Scale Invariance without Conformal Invariance in Relativistic QFT

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Planck2012
Warsaw, May 28-June 1, 2012

Phys.Lett. B704 (2011) 74-80 (1106.2540)
1107.3840
Phys.Lett. B709 (2012) 408-412 (1110.1634)
1202.4757

Two Unsolved Mysteries in QFT

- Are there QFT models in $D = 4$ with SCALE invariance but without being CONFORMAL (*i.e.*, invariant under special conformal transformations)?

  (In this talk QFT means Quantum Field Theories that are also Poincare invariant, *i.e.*, relativistic; one can ask similar questions about non-relativistic QFTs but won't here)

- Are there Renormalization Group (RG) flows in Relativistic QFTs with limit cycles or with limit ergodic flows?

  (Known to exist in non-relativistic Quantum Mechanics: Efimov cycles)

Why care?

- Classification: What are the possible behaviors (phases) of QFT models at very long distances:
  - IR-Free:
    - With mass gap: exponentially decaying correlators (eg, confinement)
    - Without mass gap: trivial correlators (eg, coulomb phase)
  - IR-Interacting
    - Interacting CFTs: simple power-law correlators
    - Interacting SwC (scale without conformal): non-simple power-law ?!

- Alternative classification: IR-limit of RG-flows (Wilson):
  - Strong (*e.g.*, QCD)
  - Fixed Point (*i.e.*, IR-CFT)
  - Limit Cycles
  - Limit ergodic flows

- We are missing quarter-to-half of the possible behaviors!
- New, unknown phenomena/behaviors?
- New, unknown applications? (e.g., "cyclunparticles")

Scale without Conformal

- Condition for Scale Invariance?

$$\partial_\mu D^\mu = 0$$

where the dilatation (scale) current is given in terms of the improved energy-momentum tensor

$$D^\mu = x_\nu T^{\mu\nu}$$

so that

$$\partial_\mu D^\mu = T^\mu_\mu$$

- Condition for Conformal Invariance?

$$\partial_\mu K^{\mu\nu} = -x^\nu T^\mu_\mu = 0$$

- It appears that in both cases the condition is

$$T^\mu_\mu = 0$$

- Improvements? If

$$T^\mu_\mu = \partial_\mu \partial_\nu L^{\mu\nu}$$

one can improve $T^{\mu\nu}$ so that scale and conformal still conserved.

- But! What if the unbroken symmetry is a combination of two broken symmetries? This happens in other familiar contexts:
    - For spontaneously broken symmetries, as in the SM: SU(2)×U(1) → U(1)$_{EM}$
    - For anomalous currents, as in B and L in SM, but not $B-L$

- *Look for a conserved current of the form*

Polchinski, Nucl.Phys. B303 (1988) 226

$$\boxed{D^\mu = x_\nu T^{\mu\nu} - V^\mu}$$

where V^μ (the "virial current") is a non-conserved current that does not depend explicitly on coordinates.

(and which is not of the form $V^\mu = \partial_\nu L^{\mu\nu}$)

THEN: We can have

$$\partial_\mu D^\mu = T^\mu_\mu - \partial_\mu V^\mu = 0$$ scale invariance

while

$$T^\mu_\mu = \partial_\mu V^\mu \neq 0$$ no conformal symmetry

<u>A scale transformation together with a U(1) rotation is still a symmetry.</u>

Immediate implication: limit cycles or ergodic limit flows.
The two mysteries (where are SwC and where are Limit Cycles/Ergodic) are linked!

Let $$Q = \int d^3x\, V^0$$ generates rotations in flavor space

Rotation on fields:

$$\Phi_I \to (e^{itQ}\Phi)_I = (e^{itQ})_I^J \Phi_J$$

A scale transformation corresponds to RG-motion of coupling constants: if $\mathcal{L}_{\text{int}} = g_{IJ\ldots}\Phi_I\Phi_J\cdots$

$$g_{IJ\ldots}(t_0) \to g_{IJ\ldots}(t)$$

This can undo the rotation (so we have a scale/rotation symmetry) if

$$g_{IJ\ldots}(t) = \left[(e^{-itQ})_I^M (e^{-itQ})_J^N \cdots\right] g_{MN\ldots}(t_0)$$

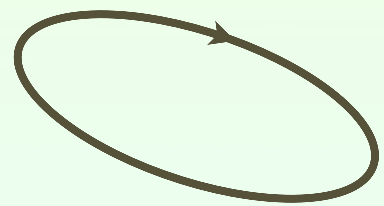
Let G_F = symmetry group of the kinetic terms ("flavor")

For fixed t,

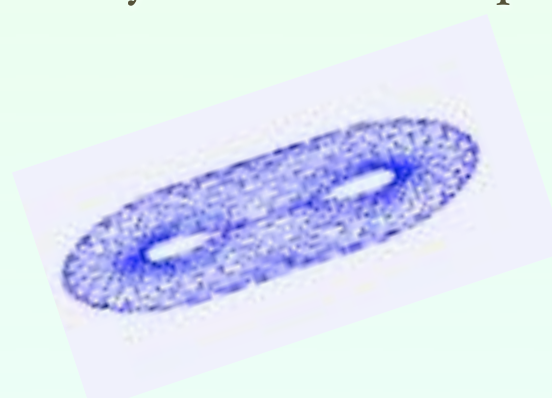
$$e^{-itQ} \in G_F$$

As a function of t: one parameter trajectory in G_F, a <u>compact space</u>

- Trajectory closes



- Trajectory comes arbitrarily close to initial point



Outline

- Introduction
- Searching for SwC models (in $D = 4 - \varepsilon$)
- Some General Properties of SI in $D = 4$
    - Scheme Dependence
    - Stability Properties
    - Correlation Functions
    - Cyclunparticles
    - Perturbative Solutions in $D = 4$
    - A word about the a-theorem
- The future

Searching For SwC Models

(SwC = Scale without Conformal Symmetry)

Want: $$D^\mu = x_\nu T^{\mu\nu} - V^\mu$$

$$\partial_\mu D^\mu = T^\mu_\mu - \partial_\mu V^\mu = 0$$

$$T^\mu_\mu = \partial_\mu V^\mu \neq 0$$

$$V^\mu \neq \partial_\nu L^{\mu\nu}$$

Considerations:

- Interacting
- Renormalizable
- Perturbative
- Enough DOFs for nontrivial virial current
- Fixed points

$D = 4 - \varepsilon$ models: scalars and spinors.

No YM: scalar and spinor coupling asymptotically free in $D = 4 - \varepsilon$

Need to be more explicit in order to write candidates for the virial current: vector operators of dimension-3 with non-vanishing divergence

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For now: focus on $D = 4 - \varepsilon$ models $\mathcal{L} = \mathcal{L}_K + \mathcal{L}_{int}$

$$\mathcal{L}_K = \sum_{a=1}^{n_s} \underbrace{\frac{1}{2}\partial^\mu \phi_a \partial_\mu \phi_a}_{\text{real scalars}} + \sum_{k=1}^{n_f} \underbrace{\bar{\psi}_k i \bar{\sigma}^\mu \partial_\mu \psi_k}_{\text{Weyl spinors}}$$

with: $$-\mathcal{L}_{int} = \frac{1}{4!}\lambda_{abcd}\phi_a\phi_b\phi_c\phi_d + \frac{1}{2} y_{a|ij}\phi_a\psi_i\psi_j + \text{h.c.}$$

Only candidate:

$$V^\mu = Q_{ab}\phi_a\partial^\mu\phi_b + iP_{ij}\bar{\psi}_i\bar{\sigma}^\mu\psi_j$$

Note that:

- $Q_{ab} = -Q_{ba}$ (symmetric combination gives total divergence)
- $P^\dagger = -P$ (current is real)
- Q and P are generators of $G_F = SO(n_s) \times U(n_f)$
- This U(1) subgroup should be broken by $\mathcal{L}_{int}$

Recall: $\partial_\mu D^\mu = T^\mu_\mu - \partial_\mu V^\mu = 0$ with $T^\mu_\mu = \partial_\mu V^\mu \neq 0$

Trace anomaly: $$T_\mu{}^\mu(x) = -\tfrac{1}{4!}\beta_{abcd}\phi_a\phi_b\phi_c\phi_d - \tfrac{1}{2}\beta_{a|ij}\phi_a\psi_i\psi_j + \text{h.c.}\,.$$

Divergence of virial: $$\partial_\mu V^\mu(x) = Q_{aa'}\partial^2\phi_a\phi_{a'} - P^*_{i'i}\bar{\psi}_i i\bar{\sigma}^\mu\partial_\mu\psi_{i'} + P_{ii'}\partial_\mu\bar{\psi}_i i\bar{\sigma}^\mu\psi_{i'}$$

Using equations of motion obtain conditions:

$$\boxed{\begin{aligned}\beta_{abcd} &= -Q_{a'a}\lambda_{a'bcd} - Q_{b'b}\lambda_{ab'cd} - Q_{c'c}\lambda_{abc'd} - Q_{d'd}\lambda_{abcd'}\\ \beta_{a|ij} &= -Q_{a'a}y_{a'|ij} - P_{i'i}y_{a|i'j} - P_{j'j}y_{a|ij'}\end{aligned}}$$

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These are not functional equations

Solution: specific values of coupling constants (and Q and P) that satisfy these equations

Precisely as in searching for conformal fixed points (with $Q = P = 0$)

We look for solutions using perturbation theory:

- beta-functions to fixed order in the loop expansion
- coupling constants on limit cycle (*e.g.*, on solution) remain small
- Q and P consistent with beta-function loop expansion

<u>ε-expansion</u>

Recall: $\beta_\lambda \sim -\varepsilon\lambda + \frac{1}{16\pi^2}\left(\lambda^2 + (y^\dagger y)^2\right) + \cdots$ and $\beta_y \sim -\varepsilon y + \frac{1}{16\pi^2} y y^\dagger y + \cdots$

Expansion ("flavor" indices implicit):

$$y = \sum_{n\geq 1} y^{(n)} \varepsilon^{n-\frac{1}{2}} \qquad \lambda = \sum_{n\geq 1} \lambda^{(n)} \varepsilon^n$$

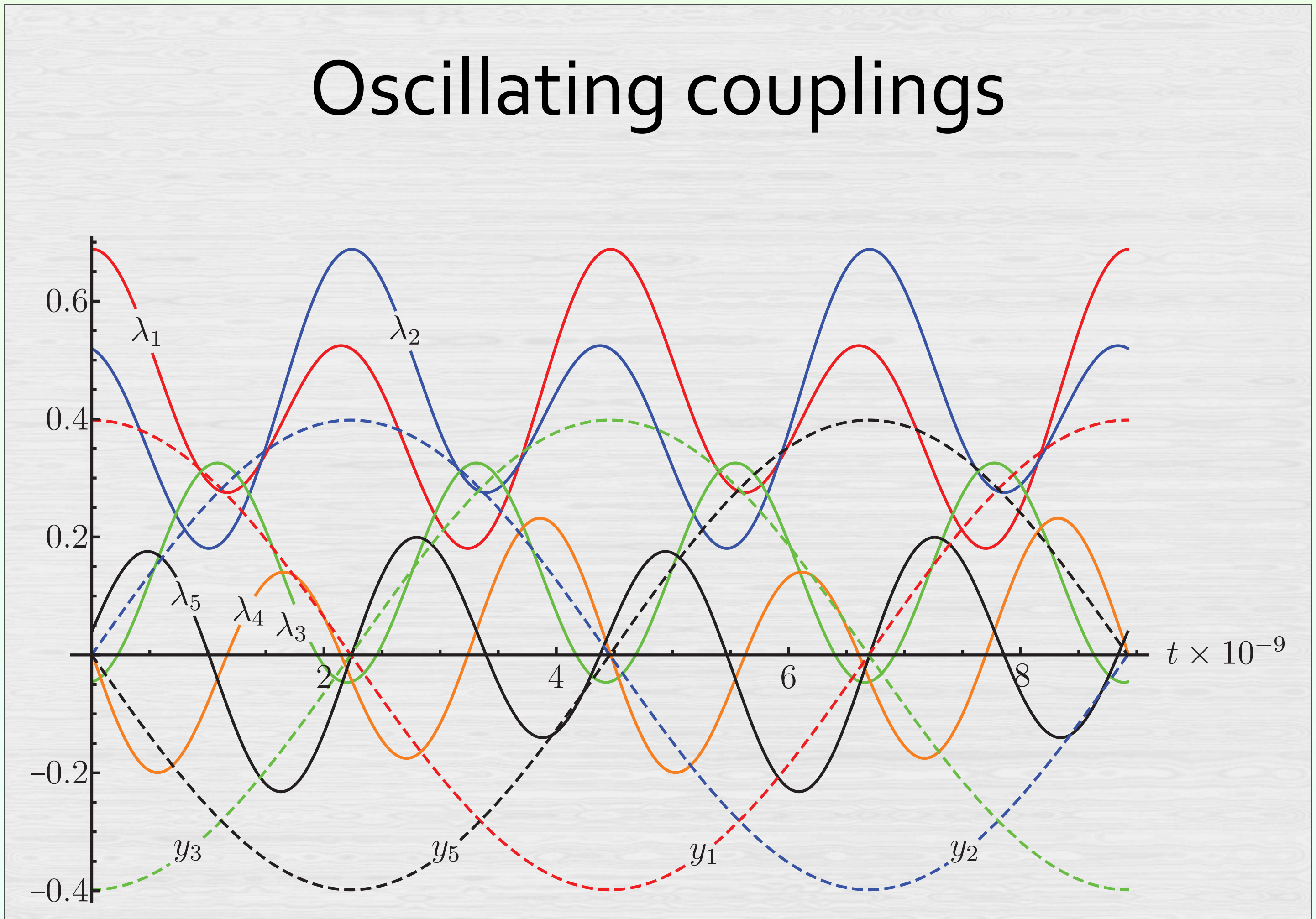
$$Q = \sum_{n\geq 2} Q^{(n)} \varepsilon^n \qquad P = \sum_{n\geq 2} P^{(n)} \varepsilon^n$$

Match powers of $\varepsilon^{\frac{1}{2}}$ on both sides of $\beta_\lambda = Q\lambda$ and $\beta_y = Qy + Py$

- Lowest order: non-linear. Many solutions. Discard "bad" ones
- Higher orders: linear

Summary of findings:

- No SwC for pure scalar theory at 2-loops
- No SwC to all orders in perturbation theory for any n_f if $n_s < 2$
- Solutions with $P = 0$ but $Q \neq 0$ (Q = order ε^3) established at 3-loops in:
  - $n_f = 1, n_s = 2,$ (unbounded tree-level scalar potential)
  - $n_f = 2, n_s = 2,$ (bounded tree-level scalar potential)



Comments:

- Tree level potential bounded vs unbounded: red-herring
  - Much like studying RG flows in ϕ^3 in $D = 6$
  - Vacuum stability determined by effective potential

- Beta-functions:
  - Used Jack & Osborn beta-functions at 2-loops Nucl.Phys. B249 (1985) 472
  - 3-loop beta functions generally unknown, but:
    - Only Yukawa's beta-function can modify Q at order ε^3
    - About 250 3-loop 1PI graphs: only 12 can modify Q at this order (see next slide)

- Computations described are all in MS-scheme
  - Result is scheme dependent in $D = 4 - \varepsilon$
  - "classical" term in beta-function ($-\varepsilon\lambda$) is not covariant

Diagrams that can contribute to $q = Q_{12}$ in the $n_s = n_f = 2$ model

2-loops

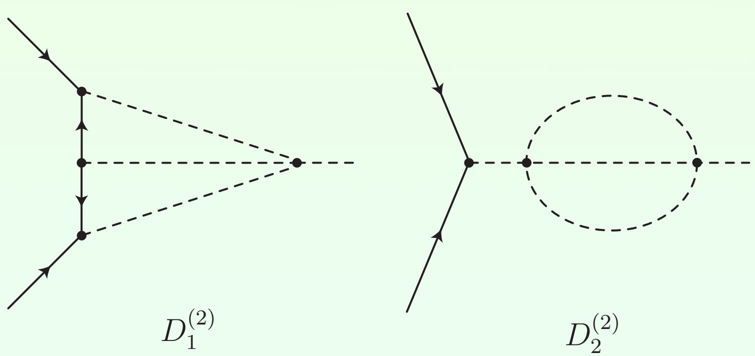


Fig. 1: Diagrams that contribute to q at two-loop order.

3-loops

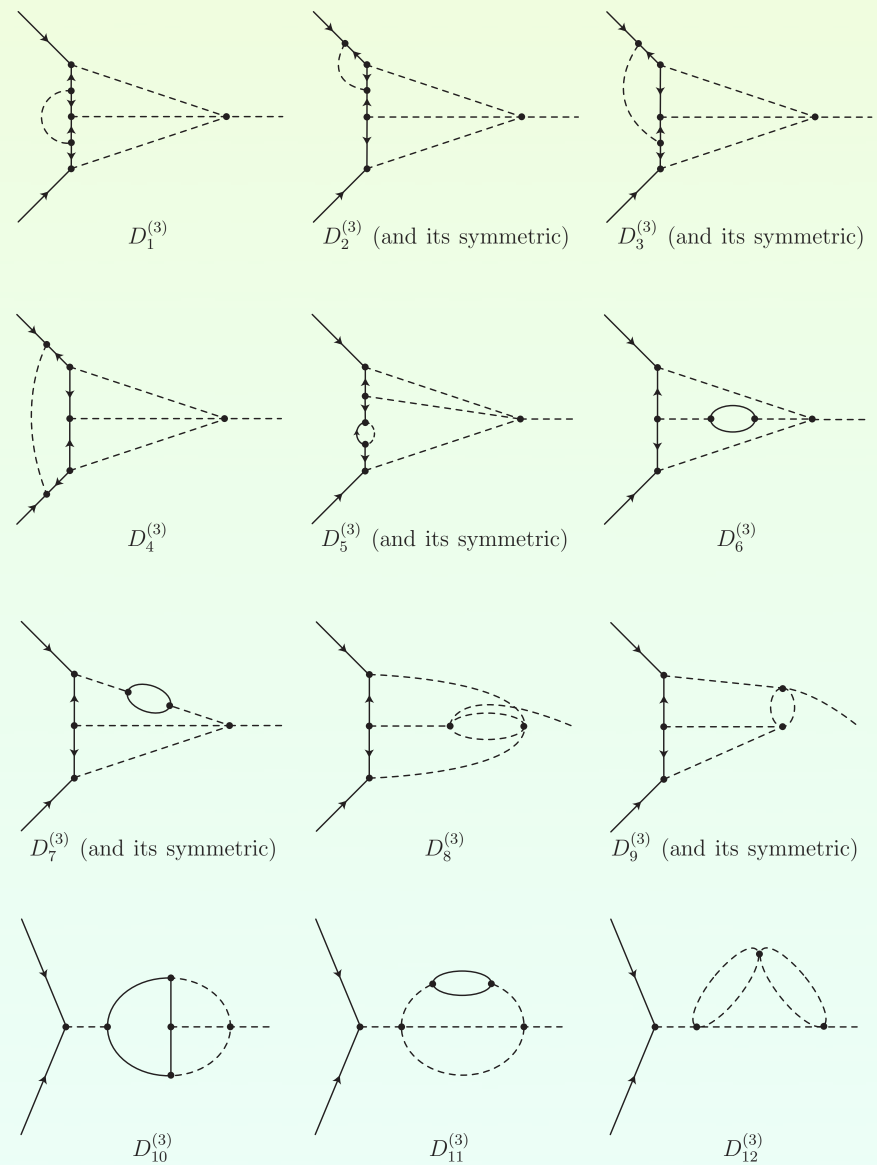


Fig. 2: Diagrams that contribute to q at three-loop order.

Some General Properties of SI Solutions

(in $D = 4$, but readily extended to other dimensions)

Scheme Dependence

Properties that have physical consequences must be independent of the scheme:

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Stability Properties

(generic vector of couplings, matrix notation)

$$\delta g(t) = [g(t) - g_*(t)]e^{-Qt}$$

Measures small deviations of flow from cycle

$$g_*(t) = g_*(0)e^{Qt}$$

Then

$$\boxed{\delta g(t) = \delta g(0)e^{-St}}$$

where

$$S = \left(\left.\frac{\partial \beta}{\partial g}\right|_{g=g_*(0)} + Q \right)$$

is the "stability matrix"
(scheme independent eigenvalues)

Limit cycle: there is always one vanishing eigenvalue

For example: in n_f, $n_s = 2$, 2 eigenvalues are 2.4, 1, 0.99, 0.74, 0.095, −0.19, 0 (in units of ε)

Correlation Functions

Determined from RGE. Less constrained than in CFTs (less symmetry)

By example here (rather than in generality).
Consider scalar and vector operators under $SO(n_s) \subset G_F$

scalar-scalar:

$$\langle \mathcal{O}(p)\mathcal{O}'(-p)\rangle = C(-p^2 - i\epsilon)^{\frac{1}{2}(\Delta+\Delta'-4)}$$

Dimensions constrained by unitarity: for (j_1, j_2) operator $\Delta \geq j_1 + j_2 + 1$
(for CFT, operators with $j_1 j_2 \neq 0$ have $\Delta \geq j_1 + j_2 + 2$)

scalar-vector:

$$\langle \mathcal{O}_a(p)\mathcal{O}'(-p)\rangle = (-p^2 - i\epsilon)^{\frac{1}{2}(\Delta'-4)}\left[(-p^2 - i\epsilon)^{\frac{1}{2}(\Delta+Q)}\right]_{ab} C_b$$

vector-vector:

$$\langle \mathcal{O}_a(p)\mathcal{O}_b(-p)\rangle = (-p^2-i\epsilon)^{-3}\left[(-p^2-i\epsilon)^{\frac{1}{2}(\Delta+Q)}C(-p^2-i\epsilon)^{\frac{1}{2}(\Delta-Q)}\right]_{ab}$$

with C an $n_s \times n_s$ matrix

and for (Lorentz) vectors:

$$\langle \mathcal{O}^\mu_a(p)\mathcal{O}^\nu_b(-p)\rangle = (-p^2-i\epsilon)^{-3}\left[(-p^2-i\epsilon)^{\frac{1}{2}(\Delta+Q)}(p^2g^{\mu\nu}C_1+p^\mu p^\nu C_2)(-p^2-i\epsilon)^{\frac{1}{2}(\Delta-Q)}\right]_{ab}$$

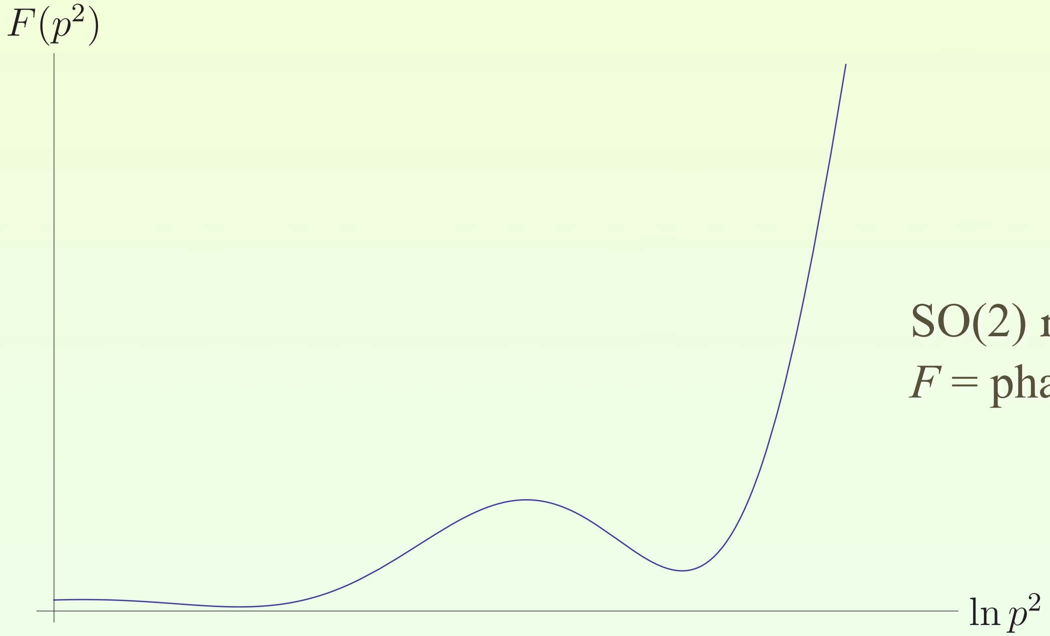
$C_{1,2}$ are $n_s \times n_s$ matrices, relation between them not forced by symmetry (as opposed to CFT case)

Cyclunparticles: As Georgi's unparticles but replacing CFT sector by SwC model

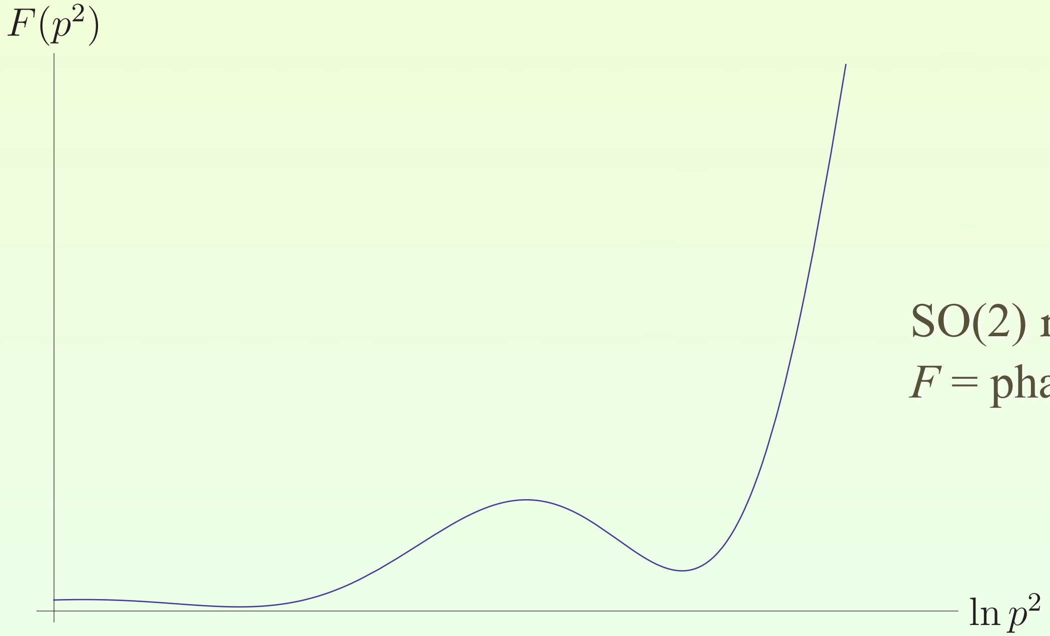
$$\mathcal{L} \supset g_a\chi\mathcal{O}_a + \text{h.c.}$$

SM source

Strongly coupled SI sector



SO(2) model with particular choice of Q_{12} and Δ
F = phase space for decay into SM+cyclunparticle



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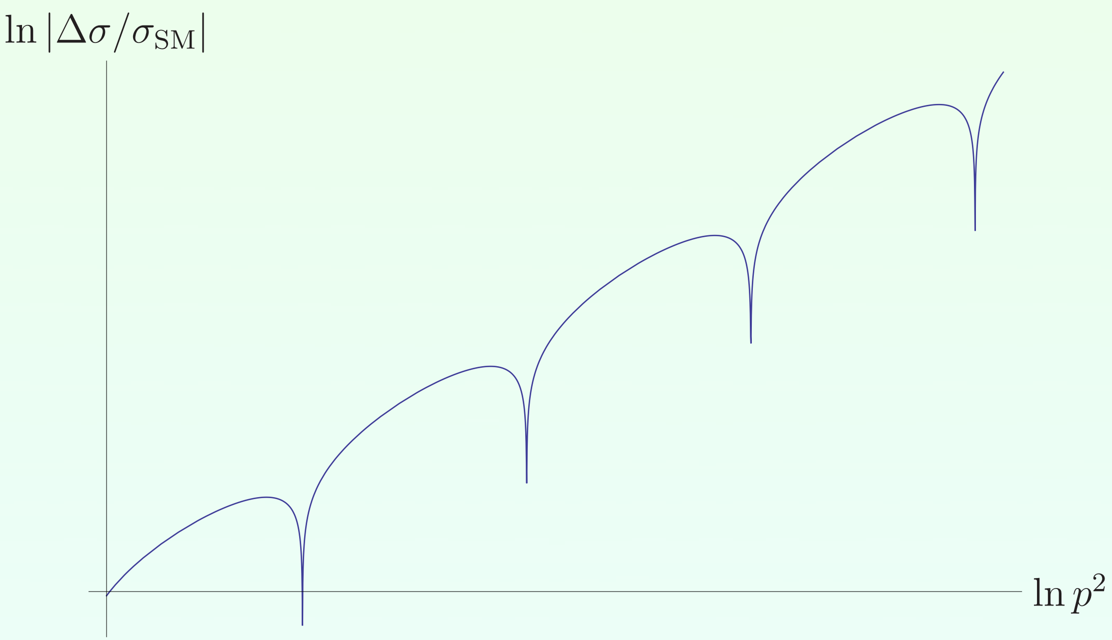
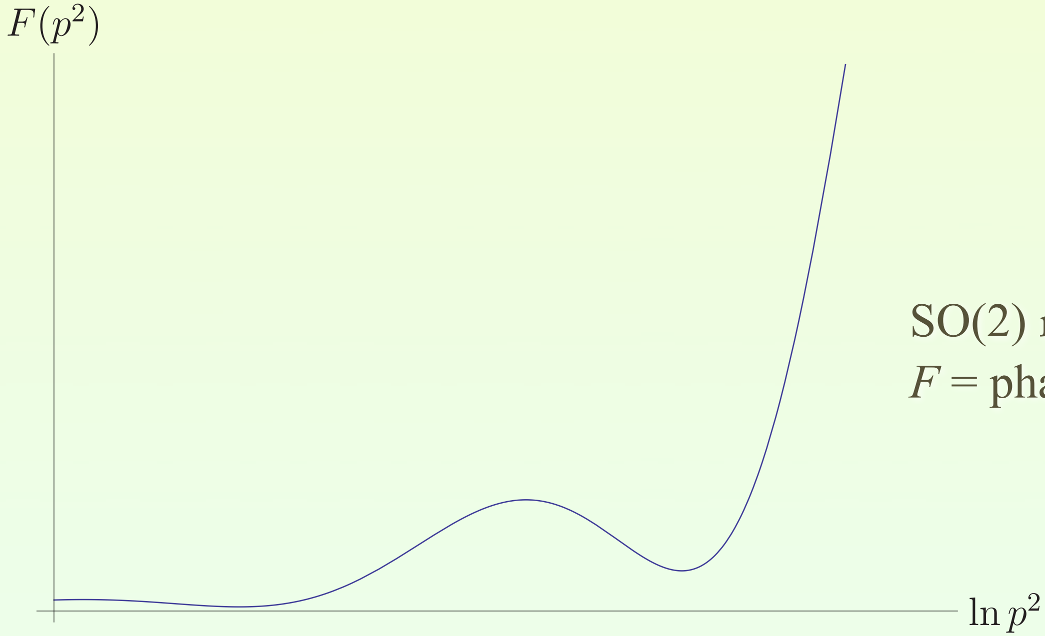


Interference in $e^+e^- \to \mu^+\mu^-$ cross section: again SO(2) model with particular choice of Q_{12} and Δ



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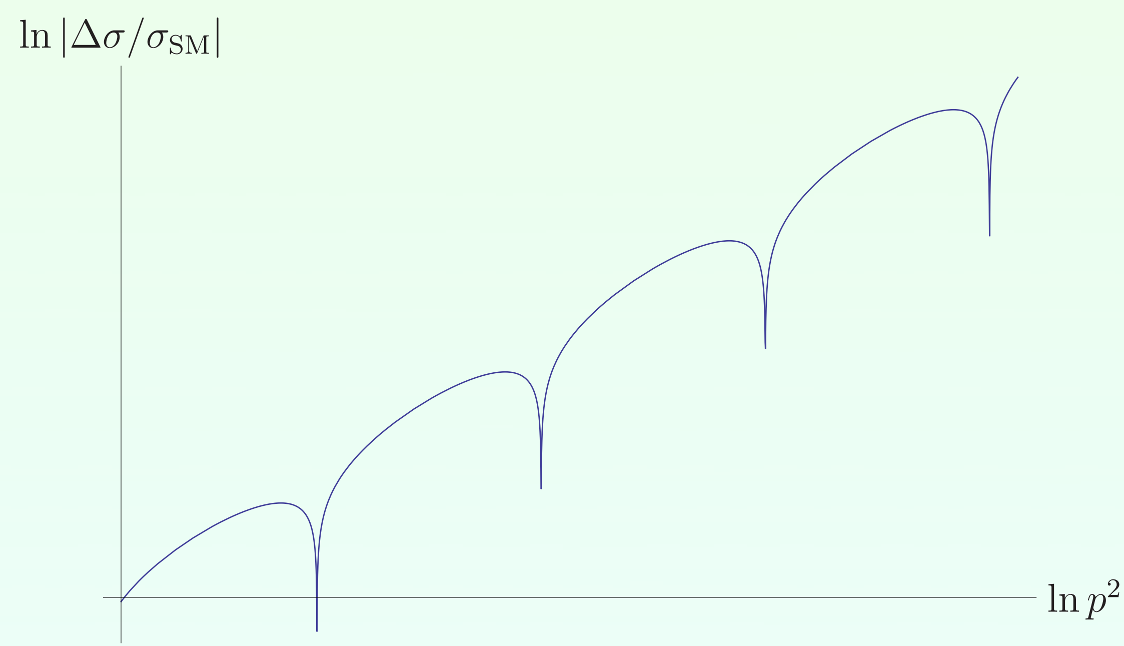

Interference in $e^+e^- \to \mu^+\mu^-$ cross section: again SO(2) model with particular choice of Q_{12} and Δ

Moral: SI is not to be confused with a CFT

Perturbative Solutions in $D = 4$

- Yang-Mills with Weyl-spinors and scalars
- Couplings:
  - YM: g
  - Scalar: λ
  - Yukawa: y
- Arrange for perturbative Caswell-Banks-Zaks fixed point g_* which drives λ and y toward fixed points and cycles

- Particular examples:
  - YM: SU(N)
  - Two real scalars, singlets under SU(N)
  - Two Weyl spinors in fundamental + two in anti-fundamental
  - The above produces at least as much complexity in flavor space as our $D = 4 - \varepsilon$, $n_s, n_f = 2,2$ model
  - Additional spinors in fundamental + anti-fundamental to achieve CBZ fixed point perturbatively
  - Have done $N = 2,3$ ($N = 2$ is questionable for perturbation theory)
  - Cycles found at 2-loops
  - Potentially undone by 3-loops (just as in $D = 4 - \varepsilon$)
  - 3-loop calculation in progress

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  - Cycles found at 2-loops
  - Potentially undone by 3-loops (just as in $D = 4 - \varepsilon$)
  - 3-loop calculation in ~~progress~~ done! PRELIMINARY: SwC found, with P = 0, Q $\neq$ 0

a-theorem

Three versions:

(i) Weak: $a_{\text{UV}} > a_{\text{IR}}$ for flows between fixed points (FP)

(ii) Strong: $da/dt \leq 0$, saturated at FP only

(iii) Strongest: gradient flow with positive definite metric

$$\beta^I = G^{IJ} \frac{\partial a}{\partial g^J} \quad \Rightarrow \quad \frac{da}{dt} = -\beta^J \frac{\partial a}{\partial g^J} = -\beta^J \beta^I G_{IJ} \leq 0$$

SFT and cycles:

(i) Weak: no conflict

(ii) Strong: no conflict if amended to "saturated at FPs and also on cycles"

(iii) Strongest: incompatible

Relation to some other work:

(iii) Strongest: not proven for any D

H. Osborn, "Weyl consistency conditions ...", Nucl.Phys. B363, 486 (1991).

Osborn: in general, for classically scale invariant $D = 4$ theory with dimensionless couplings g^I the Weyl anomaly coefficient a must satisfy

$$\frac{\partial \tilde{a}}{\partial g^I} = (G_{IJ} + \partial_{[I} w_{J]}) B^J$$

where $\tilde{a} = a + \frac{1}{8}\beta^I w_I$ and $B^J = \beta^J - (Qg)^J$

Hence $$\boxed{\frac{d\tilde{a}}{dt} = -(G_{IJ} + \partial_{[I} w_{J]}) B^J \beta^I}$$

Note that $w_I \neq 0$ even when $Q = 0$, but 2-loop exact ($\partial_{[I} w_{J]} = 0$) Also:

$$\frac{d\tilde{a}}{dt} = 0 \leftrightarrow \beta^I = 0 | B^I = 0$$ fixed points <u>or</u> cycles!

and

$$Q = 0 \quad \Rightarrow \quad \frac{d\tilde{a}}{dt} = -G_{IJ}\beta^J\beta^I \leq 0 \quad \text{with} \quad \frac{d\tilde{a}}{dt} = 0 \leftrightarrow \beta^I = 0$$

(ii) strong version:

- $D = 2$: Proof that scale implies CFT by Zamolodchikov + Polchinski
- $D = 4$: Osborn

$$Q = 0 \quad \Rightarrow \quad \frac{d\tilde{a}}{dt} = -G_{IJ}\beta^J\beta^I \leq 0 \qquad \text{with} \qquad \tilde{a} = a + \tfrac{1}{8}\beta^I w_I$$

Note that $\tilde{a} = a$ only at FPs, not along flow.

Luty *et al* claim, along flow $\dfrac{da}{dt} \leq 0$ $\Rightarrow$ Rattazzi's talk 1204.5221

Both arguments require perturbation theory.
Is there a contradiction?

With dim-3 operators Osborn finds:

$$\frac{d\tilde{a}}{dt} = -(G_{IJ} + \partial_{[I}w_{J]})B^J\beta^I$$

Luty *et al* claim inconsistency with cycles.

(i) weak version: Komargodski & Schwimmer claim proof 1107.3987 1112.4538

Obviously, lots of things left to do ...

- 3-loops
- Explore models in $D = 4$
- Find other classes of models in $D = 4$
- Supersymmetry?
- $D = 2 + \varepsilon$?
- $D = 6$?
- $D = 3$? Directly (perturbatively)
- $D = 3$ at strong coupling? (in $4 - \varepsilon \to 3$ limit)
- Flows, globally (from where to where?)
- Relation to NR-QM cycles (Efimov)?
- Gravity duals?
- Strong coupling (maybe through gravity duals?)
- ...

The End

Extra slides

Use EOM?

Trace anomaly:

$$T_\mu{}^\mu(x) = \gamma_{aa'} D^2\phi_a\phi_{a'} - \gamma^*_{i'i}\bar{\psi}_i i\bar{\sigma}^\mu D_\mu\psi_{i'} + \gamma_{ii'} D_\mu\bar{\psi}_i i\bar{\sigma}^\mu\psi_{i'}$$
$$- \tfrac{1}{4!}(\beta_{abcd} - \gamma_{a'a}\lambda_{a'bcd} - \gamma_{b'b}\lambda_{ab'cd} - \gamma_{c'c}\lambda_{abc'd} - \gamma_{d'd}\lambda_{abcd'})\phi_a\phi_b\phi_c\phi_d$$
$$- \tfrac{1}{2}(\beta_{a|ij} - \gamma_{a'a}y_{a'|ij} - \gamma_{i'i}y_{a|i'j} - \gamma_{j'j}y_{a|ij'})\phi_a\psi_i\psi_j + \text{h.c.}\,.$$

$$\partial_\mu D^\mu(x) = (\gamma_{aa'} + Q_{aa'})D^2\phi_a\phi_{a'} - (\gamma^*_{i'i} + P^*_{i'i})\bar{\psi}_i i\bar{\sigma}^\mu D_\mu\psi_{i'} + (\gamma_{ii'} + P_{ii'})D_\mu\bar{\psi}_i i\bar{\sigma}^\mu\psi_{i'}$$
$$-\tfrac{1}{4!}(\beta_{abcd} - \gamma_{a'a}\lambda_{a'bcd} - \gamma_{b'b}\lambda_{ab'cd} - \gamma_{c'c}\lambda_{abc'd} - \gamma_{d'd}\lambda_{abcd'})\phi_a\phi_b\phi_c\phi_d$$
$$-\tfrac{1}{2}(\beta_{a|ij} - \gamma_{a'a}y_{a'|ij} - \gamma_{i'i}y_{a|i'j} - \gamma_{j'j}y_{a|ij'})\phi_a\psi_i\psi_j + \text{h.c.}\,,$$

Using the EOM to eliminate anomalous dimensions is on the same footing as using EOM on the virial current

Note on strongest version in D=2

- Zamolodchikov proved it to 1st order in conformal perturbation theory
- Freedman *et al* show no longer generally possible at 2nd order — PRD 73, 066015 (2006)
- Friedan *et al* found correction to Zamolodchikov's metric and the w_I term — J Phys A: Math Theor 43 (2010) 215401

To be sure, these only guaranteed under ***natural*** scheme changes

A scheme change

$$\tilde{\lambda}_{abcd} = \lambda_{abcd} + \eta_{abcd}(\lambda, y, g)$$
$$\tilde{y}_{a|ij} = y_{a|ij} + \xi_{a|ij}(\lambda, y, g)$$

is *natural* if all couplings transform covariantly with respect to G_F (the symmetry group of the kinetic terms)

that is, if

$$\lambda_{abcd} \to R_{aa'} R_{bb'} R_{cc'} R_{dd'} \lambda_{a'b'c'd'} \quad \Rightarrow \quad \tilde{\lambda}_{abcd} \to R_{aa'} R_{bb'} R_{cc'} R_{dd'} \tilde{\lambda}_{a'b'c'd'}$$

and $\quad y_{a|ij} \to R_{aa'} \hat{R}_{ii'} \hat{R}_{jj'} y_{a'|i'j'} \quad \Rightarrow \quad \tilde{y}_{a|ij} \to R_{aa'} \hat{R}_{ii'} \hat{R}_{jj'} \tilde{y}_{a'|i'j'}$

Polchinski ('87): scalars only, 1-loop solutions: if SI then CFT

$$\beta_{abcd} = Q_{ae}\lambda_{ebcd} + \text{permutations}$$

$$\Rightarrow \qquad \sum_{a,b,c,d} \beta^2_{abcd} = \sum_{a,b,c,d} \beta_{abcd}(Q_{ae}\lambda_{ebcd} + \text{permutations})$$

now show RHS vanishes identically (for any value of coupling constant λ)

$$\beta_{abcd} = -\epsilon\lambda_{abcd} + \frac{\#}{16\pi^2}\left(\lambda_{abgh}\lambda_{cdgh} + \text{permutations}\right)$$

"classical" term: $\beta_{abcd}(Q_{ae}\lambda_{ebcd}) \propto Q_{ae}\lambda_{ebcd}\lambda_{abcd} = 0$

1-loop term: $\beta_{abcd}(Q_{ae}\lambda_{ebcd}) \propto Q_{ae}\lambda_{ebcd}\lambda_{abgh}\lambda_{cdgh} = 0$

Dorigoni&Rychkov ('10): scalar plus Weyl fermions, 1-loop: if SI then CFT

FGS ('11): obstruction to above argument appears at 2-loops (for model with Weyl+scalars)

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$a \leftrightarrow e$ symmetric

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Cyclunparticles

As Georgi's unparticles for CFTs, use SM to probe SI sector:

- weakly couple SM to SI model (possibly strongly coupled)
- use irrelevant operators to retain IR behavior
- see fractional phase space, but also possibly oscillatory behavior
- see odd scaling *and* oscillations in interference term in scattering

Cyclunparticle phase space: discontinuity of correlation function across real axis

For example, if $\mathcal{L} \supset g_a \chi \mathcal{O}_a + \text{h.c.}$

$\chi \to \chi$ forward scattering amplitude: $\mathcal{M}^{\text{fwd}} = g_a g_b |\chi|^2 \left[(-p^2 - i\epsilon)^{\frac{1}{2}(\Delta+Q)-1} C (-p^2 - i\epsilon)^{\frac{1}{2}(\Delta-Q)-1} \right]_{ab}$

Take imaginary part:

$$F(p^2) = -g_a g_b \left[(p^2)^{\frac{1}{2}(\Delta+Q)-1} \left\{ \cos\left[\left(\frac{\Delta+Q}{2} \right) \pi \right] C \sin\left[\left(\frac{\Delta-Q}{2} \right) \pi \right] \right.\right.$$
$$\left.\left. + \sin\left[\left(\frac{\Delta+Q}{2} \right) \pi \right] C \cos\left[\left(\frac{\Delta-Q}{2} \right) \pi \right] \right\} (p^2)^{\frac{1}{2}(\Delta-Q)-1} \right]_{ab}$$