \times 8) \times 10^{-10}$ at 2σ
- $b \to s\,\gamma$ $3.03 < 10^4$ Br$(b \to s\gamma) < 4.07$ at 2σ
- $B_s \to \mu^+\mu^-$ Br$(B_s \to \mu^+\mu^-) < 1.08 \times 10^{-8}$ at 2σ
- ρ-parameter $-0.0007 < \delta\rho < 0.0033$ at 2σ
- dark matter $\Omega h^2 = 0.1099 \pm 3 \times 0.0062$ at 3σ
- CMS: $m_h = 125$ GeV (2.9σ), Atlas: $m_h = 126$ GeV (2.5σ), combined: $122.5 \leq M_h \leq 127.5$ GeV.
- δa_μ, Higgs mass m_h not imposed.

• Tools: micrOMEGAs 2.4.5, SoftSUSY 3.2.4. Random scan. $\Rightarrow \Delta_q$, Δ_{max} at 2-loop LL.

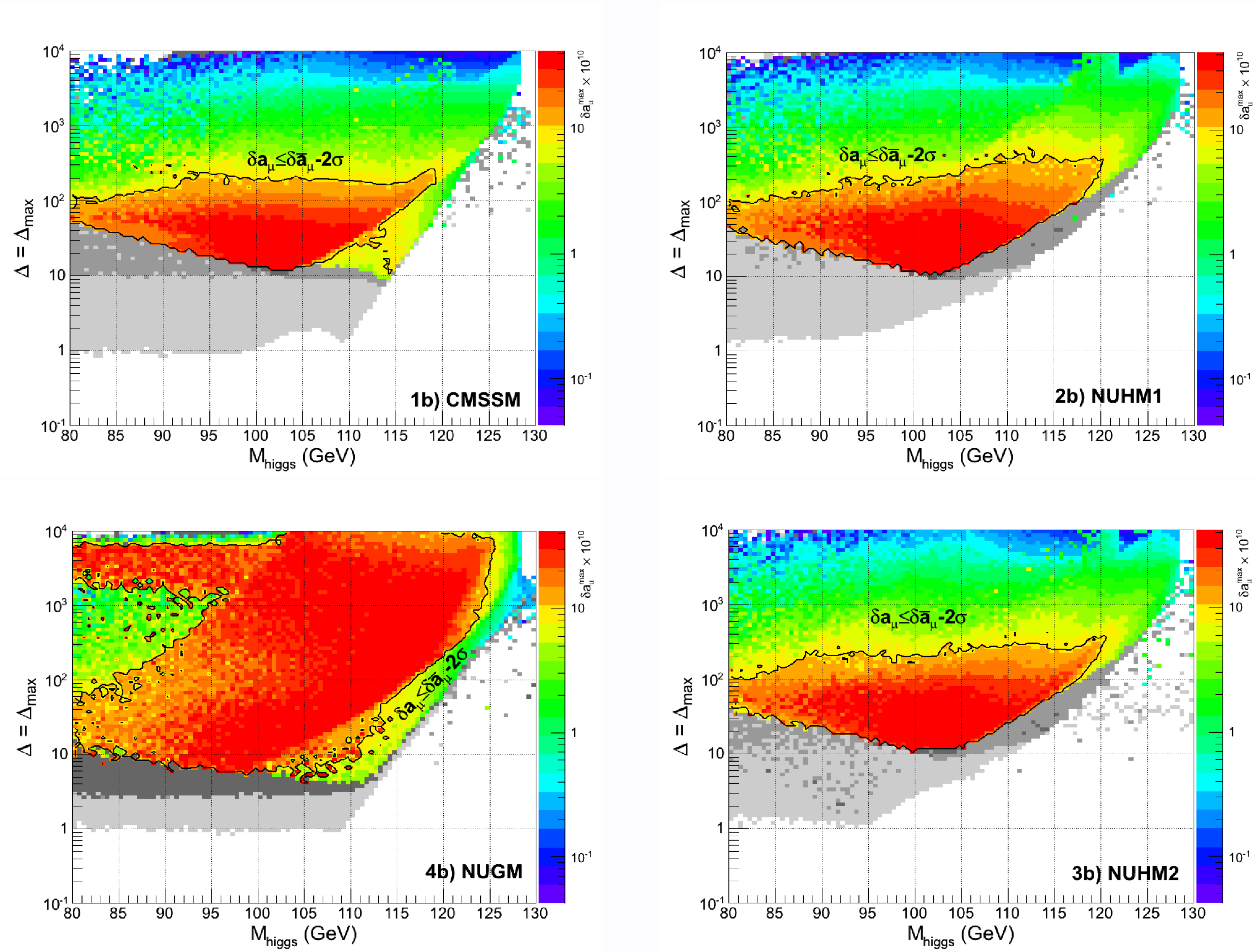
- Impact of m_h, δa_μ on Δ_q. [2-loop, all $\{\gamma, \tan\beta\}$ all values] D.G., H. M. Lee, M. Park, arXiv:1203:0569



- grey 0: excluded by SUSY; grey 1: $b \to s\gamma$, $B_s \to \mu^+\mu^-$, $\delta\rho$; grey 2: excluded by $\delta a_\mu > 0$.

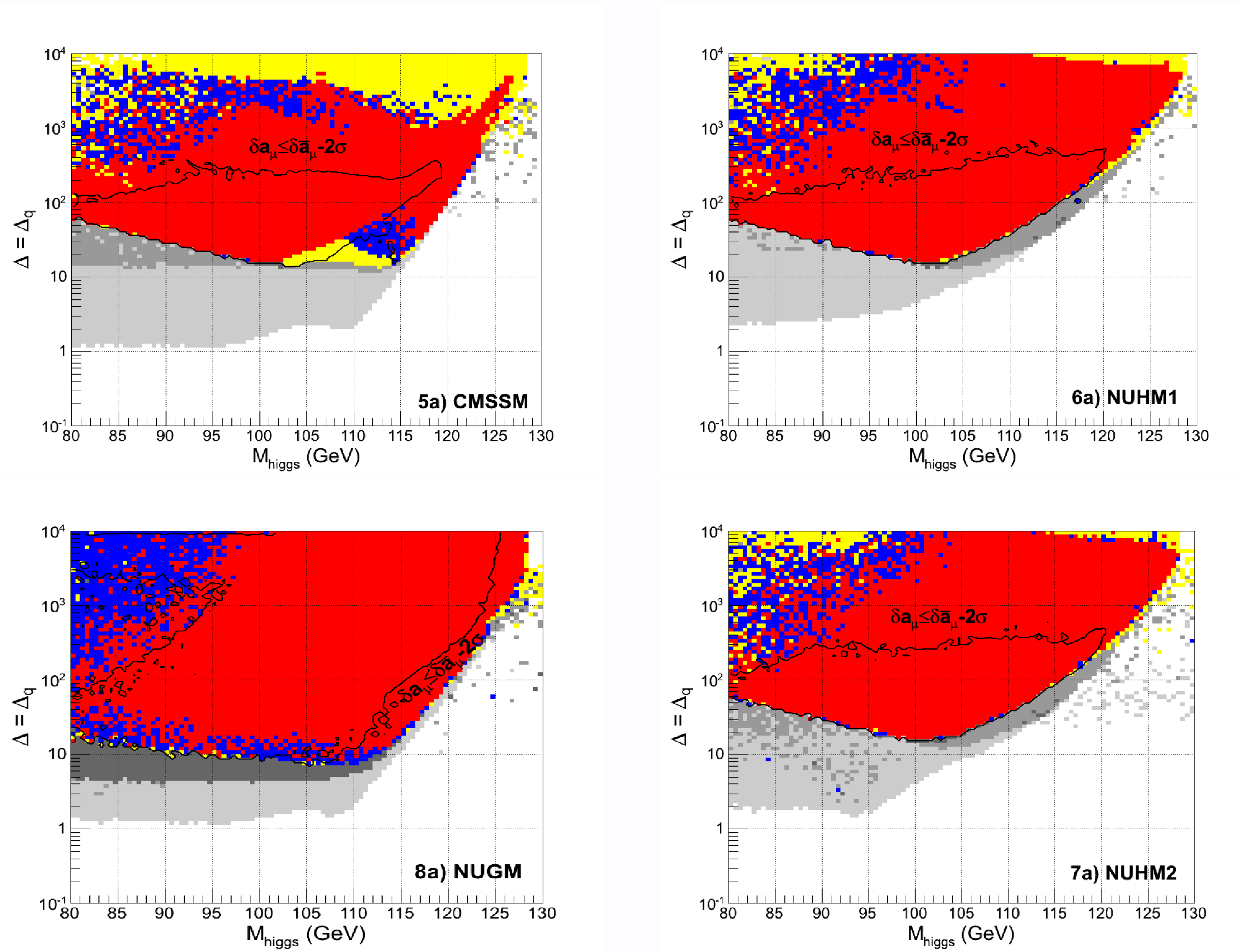
$\Rightarrow$ m_h strongest impact: $\Delta_q \sim e^{m_h}$. $\Delta_q \sim 1000$ (!) near 125 GeV. NUGM better behaviour.

- Impact of m_h, δa_μ on Δ_{max}. [2-loop, all $\{\gamma, \tan\beta\}$ all values]. D.G., H. M. Lee, M. Park, arXiv:1203:0569



- δa_μ: 2σ contour (red) [smaller δa_μ outside]. $\Rightarrow$ Δ_{max} identical behaviour to Δ_q (marginally smaller)

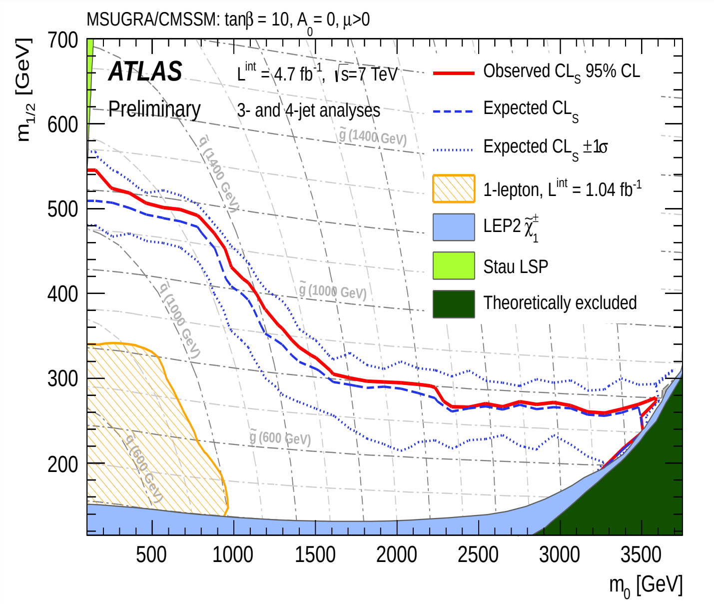
- Impact of m_h, dark matter on Δ_q: [2-loop, $\{\gamma, \tan\beta\}$ all values]. D.G., H. M. Lee, M. Park, arXiv:1203:0569

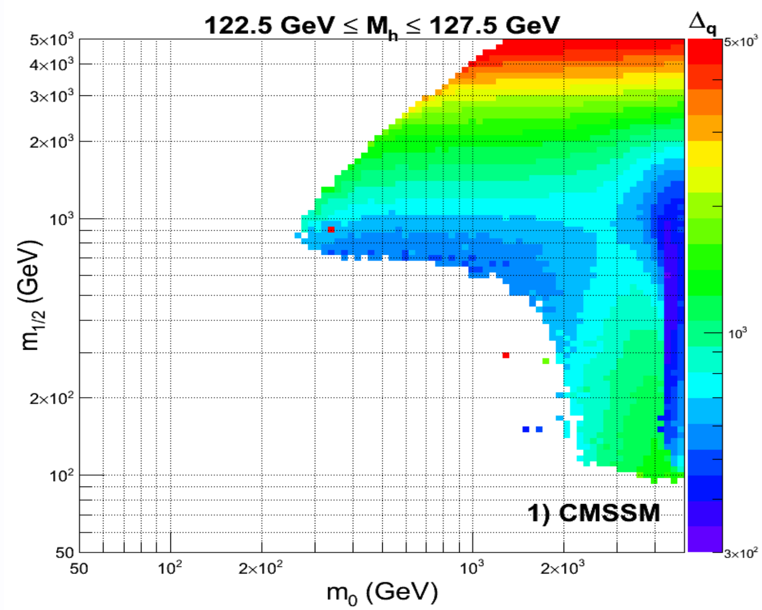


- blue: consistent with Ωh^2; Red: saturate Ωh^2 within 3σ. Grey areas ruled out by data.

$\Rightarrow$ Δ_{max} (not shown) nearly identical plots to Δ_q, (marginally smaller for same Higgs mass).

- SUSY searches from ATLAS and impact on parameter space (CMSSM only):



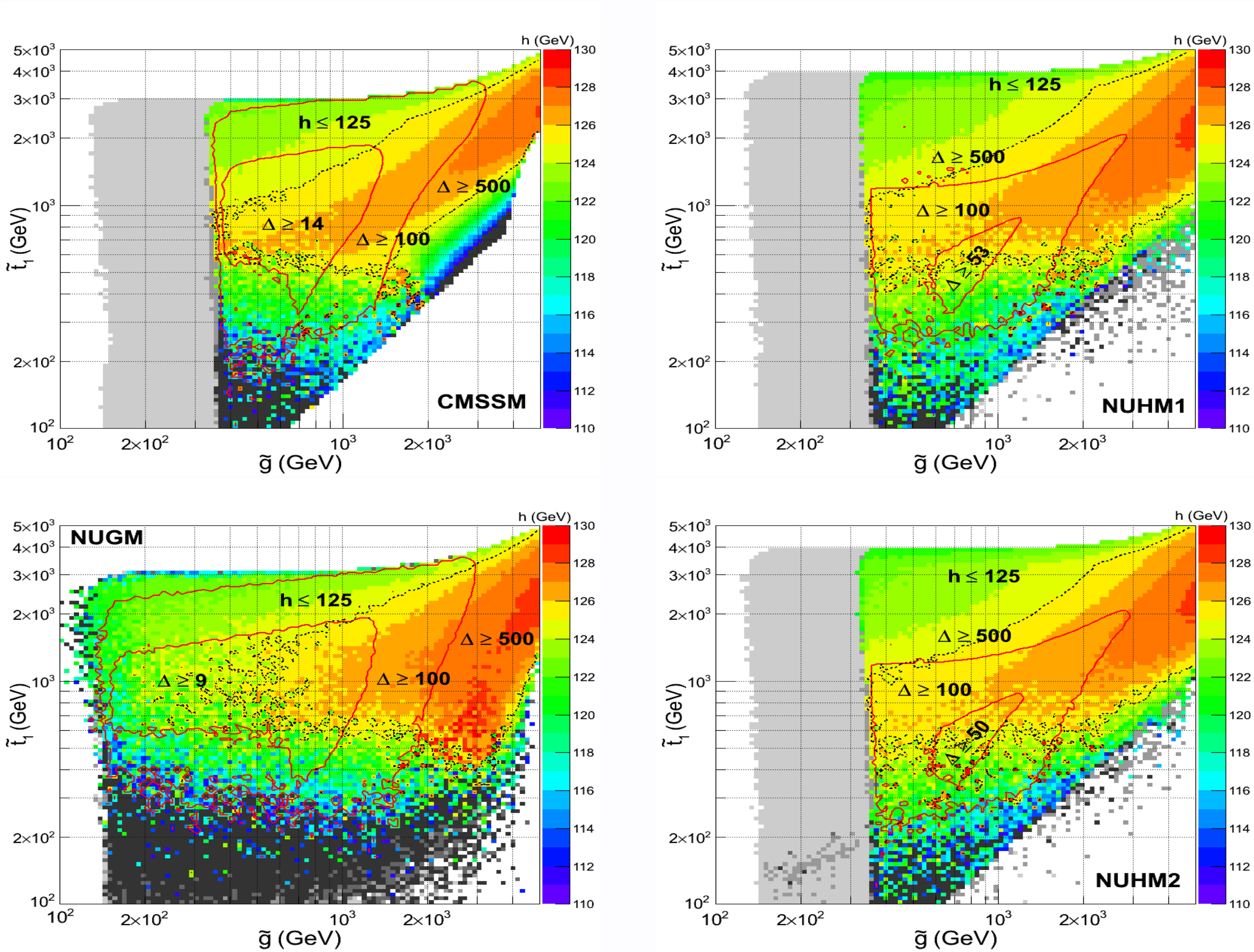


ATALS-CONF-2012-041

CMSSM, $122.5 \leq m_h \leq 127.5$ GeV and Δ_q cost.
[all values for γ, $\tan\beta$]. $\Delta_q > 500$.

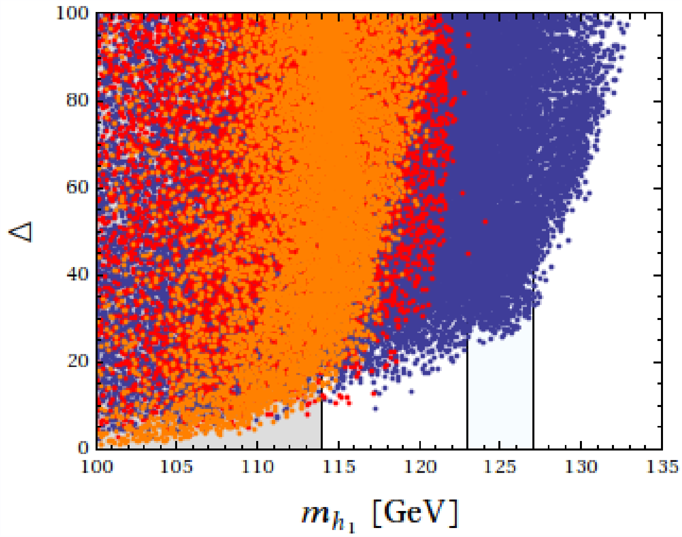
D.G., Hyun Min Lee, Myeonghun Park (2012)

- Stop vs Gluino with largest m_h and min Δ_q. $[\{\gamma, \tan\beta\}$ all values$]$ D.G., H. M. Lee, M. Park, arXiv:1203:0569



⇒ constraints on m_h strongly reduce the viable regions.

- Fine tuning beyond MSSM: NMSSM, GNMSSM, ...



MSSM

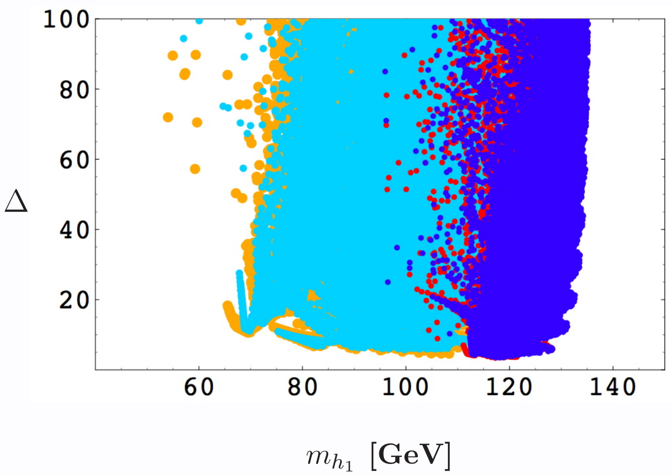
NMSSM: $W = W_Y + \lambda S H_1 H_2 + \kappa S^3$

GNMSSM: $W = W_Y + (\mu + \lambda S)\, H_1 H_2 + M_*\, S^2 + \kappa S^3$

$\Delta < 50$ for $m_h \leq 130\, GeV$.

G.G. Ross et al, arXiv:1205.1509

U. Ellwanger et al arXiv:1107.2472



Decoupling limit: MSSM with a massive gauge singlet:

$\Rightarrow$ MSSM + d=5 operator:

$$W = W_Y + \mu\, H_1 H_2 + \lambda\, S\, H_1\, H_2 + M_*\, S^2$$
$$\Rightarrow W = W_Y + \mu\, H_1 H_2 + \zeta_0\, (H_1 H_2)^2, \quad \zeta_0 \sim \lambda / M_*$$
$$\Delta < 20 \;\text{ for }\; m_h \leq 130\, GeV.$$

S. Cassel, D.G., G.G. Ross, NPB 825(2010)

$\Rightarrow$ massive gauge singlet $\Rightarrow$ reduces fine tuning considerably.

- Fine tuning beyond MSSM: MSSM + d=6 operators

$$\mathcal{O}_j = \int d^4\theta \; \mathcal{Z}_j \, (H_j^\dagger \, e^{V_j} \, H_j)^2, \quad (j=1,2). \qquad \mathcal{O}_3 = \int d^4\theta \;\; \mathcal{Z}_3 \, (H_1^\dagger \, e^{V_1} \, H_1) \, (H_2^\dagger \, e^{V_2} \, H_2)$$

$$\mathcal{O}_4 = \int d^4\theta \; \mathcal{Z}_4 \, (H_2 \, H_1) \, (H_2 \, H_1)^\dagger, \qquad \mathcal{O}_k = \int d^4\theta \;\; \mathcal{Z}_k \, (H_k^\dagger \, e^{V_k} \, H_k) \, H_2 H_1 + h.c., \;\text{(k=5,6)}$$

$$\mathcal{O}_7 = \int d^2\theta \; \mathcal{Z}_7 \, \mathrm{Tr} \, W_i^\alpha \, W_{i,\alpha} \, (H_2 \, H_1) + h.c.,$$

where $\mathcal{Z}_j(S, S^\dagger) = \alpha_{j0} + \alpha_{j1} \, S + \alpha_{j1}^* \, S^\dagger + \alpha_{j2} \, m_0^2 \, S \, S^\dagger, \qquad \alpha_{jk} \sim 1/M_*^2, \;\; S = m_0 \, \theta\theta$

$\mathcal{O}_{1,2,3}$: generated by massive T, U(1); $\mathcal{O}_4$: singlet, T. $\mathcal{O}_{5,6}$: 2 D, singlet.

$$\Rightarrow \delta \, m_h^2 = -2 \, v^2 \left[(\alpha_{30} + \alpha_{40}) \mu_0^2 - \alpha_{20} \, m_Z^2 \right] - \frac{(2 \, \zeta_0 \, \mu_0)^2 \;\, v^4}{m_A^2 - m_Z^2} + \frac{v^2 \cot \beta}{m_A^2 - m_Z^2} \Big[4 \, m_A^2 \, \mu_0^2 \; (2\alpha_{50} + \alpha_{60})$$
$$- (2\alpha_{60} - 3\alpha_{70}) \, m_A^2 \, m_Z^2 - (2\alpha_{60} + \alpha_{70}) \, m_Z^4 \Big] + \mathcal{O}\big(1/(M_*^2 \tan^2 \beta)\big)$$

$\Rightarrow \alpha_{j0}$ (choice?) $\Rightarrow$ increase m_h, reduce fine-tuning by:

$$\Delta_q(m_h) \approx \exp(-\delta m_h/\text{GeV}) \, \Delta_q(m_h)\big|_{\text{CMSSM}}$$

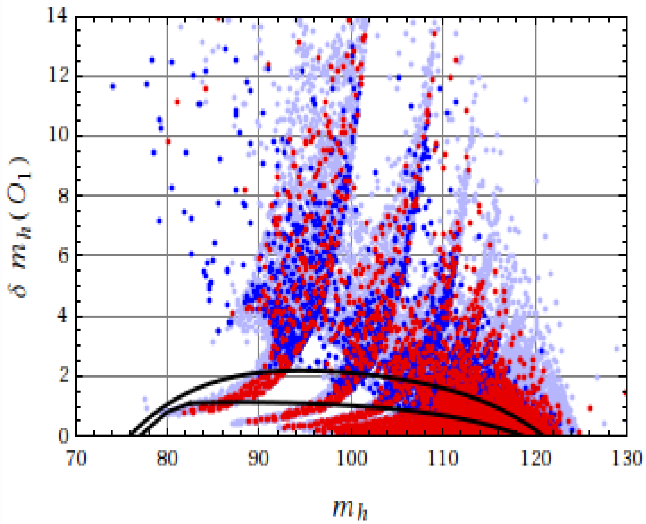
D.G. et al, NPB 848(2011), NPB 831(2010),

M. Carena et al, PRD 85(2012), PRD 81, 82(2010),

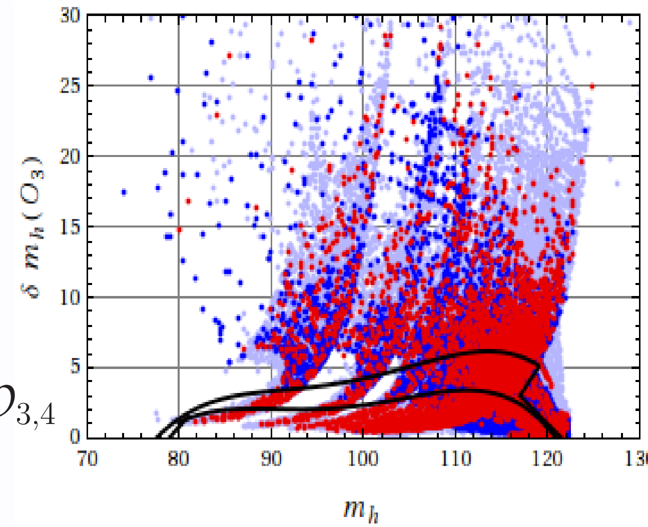
F. Boudjema et al, PRD 85 (2012)

- Corrections to m_h from MSSM + d=6 operators D.G., I. Antoniadis, E. Dudas, P. Tziveloglu, NPB B848(2011)1.

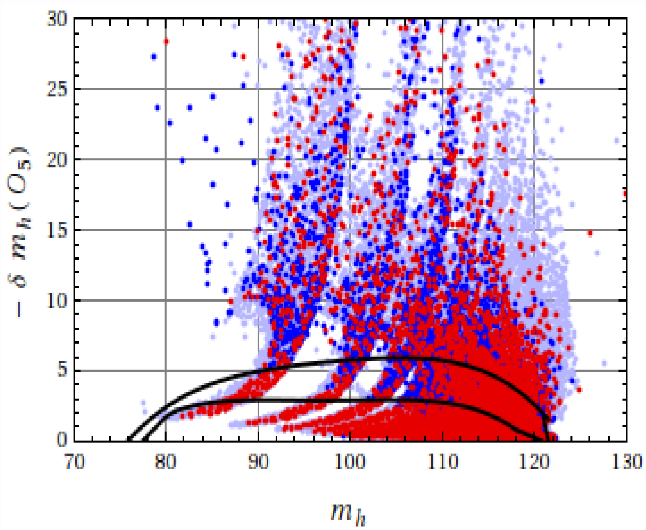
$\mathcal{O}_{1,2}$



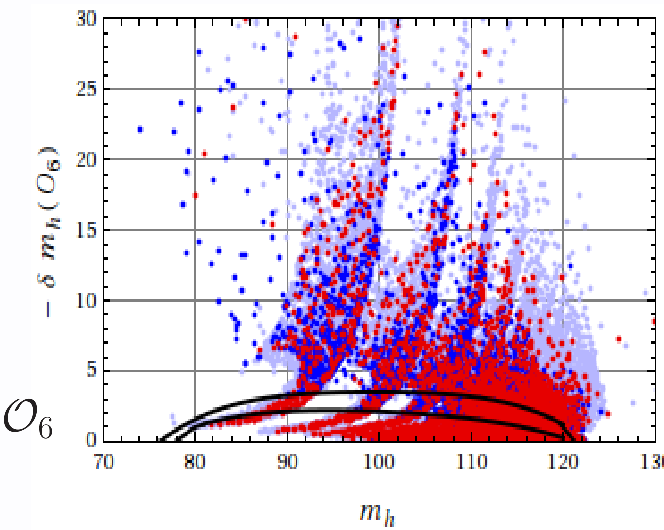
$\mathcal{O}_{3,4}$



$\mathcal{O}_5$



$\mathcal{O}_6$



$\delta m_h = \left(m_h^2 + \delta m_h^2\right)^{1/2} - m_h$, m_h: 2-loop LL CMSSM. $M_* = 8$ TeV. take: $\alpha_{j0} \leq 1/4$

$\Rightarrow$ top curve: $\Delta < 200$: $m_h < 122$ GeV, $\delta m_h < 6$ GeV. $\pm 1\,\mathrm{GeV}\ (\delta m_h) \leftrightarrow \mp 1$ TeV (δM_*).

$\Rightarrow$ largest δm_h: $\mathcal{O}_3$, $\mathcal{O}_4$. $\Delta_q(128\,\mathrm{GeV}) \approx e^{-6}\, \Delta_q(128\,\mathrm{GeV})_{\mathrm{MSSM}} = \mathcal{O}(10)$.

- Final Remarks:

⇒ Fine tuning: - found mathematical support (Δ'_q) and
- new interpretation from Bayesian approach, due to EW/theoretical constraints.

⇒ large L/Δ_q required, with Δ_q in "quadrature".

⇒ Numerical results: Δ_q: of all EW data, m_h strongest constraint.

⇒ Δ_q, Δ_{max}: similar, large: ~ 1000 in CMSSM, NUHM1, NUHM2, NUGM for $m_h \sim 125$ GeV.

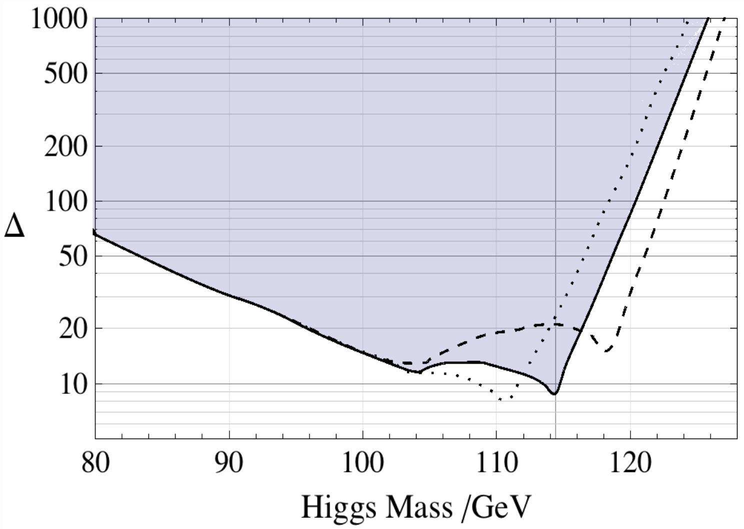
⇒ expect L/Δ_q worsen for $m_h > 115$ as $\Delta_q \sim \exp(m_h/\text{GeV})$.

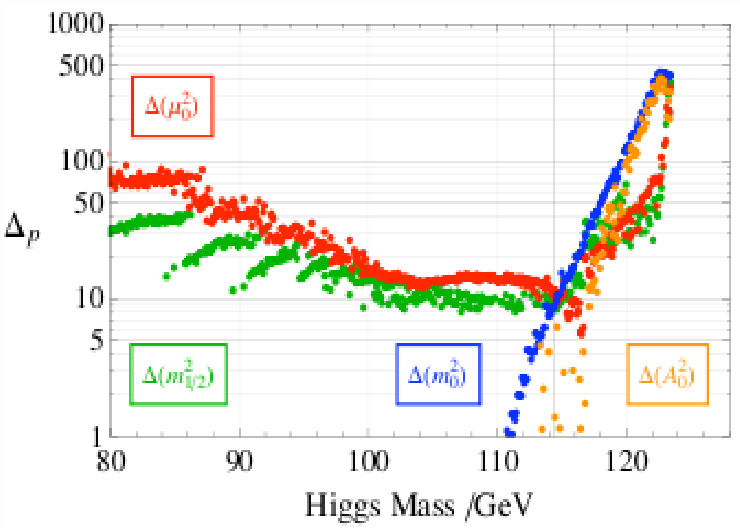
⇒ GNMSSM preferable?.

⇒ Beyond MSSM: with d=5, d=6 operators. ⇒ increase m_h, reduce Δ by $\exp(\delta m_h/\text{GeV})$.
⇒ best scenario: extra massive singlet, extra U(1).

- CMSSM - impact of 1σ shifts of $\alpha_3(M_z)$, m_{top} on Δ:

S. Cassel, D.G., G.G. Ross (2010)





$(\alpha_3, m_t) = (0.1182, 173.1)$.

Δ_p, $p = \mu_0^2, m_0^2, A_0^2, B_0^2, m_{1/2}^2$,

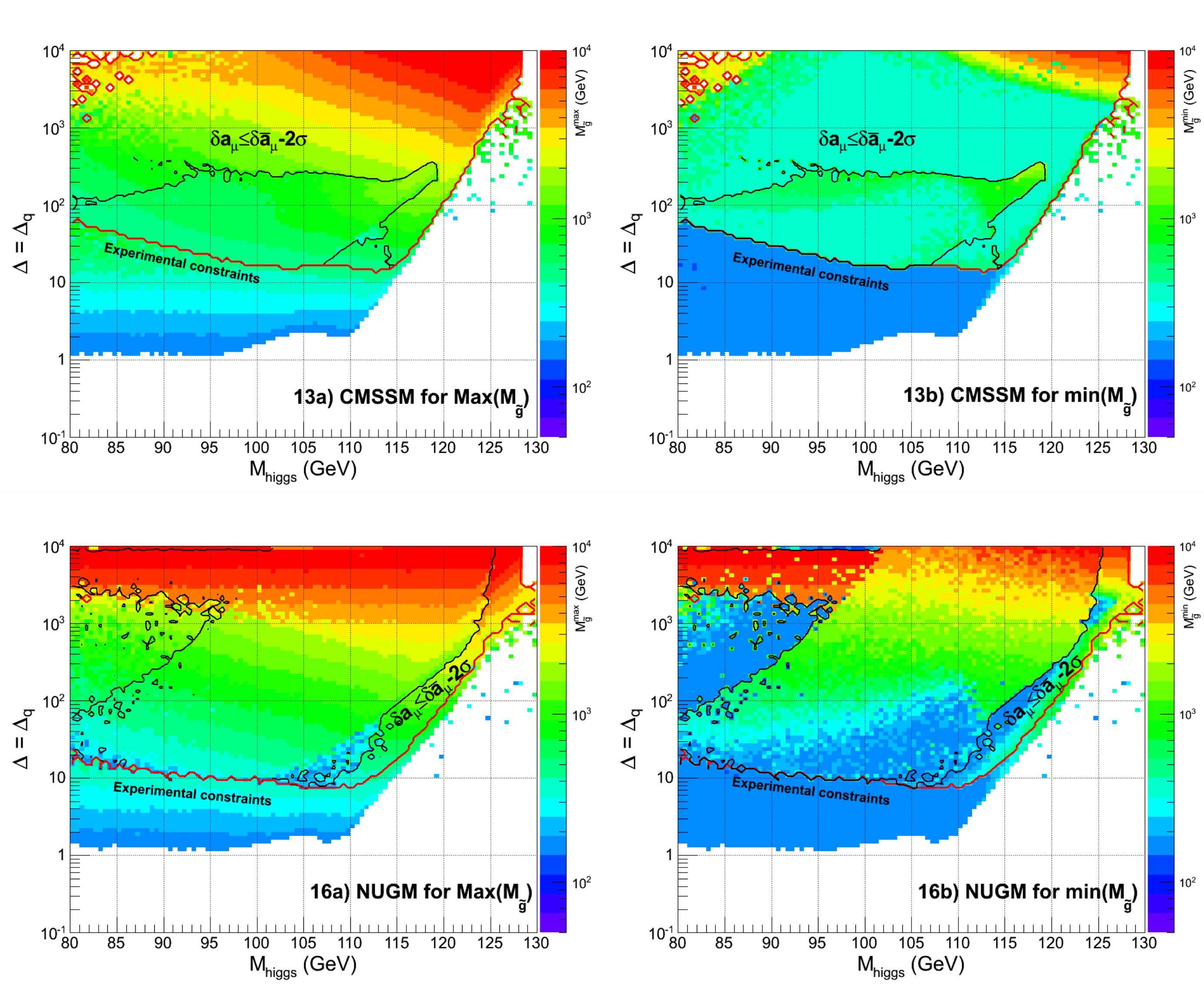
$\Delta_{\mu_0^2}$, $\Delta_{m_0^2}$ dominant (EW vs. QCD)

$\Rightarrow$ if $\alpha_3 - 1\sigma$, $m_t + 1\sigma$ $\Rightarrow$ dashed line: reduce Δ by a factor of ≈ 2 (same higgs mass)

$\Rightarrow$ if $\alpha_3 + 1\sigma$, $m_t - 1\sigma$ $\Rightarrow$ dotted line: QCD does not like large m_h (fine-tuning cost)

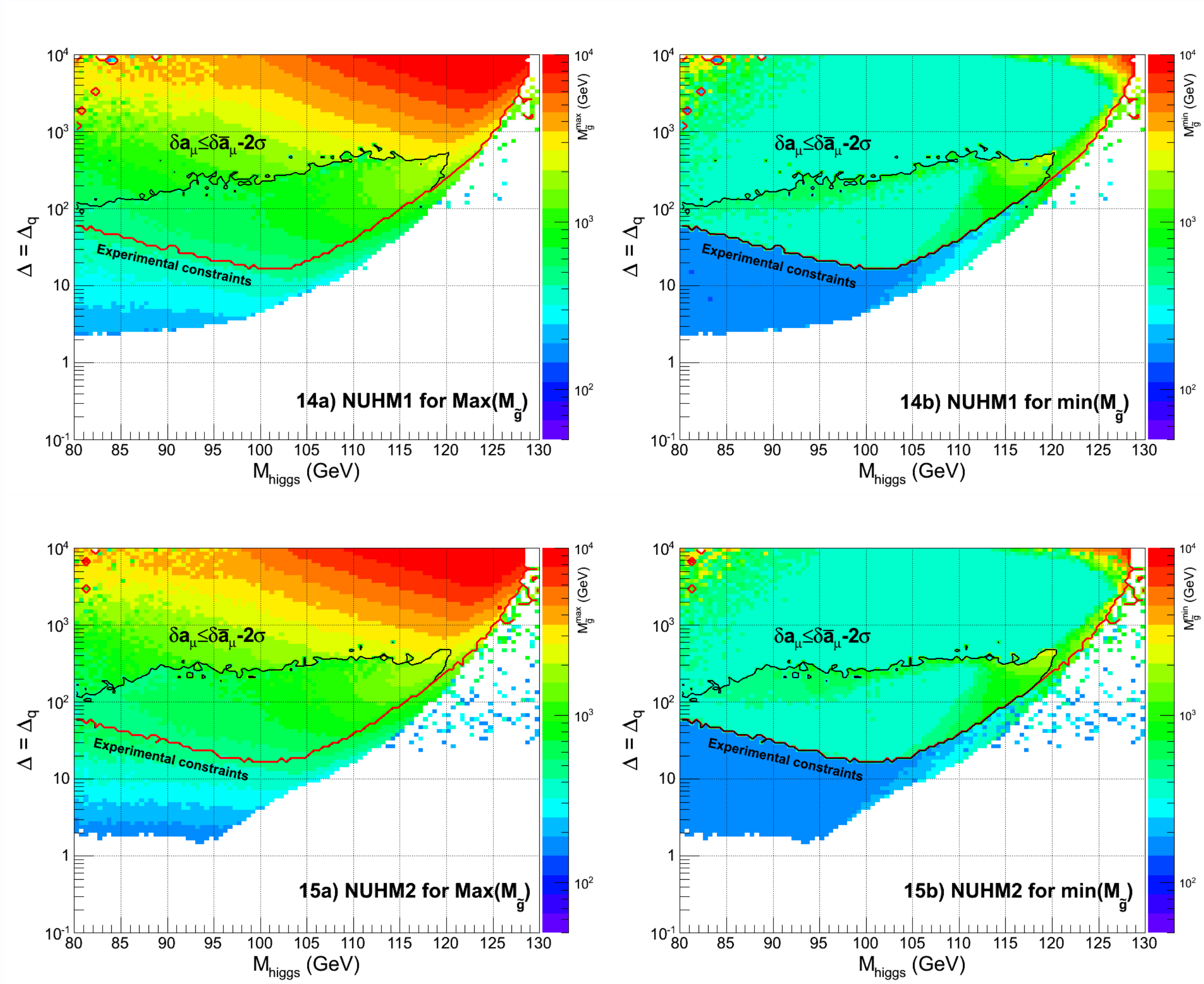
- Similar impact if using Δ_q. Also expected for NUHM1, NUHM2, NUGM.

- Δ_q vs m_h and gluino mass range, δa_μ: $[\{\gamma, \tan\beta\}$ all values$]$ D.G., H. M. Lee, M. Park, arXiv:1203:0569



largest gluino mass lowest gluino mass.

- Δ_q vs m_h and gluino mass range, δa_μ: $[\{\gamma, \tan\beta\}$ all values] D.G., H. M. Lee, M. Park, arXiv:1203:0569

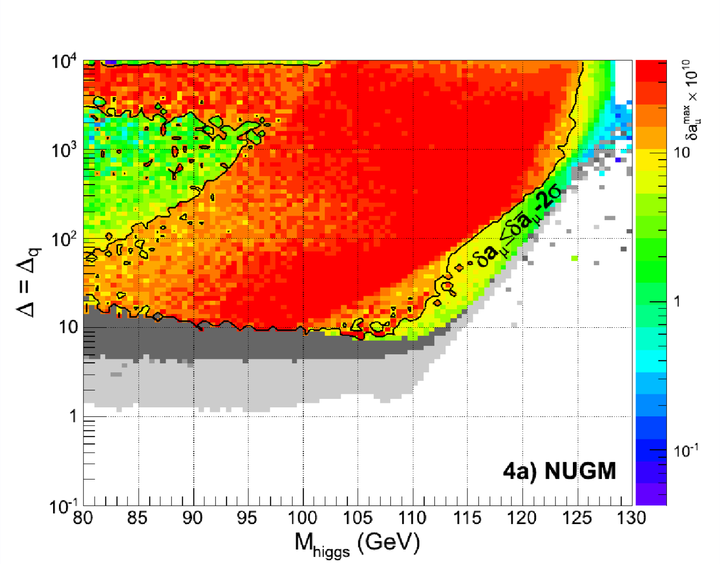


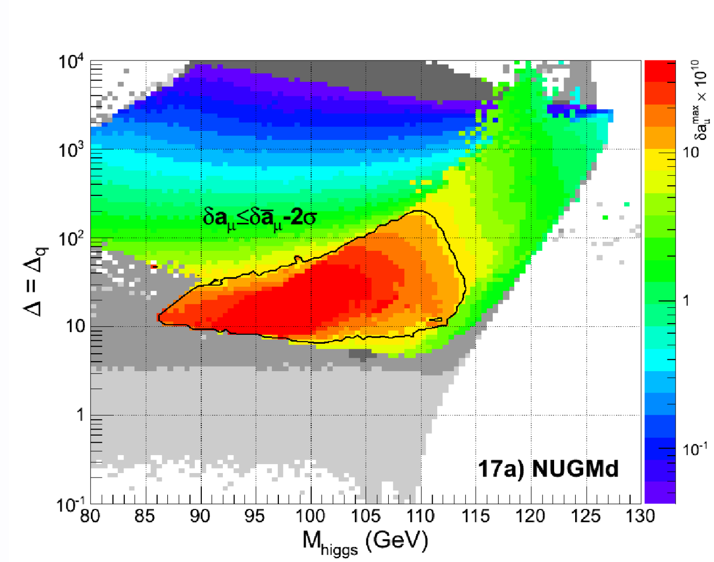
largest gluino mass

lowest gluino mass.

• NUGM versus NUGM with GUT-like gaugino mass relation (NUGMd):

D.G., H. M. Lee, M. Park, arXiv:1203:0569





non-univ gaugino mass with a GUT relation:

$$m_{\lambda_1} = 5/3 m_{1/2}, m_{\lambda_2} = m_{1/2}, m_{\lambda_3} = (1/3)\, m_{1/2}$$

Horton, Ross, arXiv:0908.0857 [hep-ph]

• NUGM better behaviour if δa_μ considered, near 125 GeV for m_h.