Vacuum Alignment from Group Theory

Martin Holthausen

based on
MH, Michael A. Schmidt
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MH, Manfred Lindner, Michael A. Schmidt
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Lepton mixing from discrete groups

G_f e.g. $A_4 \cong (Z_2 \times Z_2) \rtimes Z_3 = \langle S,T \rangle$

$S_4 \cong (Z_2 \times Z_2) \rtimes S_3 = \langle S,T,U \rangle$

$L \to \rho(g) L$

complete flavour group

residual symmetry of ($m_e m_e^+$) $e \to \rho(g_e) e$

$$\rho(g_e)^T M_e M_e^\dagger \rho(g_e)^* = M_e M_e^\dagger$$

$$G_e = \langle T \rangle = Z_3$$

$$\rho(T) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

(Z3 smallest choice, but can also be continuous)

residual symmetry of m_ν $\nu \to \rho(g_\nu)$

$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$$

$$G_\nu = \langle S,U \rangle = Z_2 x Z_2$$

$$\rho(S) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \rho(U) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(Z2xZ2 most general choice if mixing angles do not depend on masses & Majorana νs)

misaligned non-commuting symmetries lead to

$$\Omega_e^\dagger \rho(g_e) \Omega_e = \rho(g_e)_{diag}$$

$$\Omega_\nu^\dagger \rho(g_\nu) \Omega_\nu = \rho(g_\nu)_{diag}$$

$$U_{PMNS} = \Omega_e^\dagger \Omega_U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{1}{3}, \quad \sin^2\theta_{23} = \frac{1}{2}, \quad \sin^2\theta_{13} = 0$$

[He, Keum, Volkas '06; Lam'07,'08; Altarelli,Feruglio'05, Feruglio, Hagedorn,Toroop'11]

tri-bimaximal mixing(TBM)

mixing matrix determined from symmetry up to interchanging of rows/columns and diagonal phase matrix

for impact of measurement $\sin^2\Theta_{13} \approx .03$, see talks by Feruglio, parallel session tomorrow

An A_4 Prototype model

- (A_4,Z_4) charge assignments: $L\sim(3,i)$, $e^c\sim(1_1,-i)$, $\mu^c\sim(1_2,-i)$, $\tau^c\sim(1_3,-i)$, $\chi\sim(3,1)$, $\Phi\sim(3,-1)$, $\xi\sim(1,-1)$
- auxiliary Z_4 separates neutral and charged lepton sectors at LO

$$y_e(\ell\chi)_{\underline{\mathbf{1}}_1}e^c + y_\mu(\ell\chi)_{\underline{\mathbf{1}}_3}\mu^c + y_\tau(\ell\chi)_{\underline{\mathbf{1}}_2}\tau^c$$

$\langle\chi\rangle\sim(1,1,1)$ $\qquad A_4 \qquad$ $\langle\Phi\rangle\sim(1,0,0)$

$$(LL)_{\underline{\mathbf{3}}}\phi + (LL)_{\underline{\mathbf{1}}_1}\xi$$

$Z_3=\langle T\rangle$ $\qquad\qquad$ $Z_2=\langle S\rangle$

$$M_E \sim \begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix} \quad \xleftrightarrow{\text{TBM}} \quad M_\nu \sim \begin{pmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{a} & \tilde{d} \\ 0 & \tilde{d} & \tilde{a} \end{pmatrix}$$

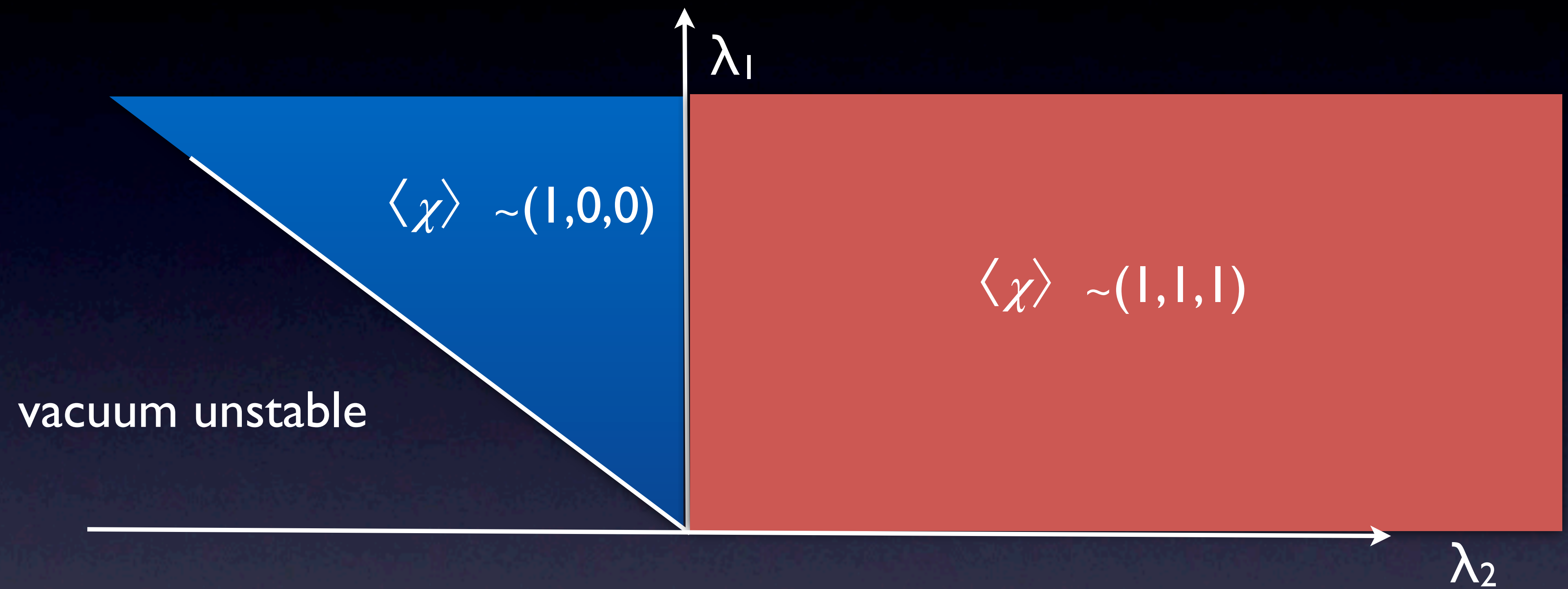
(symmetry U accidental)

Vacuum alignment crucial!

[e.g. Ma,Rajasekaran'01, Babu, Ma, Valle '03, Altarelli,Feruglio, '05,'06]

Can Vacuum Alignment be realised?

$$V_\chi = m_0^2\,(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} + \lambda_1\,(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} + \lambda_2\,(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{2}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{3}}$$



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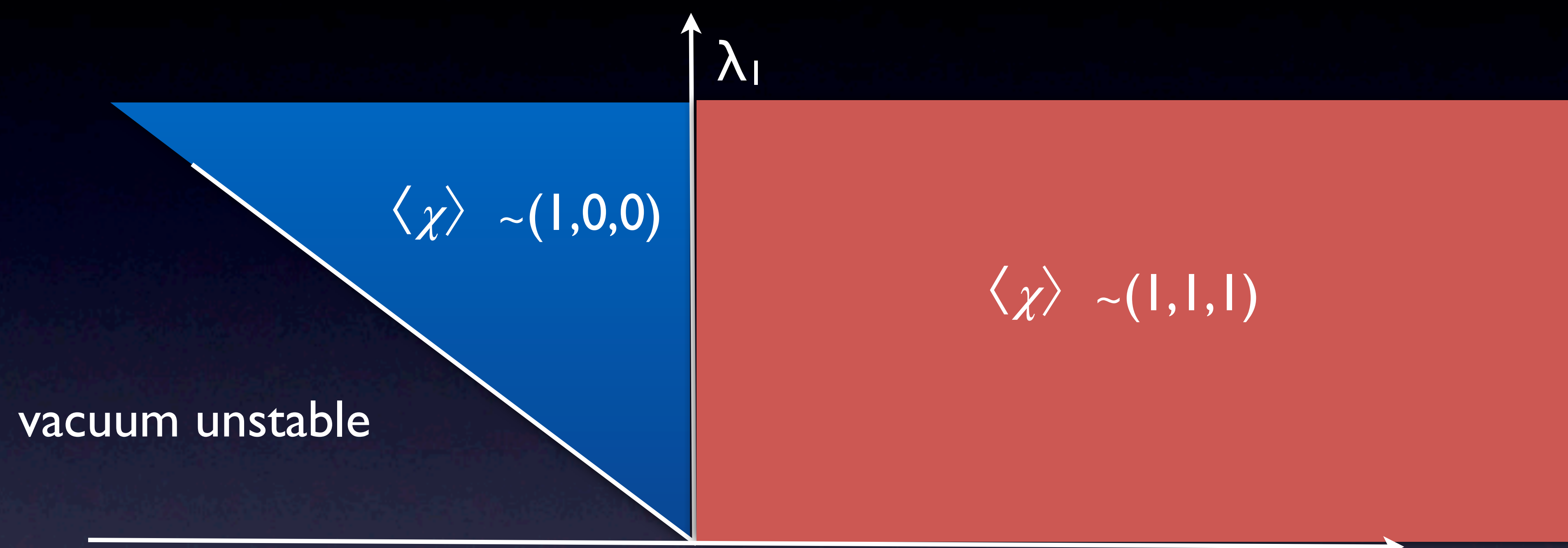


Effect of breaking to Z_2 in another sector can be included by adding:

$$V_{soft,Z_2} = m_A^2\chi_1^2 + m_B^2\chi_2^2 + m_C^2\chi_2\chi_3$$

Minimization conditions then give:

$$0 = \left[\frac{\partial V}{\partial \chi_1}\right]_{\chi_i=v'} = \frac{2}{\sqrt{3}}\left(m_0^2 + \sqrt{3}m_A^2\right)\,v' + 4\lambda_1 {v'}^3$$

$$0 = \left[\frac{\partial}{\partial \chi_2}V - \frac{\partial}{\partial \chi_3}V\right]_{\chi_i=v'} = 2\,m_B^2\,v'$$

$$0 = \left[\frac{\partial}{\partial \chi_1}V - \frac{\partial}{\partial \chi_3}V\right]_{\chi_i=v'} = \left(2\,m_A^2 - m_C^2\right)\,v'$$

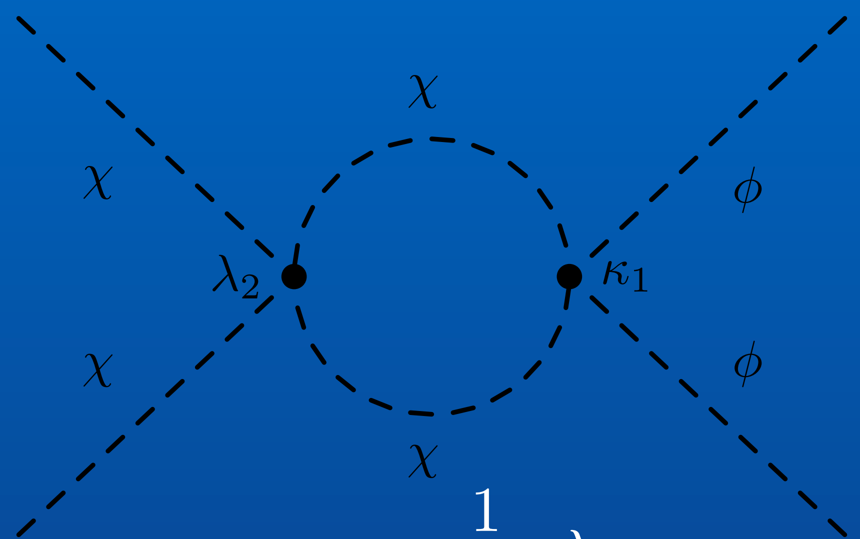
- This thus requires $m_A = m_B = m_C = 0$, i.e. all non-trivial contractions between Φ and χ have to vanish in the potential.

Can Vacuum Alignment be realized?

- To get the correct vacuum alignment, one thus needs to fine-tune the couplings

$$V_{\mathrm{mix}}(\chi,\phi) = \kappa_{\underline{\mathbf{3}}_\mathbf{1}}(\phi\phi)_{\underline{\mathbf{3}}_\mathbf{1}}(\chi\chi)_{\underline{\mathbf{3}}_\mathbf{1}} + \left(\kappa_{\underline{\mathbf{1}}_\mathbf{2}}(\phi\phi)_{\underline{\mathbf{1}}_\mathbf{2}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{3}} + \mathrm{h.c.}\right) + \rho_{\underline{\mathbf{3}}_\mathbf{1}}\phi(\chi\chi)_{\underline{\mathbf{3}}_\mathbf{1}}$$

- even if one sets the couplings to zero, they will be generated at one-loop level



$$\kappa_{\underline{\mathbf{1}}_\mathbf{2}} \sim \frac{1}{16\pi^2}\lambda_2\kappa_1$$

$\kappa_1(\phi\phi)_{\underline{\mathbf{1}}_\mathbf{1}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}}$

flavour conserving

- one needs a symmetry to enforce $V=V_\Phi(\Phi)+V_\chi(\chi)+(\Phi\Phi)_1(\chi\chi)_1$.

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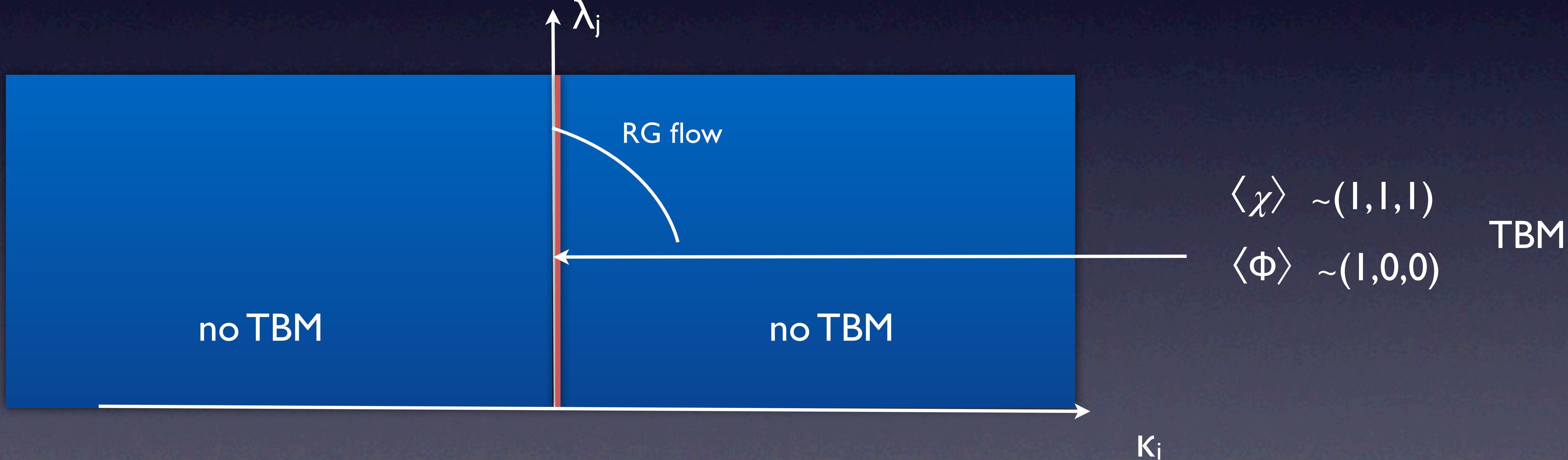

- for a natural model the realizes vacuum alignment, we need to have a finite portion of parameter space in which TBM vacuum is realized

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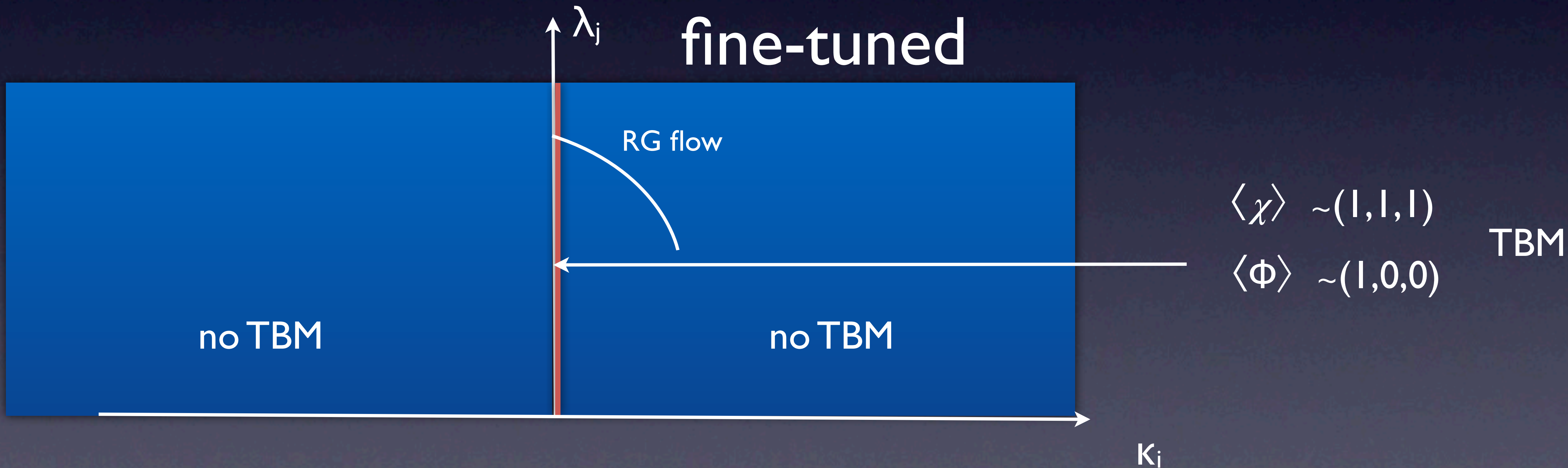

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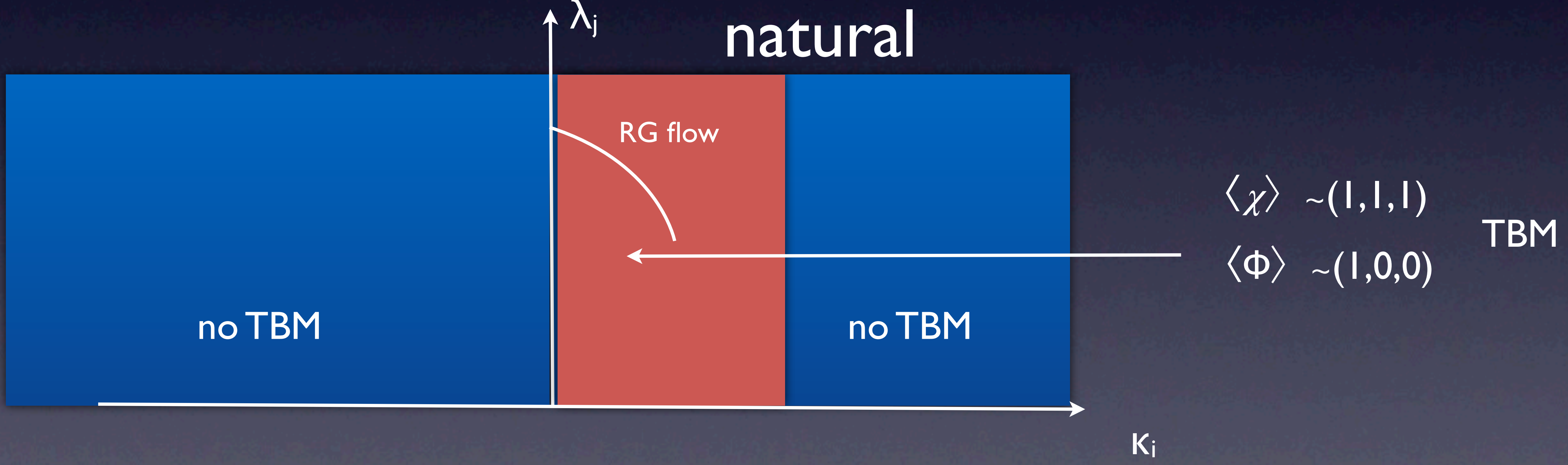

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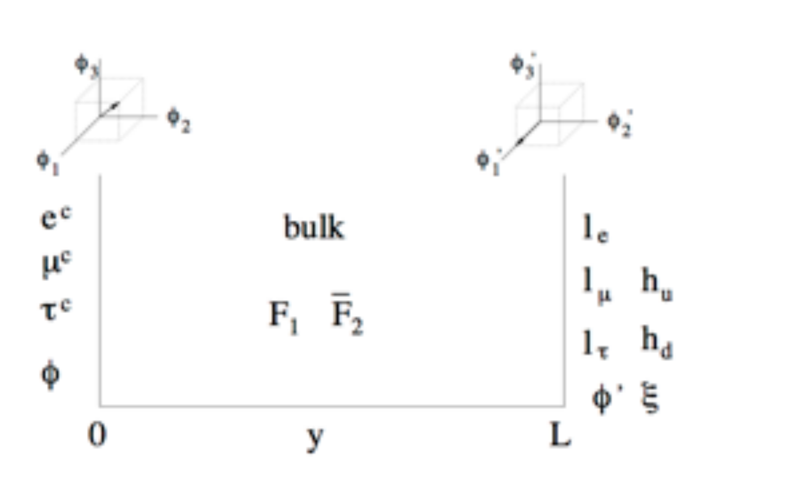
Solutions in the Literature

In models with extra dimensions(ED), it is possible to locate the various fields at different locations in the ED, thereby forbidding the cross-couplings.

Altarelli, Feruglio 2005



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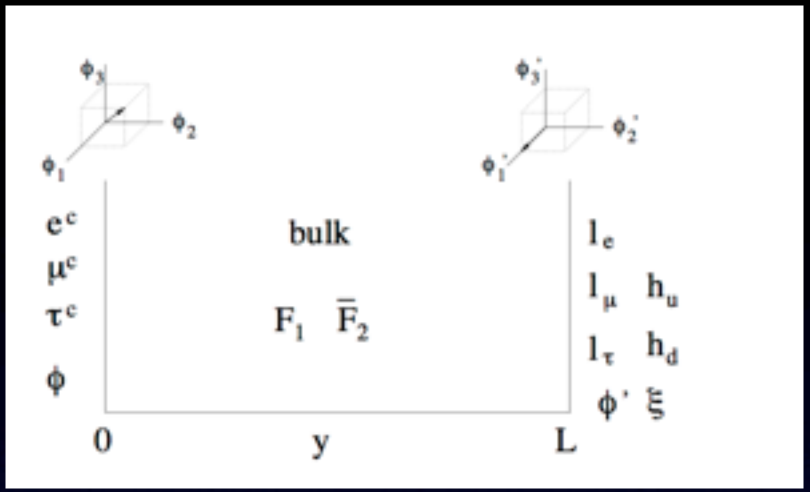

In SUSY, one has to introduce a continuous R-symmetry and additional fields with R-charge 2(driving fields). These fields enter the superpotential only linearly and allow the vacuum alignment.

Field	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	3	1	1	3	3	1
Z_3	1	ω	ω	ω	1	ω	ω
$U(1)_R$	0	0	0	0	2	2	2

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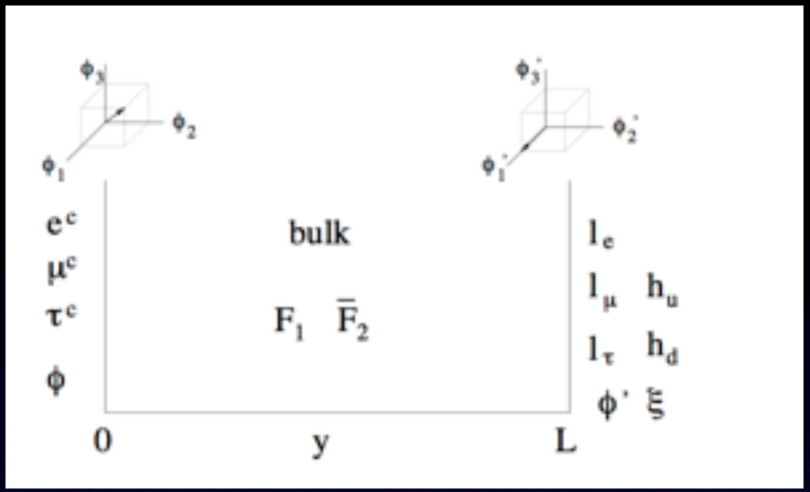

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A_4	3	3	1	1	3	3	1
Z_3	1	ω	ω	ω	1	ω	ω
$U(1)_R$	0	0	0	0	2	2	2

Altarelli, Feruglio 2006

Babu and Gabriel(2010) proposed the flavour group $(S_3)^4 \rtimes A_4$, which has the properties

- leptons transform only under A_4 subgroup
- if one takes $\Phi \sim 16$, vacuum alignment possible as $V=V(\Phi)+V(\chi)+(\Phi\,\Phi)_1(\chi\chi)_1$
- neutrino masses then generated by coupling to $\langle \Phi^4 \rangle \sim (1,0,0)$

Group extensions and Vacuum alignment

- To solve the vacuum alignment problem, we extend the flavour group H [e.g. the successful groups H=A_4,T_7,S_4,T' or $\Delta(27)$].
- we require the following:
  - lepton structure should be same → irreps of H should be promoted to irreps of G, we therefore need a surjective homomorphism $\xi : G \rightarrow H$ such that $\rho^G = \rho^H \circ \xi$, $l \sim \underline{\mathbf{3}}^G$, $\chi \sim \underline{\mathbf{3}}^G$
  - there should be an irrep Φ, the product Φ^n should contain a $\underline{\mathbf{3}}^G$
- renormalizable scalar potential should be of form: $V=V(\Phi)+V(\chi)+(\Phi\Phi)_1(\chi\chi)_1$.

$(l\chi)e^c$

$$
\begin{tikzcd}
 & G \arrow[dl, "\langle\chi\rangle \sim (1,1,1)"'] \arrow[dr, "\langle\Phi^n\rangle \sim (1,0,0)"] & \\
Z_3 = \langle T \rangle \arrow[rr, leftrightarrow, "\mathrm{TBM}"'] & & Z_2 = \langle S \rangle
\end{tikzcd}
$$

$(ll)_3\Phi^n+(ll)_1\Phi^n$

Scan for Small Groups

- using the computer algebra system GAP and its SmallGroups catalogue, we have checked all groups with size smaller than 1000 (11,758,814 groups) and we have found a number of candidates:

- no candidates for T_7 or $\Delta(27)$, maybe because here 3 is complex and there are more couplings that have to be forbidden (also smaller number of possible extensions)

- all candidates in list have non-trivial centre(=elements that commute with all other elements), not necessary true for all groups(see e.g. $(S_3)^4 \rtimes A_4$ studied in Babu/Gabriel 2010)

Subgroup H	Order of G	GAP	Structure Description	$Z(G)$
A_4	96	204	$Q_8 \rtimes A_4$	Z_2
	288	860	$T' \rtimes A_4$	Z_2
	384	617, 20123	$((Z_2 \times Q_8) \rtimes Z_2) \rtimes A_4$	Z_2
	576	8273	$(Z_2.S_4) \rtimes A_4$	Z_2
	768	1083945	$(Z_4.Z_4^2) \rtimes A_4$	Z_4
		1085279	$((Z_2 \times Q_{16}) \rtimes Z_2) \rtimes A_4$	Z_2
S_4	192	1494	$Q_8 \rtimes S_4$	Z_2
	384	18133, 20092	$(Z_2 \times Q_8) \rtimes S_4$	Z_2
		20096	$((Z_4 \times Z_2) \rtimes Z_2) \rtimes S_4$	Z_4
	576	8282	$T' \rtimes S_4$	Z_2
		8480	$(Z_3 \times Q_8) \rtimes S_4$	Z_6
	768	1086052, 1086053	$((Z_2 \times Q_8) \rtimes Z_2) \rtimes S_4$	Z_2
	960	11114	$(Z_5 \times Q_8) \rtimes S_4$	Z_{10}
T'	192	1022	$Q_8 \rtimes T'$	Z_2^2
	648	533	$\Delta(27) \rtimes T'$	Z_3
	768	1083573, 1085187	$((Z_2 \times Q_8) \rtimes Z_2) \rtimes T'$	Z_2^2

Groups of the Structure $G \simeq N \rtimes H$, H is subgroup of G defined by an homomorphism φ:H→Aut(N) that defines the product (n1,h1)*(n2,h2)= (n1φh1(n2), h1 h2)

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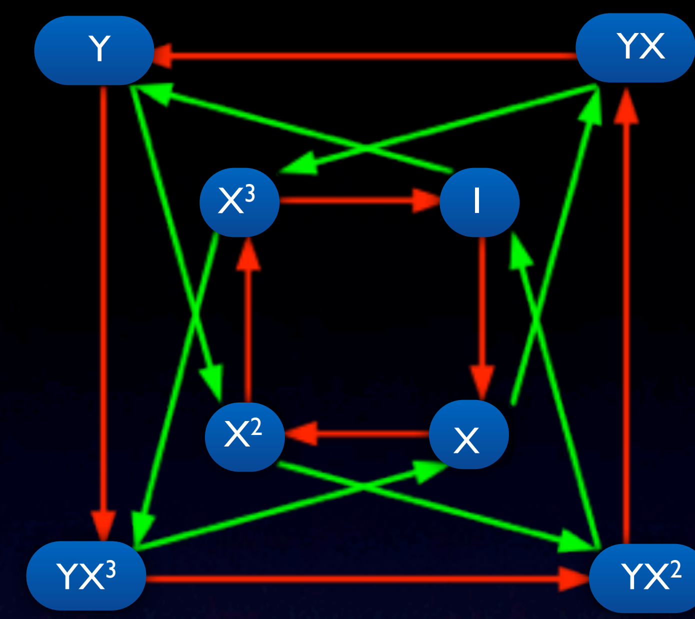
- all candidates in list have non-trivial centre(=elements that commute with all other elements), not necessary true for all groups(see e.g. $(S_3)^4 \rtimes A_4$ studied in Babu/Gabriel 2010)

Quotient Group H	Order of G	GAP	Structure Description
A_4	96	201	$Z_2.(Z_2^2 \times A_4)$
	144	127	$Z_2.(A_4 \times S_3)$
	192	1017	$Z_2.(D_8 \times A_4)$
S_4	96	67, 192	$Z_4.S_4$
	144	121, 122	$Z_6.S_4$
	192	187, 963	$Z_8.S_4$
	192	987, 988	$Z_2.((Z_2^2 \times A_4) \rtimes Z_2)$
	192	1483,1484	$Z_2.(Z_2^2 \times S_4)$
	192	1492	$Z_2.((Z_2^4 \rtimes Z_3) \rtimes Z_2)$
T'	192	1007	$Z_2^2.(Z_2^2 \times A_4)$

Groups N.H for which H is not a subgroup of G, with $G/N \simeq H$,
as this is enough to ensure the existence of the relevant representations(e.g. $T'/Z_2=A_4$).

Smallest Group



The smallest candidate group that contains A_4 as a subgroup is the semidirect product of the quaternion group Q_8

$$\langle X, Y | X^4 = 1,\ X^2 = Y^2,\ Y^{-1}XY = X^{-1} \rangle$$

with A_4

$$\langle S, T | S^2 = T^3 = (ST)^3 = 1 \rangle$$

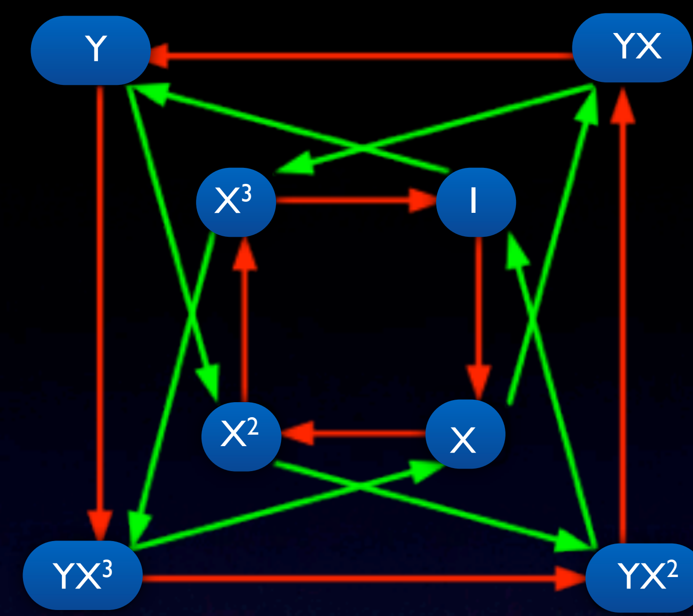
defined by the additional relations

$$SXS^{-1} = X, \qquad SYS^{-1} = Y^{-1}, \qquad TXT^{-1} = YX, \qquad TYT^{-1} = X\ .$$

Representations:

		1	T	SYX	SY	Y^2	T^2	TY	S	SX	X	$STYT$
	$\underline{\mathbf{1}}_1$	1	1	1	1	1	1	1	1	1	1	1
unfaithful A_4 reps for	$\underline{\mathbf{1}}_2$	1	ω	1	1	1	ω^2	ω	1	1	1	ω^2
leptons, χ	$\underline{\mathbf{1}}_3$	1	ω^2	1	1	1	ω	ω^2	1	1	1	ω
	$\underline{\mathbf{3}}_1$	3	.	-1	-1	3	.	.	-1	-1	3	.
	$\underline{\mathbf{3}}_2$	3	.	3	-1	3	.	.	-1	-1	-1	.
	$\underline{\mathbf{3}}_3$	3	.	-1	3	3	.	.	-1	-1	-1	.
	$\underline{\mathbf{3}}_4$	3	.	-1	-1	3	.	.	3	-1	-1	.
	$\underline{\mathbf{3}}_5$	3	.	-1	-1	3	.	.	-1	3	-1	.
faithful rep for Φ	$\underline{\mathbf{4}}_1$	4	1	.	.	-4	1	-1	.	.	.	-1
	$\underline{\mathbf{4}}_2$	4	ω^2	.	.	-4	ω	$-\omega^2$	.	.	.	$-\omega$
	$\underline{\mathbf{4}}_3$	4	ω	.	.	-4	ω^2	$-\omega$	.	.	.	$-\omega^2$

Smallest Group



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Representations:

$$\underline{\mathbf{3}}_{\mathbf{i}} \times \underline{\mathbf{3}}_{\mathbf{i}} = \underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}} + \underline{\mathbf{3}}_{\mathbf{i}\,S} + \underline{\mathbf{3}}_{\mathbf{i}\,A}$$

$$\underline{\mathbf{3}}_{\mathbf{i}} \times \underline{\mathbf{3}}_{\mathbf{j}} = \sum_{\substack{k=1 \\ k \neq i,j}}^{5} \underline{\mathbf{3}}_{\mathbf{k}} \qquad (i \neq j)$$

$$\underline{\mathbf{3}}_{\mathbf{i}} \times \underline{\mathbf{4}}_{\mathbf{j}} = \underline{\mathbf{4}}_{\mathbf{1}} + \underline{\mathbf{4}}_{\mathbf{2}} + \underline{\mathbf{4}}_{\mathbf{3}}$$

$$\underline{\mathbf{4}}_{\mathbf{1}} \times \underline{\mathbf{4}}_{\mathbf{1}} = \underline{\mathbf{1}}_{\mathbf{1}\,S} + \underline{\mathbf{3}}_{\mathbf{1}\,A} + \underline{\mathbf{3}}_{\mathbf{2}\,S} + \underline{\mathbf{3}}_{\mathbf{3}\,S} + \underline{\mathbf{3}}_{\mathbf{4}\,S} + \underline{\mathbf{3}}_{\mathbf{5}\,A}$$

$$\underline{\mathbf{4}}_{\mathbf{1}} \times \underline{\mathbf{4}}_{\mathbf{2}} = \underline{\mathbf{1}}_{\mathbf{2}\,S} + \underline{\mathbf{3}}_{\mathbf{1}\,A} + \underline{\mathbf{3}}_{\mathbf{2}\,S} + \underline{\mathbf{3}}_{\mathbf{3}\,S} + \underline{\mathbf{3}}_{\mathbf{4}\,S} + \underline{\mathbf{3}}_{\mathbf{5}\,A}$$

		S	T	X	Y
A_4 reps	$\underline{\mathbf{1}}_{\mathbf{1}}$	1	1	1	1
	$\underline{\mathbf{1}}_{\mathbf{2}}$	1	ω	1	1
	$\underline{\mathbf{1}}_{\mathbf{3}}$	1	ω^2	1	1
	$\underline{\mathbf{3}}_{\mathbf{1}}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Φ	$\underline{\mathbf{4}}_{\mathbf{1}}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$

faithful representation Φ is what we were looking for.
(Φ Φ) only contains non-trivial contraction of the A_4 subgroup.

The model

particle	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
ϕ_2	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	-1

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ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
ϕ_2	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	-1

$$\langle\chi\rangle = (v', v', v')^T,$$

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}(a, a, b, -b)^T,$$

$$\langle\phi_2\rangle = \frac{1}{\sqrt{2}}(c, c, d, -d)^T$$

$$\langle(\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}}\rangle = \frac{1}{2}(bc - ad, 0, 0)^T$$

$$\langle(\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}}\rangle = \frac{1}{2}(ac + bd)$$

The model

particle	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
ϕ_2	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	-1

VEVs:

$$\langle\chi\rangle = (v', v', v')^T,$$

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}(a, a, b, -b)^T,$$

$$\langle\phi_2\rangle = \frac{1}{\sqrt{2}}(c, c, d, -d)^T$$

$$\langle(\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}}\rangle = \frac{1}{2}(bc - ad, 0, 0)^T$$

$$\langle(\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}}\rangle = \frac{1}{2}(ac + bd)$$

LO charged lepton masses:

$$\mathcal{L}_e^{(5)} = y_e(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} e^c \tilde{H}/\Lambda + y_\mu(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{3}}} \mu^c \tilde{H}/\Lambda + y_\tau(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}} \tau^c \tilde{H}/\Lambda + \text{h.c.}\ ,$$

$$\downarrow$$

$$M_E \sim \begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix}$$

The model

particle	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
ϕ_2	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	-1

VEVs:

$$\langle\chi\rangle = (v', v', v')^T,$$

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}(a, a, b, -b)^T,$$

$$\langle\phi_2\rangle = \frac{1}{\sqrt{2}}(c, c, d, -d)^T$$

$$\langle(\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}}\rangle = \frac{1}{2}(bc - ad, 0, 0)^T$$

$$\langle(\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}}\rangle = \frac{1}{2}(ac + bd)$$

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LO neutral lepton masses:

$$\mathcal{L}_\nu^{(7)} = x_a(\ell H \ell H)_{\underline{\mathbf{1}}_{\mathbf{1}}} (\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}}/\Lambda^3 + x_d(\ell H \ell H)_{\underline{\mathbf{3}}_{\mathbf{1}}} \cdot (\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}}/\Lambda^3 + \text{h.c.}\ .$$

$$M_E \sim \begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix} \qquad \longleftrightarrow \qquad M_\nu \sim \begin{pmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{a} & \tilde{d} \\ 0 & \tilde{d} & \tilde{a} \end{pmatrix}$$

TBM

(symmetry U accidental)

The model

particle	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
ϕ_2	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	-1

VEVs:

$$\langle \chi \rangle = (v', v', v')^T,$$
$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}}(a, a, b, -b)^T,$$
$$\langle \phi_2 \rangle = \frac{1}{\sqrt{2}}(c, c, d, -d)^T$$
$$\langle (\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}} \rangle = \frac{1}{2}(bc - ad, 0, 0)^T$$
$$\langle (\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}} \rangle = \frac{1}{2}(ac + bd)$$

LO charged lepton masses:

$$\mathcal{L}_e^{(5)} = y_e(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} e^c \tilde{H}/\Lambda + y_\mu(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{3}}} \mu^c \tilde{H}/\Lambda + y_\tau(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}} \tau^c \tilde{H}/\Lambda + \text{h.c.}\ ,$$

LO neutral lepton masses:

$$\mathcal{L}_\nu^{(7)} = x_a(\ell H \ell H)_{\underline{\mathbf{1}}_{\mathbf{1}}} (\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}}/\Lambda^3 + x_d(\ell H \ell H)_{\underline{\mathbf{3}}_{\mathbf{1}}} \cdot (\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}}/\Lambda^3 + \text{h.c.}\ .$$

- additional 4_1 necessary to get correct symmetry breaking (otherwise only breaking to A_4)
- same # of d.o.f. as in case of complex triplet and singlet, no additional driving fields necessary
- low flavour symmetry breaking scale possible, testable

Scalar Potential & Vacuum Alignment

The most general scalar potential invariant under the flavour symmetry is given by

$$V(\chi,\phi_1,\phi_2) = V_\chi(\chi) + V_\phi(\phi_1,\phi_2) + V_{\rm mix}(\chi,\phi_1,\phi_2)$$

with

$$\boxed{\begin{aligned}
V_\phi(\phi_1,\phi_2) =& \mu_1^2(\phi_1\phi_1)_{\underline{\mathbf{1}}_\mathbf{1}} + \alpha_1(\phi_1\phi_1)^2_{\underline{\mathbf{1}}_\mathbf{1}} + \sum_{i=2,3}\alpha_i(\phi_1\phi_1)_{\underline{\mathbf{3}}_\mathbf{i}}\cdot(\phi_1\phi_1)_{\underline{\mathbf{3}}_\mathbf{i}} \\
&+\mu_2^2(\phi_2\phi_2)_{\underline{\mathbf{1}}_\mathbf{1}} + \beta_1(\phi_2\phi_2)^2_{\underline{\mathbf{1}}_\mathbf{1}} + \sum_{i=2,3}\beta_i(\phi_2\phi_2)_{\underline{\mathbf{3}}_\mathbf{i}}\cdot(\phi_2\phi_2)_{\underline{\mathbf{3}}_\mathbf{i}} \\
&+\gamma_1(\phi_1\phi_1)_{\underline{\mathbf{1}}_\mathbf{1}}(\phi_2\phi_2)_{\underline{\mathbf{1}}_\mathbf{1}} + \sum_{i=2,3,4}\gamma_i(\phi_1\phi_1)_{\underline{\mathbf{3}}_\mathbf{i}}\cdot(\phi_2\phi_2)_{\underline{\mathbf{3}}_\mathbf{i}} \\
V_\chi(\chi) =& \mu_3^2(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} + \rho_1(\chi\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} + \lambda_1(\chi\chi)^2_{\underline{\mathbf{1}}_\mathbf{1}} + \lambda_2(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{2}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{3}} \\
V_{\rm mix}(\chi,\phi_1,\phi_2) =& \zeta_{13}(\phi_1\phi_1)_{\underline{\mathbf{1}}_\mathbf{1}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} + \zeta_{23}(\phi_2\phi_2)_{\underline{\mathbf{1}}_\mathbf{1}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}}
\end{aligned}}$$

- Potential has an accidental symmetry $[(Q_8 \rtimes A_4)\times A_4]\times Z_4$
  - invariant under independent transformations of Φ and χ
- note that couplings such as $\chi\cdot(\phi_1\phi_2)_{\underline{\mathbf{3}}_\mathbf{1}}$ are forbidden by the auxiliary Z4 symmetry that separates the charged and neutral lepton sectors

Scalar Potential & Vacuum Alignment

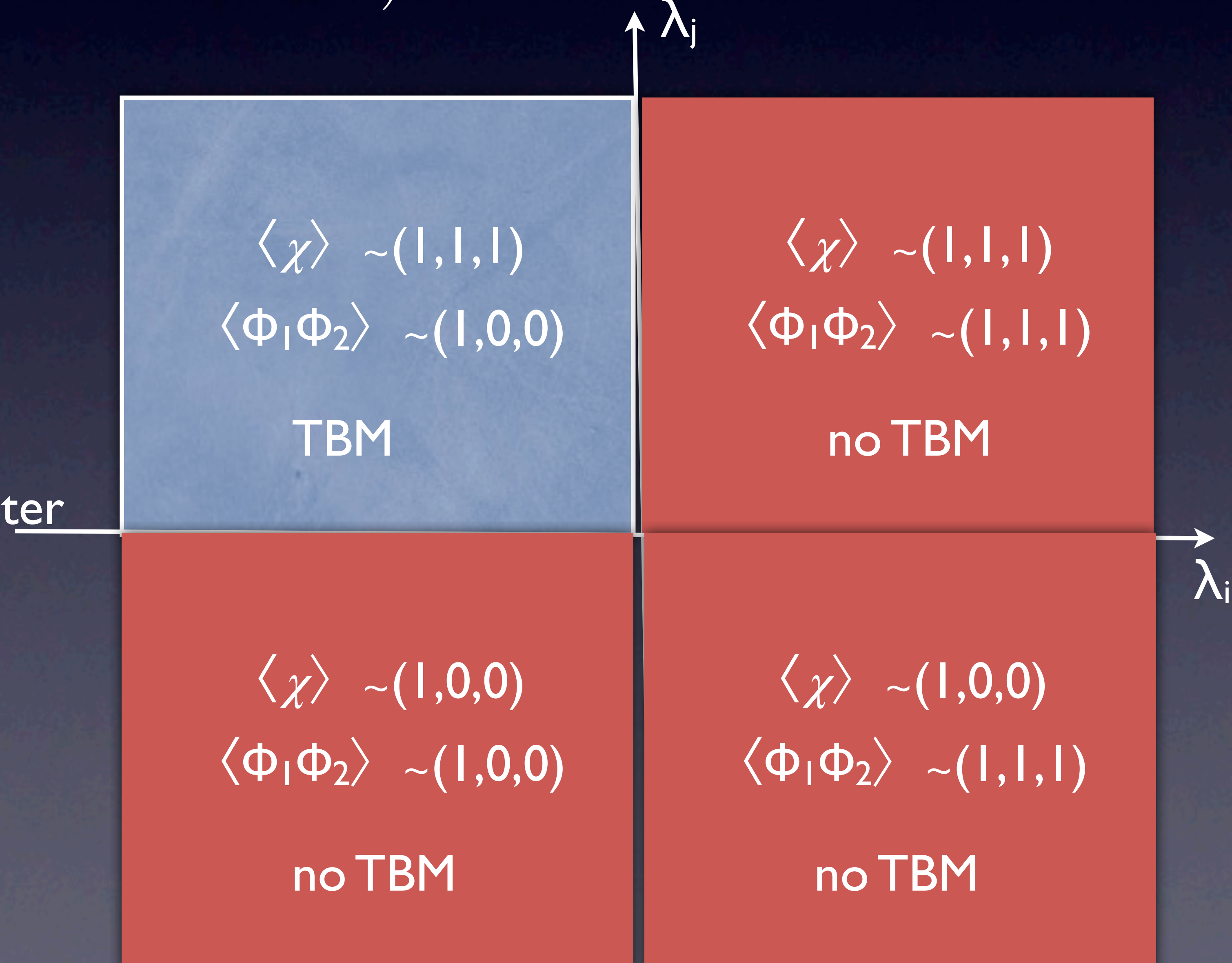
Minimum Conditions

$$a\left(\alpha_+\left(a^2+b^2\right)+\alpha_-\left(a^2-b^2\right)+\gamma_+\left(c^2+d^2\right)+\gamma_-\left(c^2-d^2\right)+U_1\right)+\Gamma bcd=0$$
$$b\left(\alpha_+\left(a^2+b^2\right)-\alpha_-\left(a^2-b^2\right)+\gamma_+\left(c^2+d^2\right)-\gamma_-\left(c^2-d^2\right)+U_1\right)+\Gamma acd=0$$
$$c\left(\beta_+\left(c^2+d^2\right)+\beta_-\left(c^2-d^2\right)+\gamma_+\left(a^2+b^2\right)+\gamma_-\left(a^2-b^2\right)+U_2\right)+\Gamma abd=0$$
$$d\left(\beta_+\left(c^2+d^2\right)-\beta_-\left(c^2-d^2\right)+\gamma_+\left(a^2+b^2\right)-\gamma_-\left(a^2-b^2\right)+U_2\right)+\Gamma abc=0$$
$$v'\left(4\sqrt{3}\lambda_1 v'^2+3\rho_1 v'+2\mu_3^2+\zeta_{13}(a^2+b^2)+\zeta_{23}(c^2+d^2)\right)=0$$

with

$$\xi_+=\frac{\xi_1}{2},\xi_-=\frac{\xi_2+\xi_3}{2\sqrt{3}}\text{ for }\xi=\alpha,\beta$$
$$\gamma_+=\frac{\sqrt{3}\gamma_1+\gamma_4}{4\sqrt{3}},\quad \gamma_-=\frac{\gamma_2+\gamma_3}{4\sqrt{3}}\quad\text{and}\quad\Gamma=\frac{\gamma_4}{\sqrt{3}}$$

- eleven minimization conditions reduce to these 5 equations for 5 VEVs there is therefore generally a solution
- we have performed a numerical study to show that there is finite region of parameter space where the desired vacuum configuration is the global minimum



Higher Order Corrections

- NLO Corrections to vacuum potential

$$V^{(5)} = \sum_{L,M=1}^{2} \sum_{i,j=2}^{4} \frac{\delta_{ij}^{(LM)}}{\Lambda} \chi \cdot \left\{ (\phi_L \phi_L)_{\underline{\mathbf{3}}_{\mathbf{i}}} \cdot (\phi_M \phi_M)_{\underline{\mathbf{3}}_{\mathbf{j}}} \right\}_{\underline{\mathbf{3}}_{\mathbf{1}}} + \\ + \frac{\chi^3}{\Lambda} \left(\delta_1^{(3)} \chi^2 + \delta_2^{(3)} (\phi_1 \phi_1)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \delta_3^{(3)} (\phi_2 \phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}} \right)$$

$\delta_{ij}^{(LM)} = 0$ for $i \geq j$

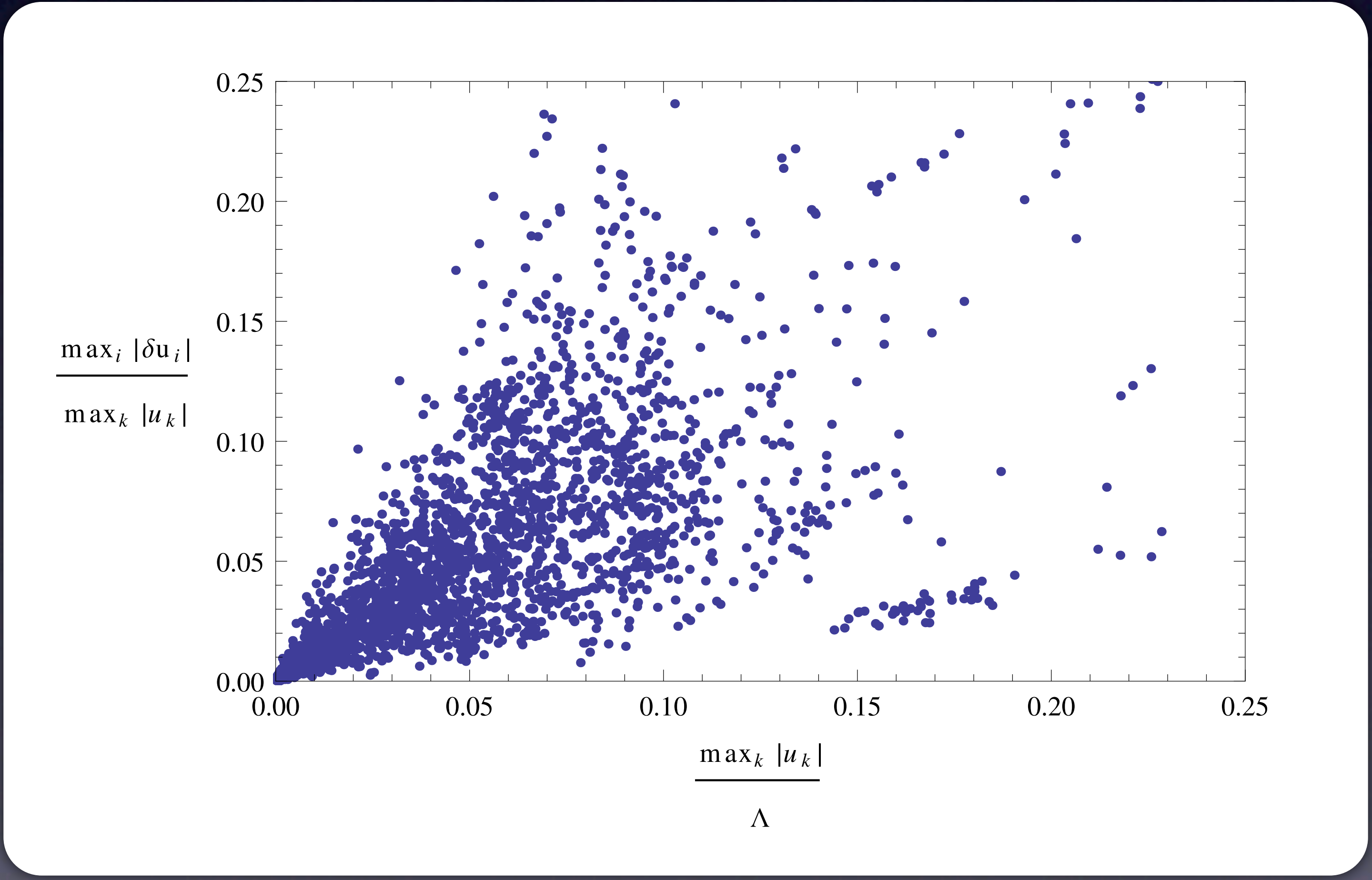
- leads to shifts in VEVs

$$\langle \chi \rangle = (v' + \delta v'_1, v' + \delta v'_2, v' + \delta v'_2)^T,$$
$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} (a + \delta a_1, a + \delta a_2, b + \delta a_3, -b + \delta a_4)^T,$$
$$\langle \phi_2 \rangle = \frac{1}{\sqrt{2}} (c + \delta b_1, c + \delta b_2, d + \delta b_3, -d + \delta b_4)^T$$

- generic size of shifts

$$\frac{\delta u}{u} \sim \frac{u}{\Lambda}$$

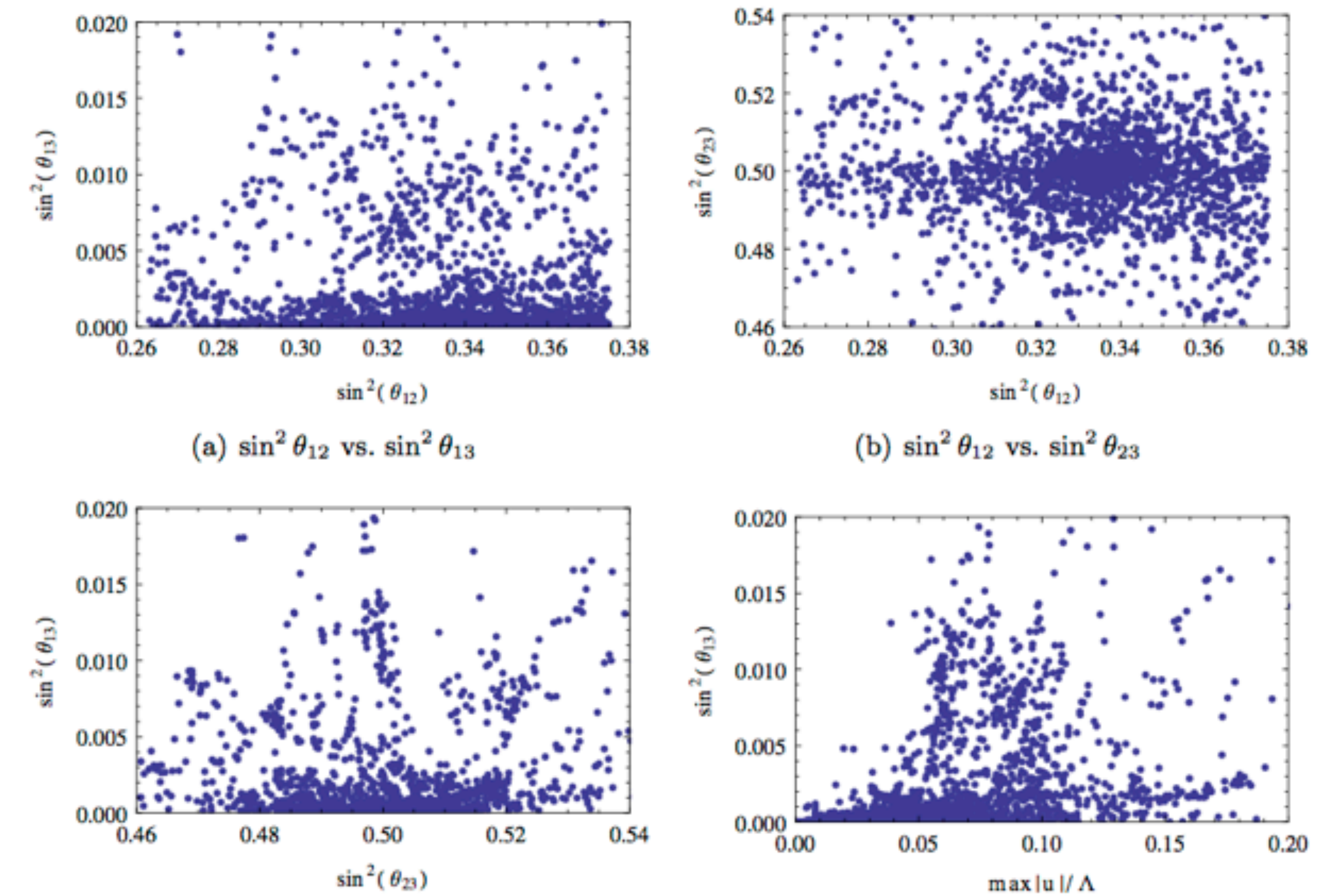
$$\langle \chi_2 \rangle - \langle \chi_3 \rangle = \mathcal{O}(1/\Lambda^2)$$



generic size of shifts for scalar potential parameters of order one

VEV alignment not destroyed!

Higher Order Corrections



(a) $\sin^2\theta_{12}$ vs. $\sin^2\theta_{13}$

(b) $\sin^2\theta_{12}$ vs. $\sin^2\theta_{23}$

(c) $\sin^2\theta_{13}$ vs. $\sin^2\theta_{23}$

(d) $\sin^2\theta_{13}$ vs. $\frac{\max_i u_i}{\Lambda}$

- $\sin^2\Theta_{13}\approx.03$ can be accommodated at NLO
- or by introducing additional non-trivial singlet field $\xi \sim (1_2,i)$ giving trimaximal mixing[does not destroy VEV alignment]

[Lin'10, Shimizu,Tanimoto, Watanabe'11,Luhn,King'11]

Flavour Breaking at the Electroweak Scale

- mechanism allows for low scale flavour breaking, testability of alignment sector

- Change model such that χ is an EW doublet, Φs singlet, add messenger fields to make it renormalizable

fermion	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
S	1	0	$\underline{\mathbf{3}}_2$	-1

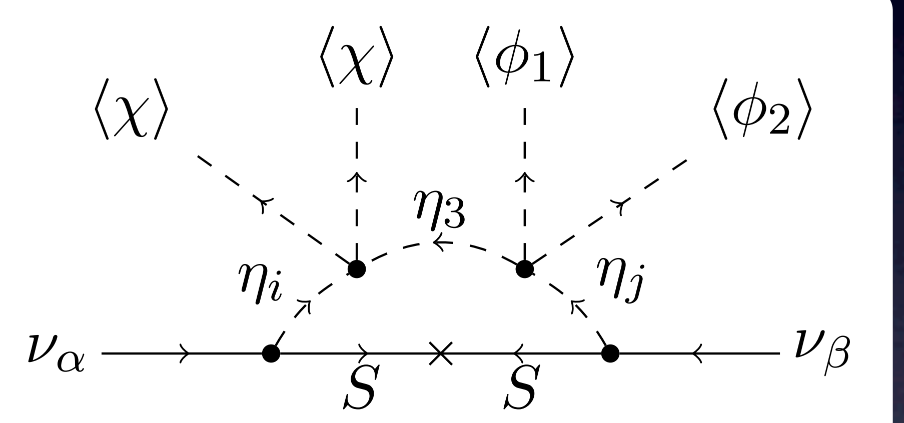
scalars	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
η_1	2	1/2	$\underline{\mathbf{3}}_5$	i
η_2	2	1/2	$\underline{\mathbf{3}}_4$	i
η_3	2	1/2	$\underline{\mathbf{3}}_5$	$-$i

- neutrino masses are generated at one-loop level

$$\mathcal{L}_\nu = h_1 L \eta_1 S + h_2 L \eta_2 S + \sqrt{3}\, M_S S S + \text{h.c.}\ .$$

$$V_{\eta,\phi} = \lambda_3 (\phi_1 \phi_2)_{\underline{\mathbf{1}}_1} (\eta_3^\dagger \eta_1)_{\underline{\mathbf{1}}_1} + \lambda_4 (\phi_1 \phi_2)_{\underline{\mathbf{3}}_1} (\eta_3^\dagger \eta_2)_{\underline{\mathbf{3}}_1} + \text{h.c.}$$

$$V_{\eta,\chi} = \lambda_1 (\chi^t \tau_2 \vec{\tau} \chi)_{\underline{\mathbf{1}}_1} (\eta_1^t \tau_2 \vec{\tau} \eta_3)^*_{\underline{\mathbf{1}}_1} + \lambda_2 e^{\mathrm{i}\alpha_\lambda} (\chi^t \tau_2 \vec{\tau} \chi)_{\underline{\mathbf{3}}_1} (\eta_2^t \tau_2 \vec{\tau} \eta_3)^*_{\underline{\mathbf{3}}_1} + \text{h.c.}$$



- neutrino masses small $m_\nu \sim \frac{1}{16\pi^2} h^2 \left(\frac{\delta M_\eta^2}{M_\eta^2}\right)^2 \frac{M_\eta^2}{M_S} \sim 1\,\text{eV}$ for $h \sim \frac{\delta M_\eta^2}{M_\eta^2} \sim 10^{-2}, M_\eta \sim 100\,\text{GeV}, M_S \sim 1\,\text{TeV}$

- flavour structure

$$m_\nu = \begin{pmatrix} \hat{a} & \hat{e}\, e^{\mathrm{i}\alpha_\lambda} & \hat{e}\, e^{\mathrm{i}\alpha_\lambda} \\ . & \hat{a} + \hat{b}\, e^{\mathrm{i}\alpha_\lambda} & \hat{d} + \hat{e}\, e^{\mathrm{i}\alpha_\lambda} \\ . & . & \hat{a} \end{pmatrix}$$

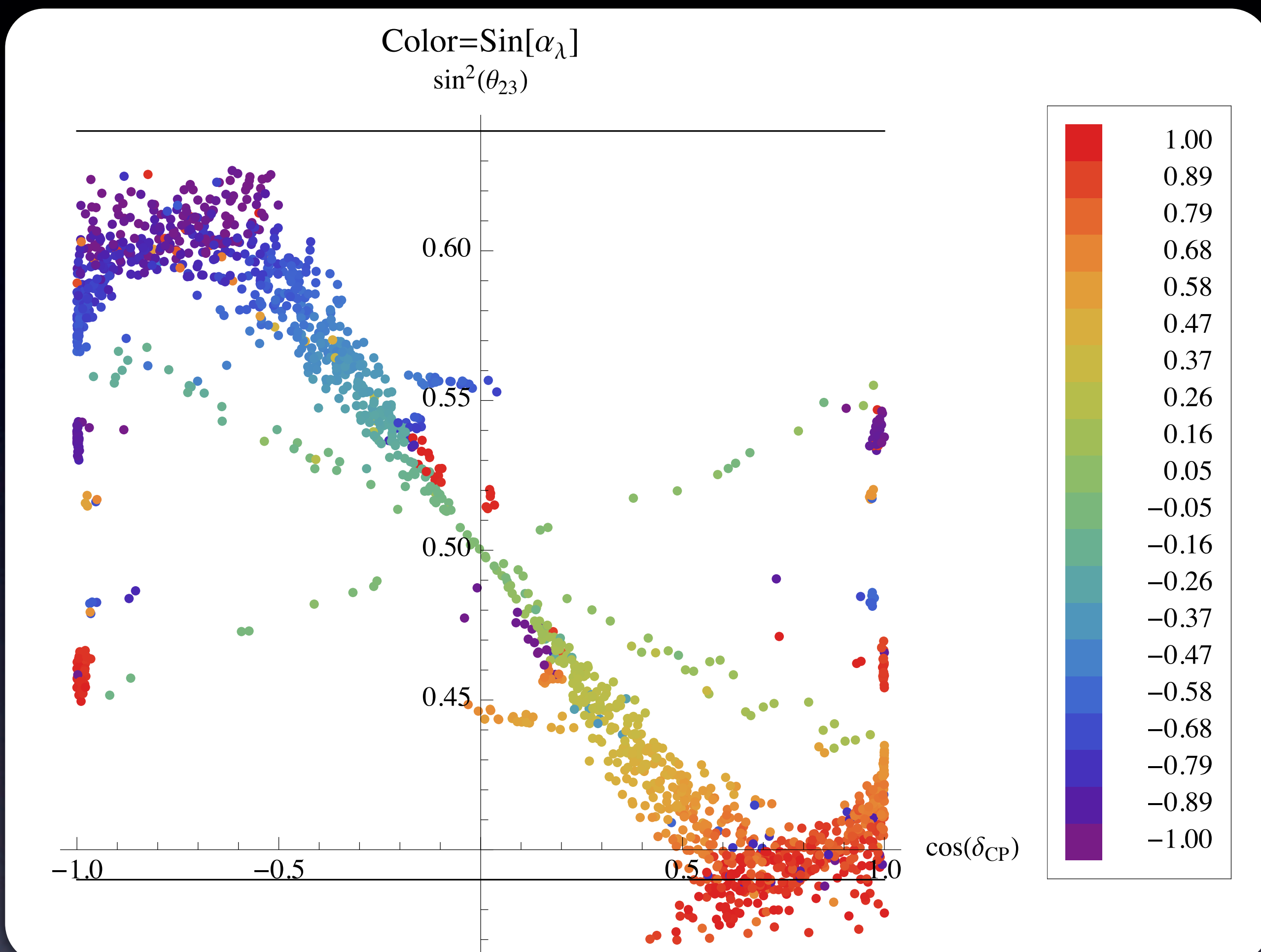
$$I(m_1, m_2, m_3, m_4) = -\frac{1}{16\pi^2} \sum_i \frac{{m_i}^2 \log\left(\frac{{m_i}^2}{\mu^2}\right)}{\Pi_{k\neq i}\,({m_i}^2 - {m_k}^2)}\ .$$

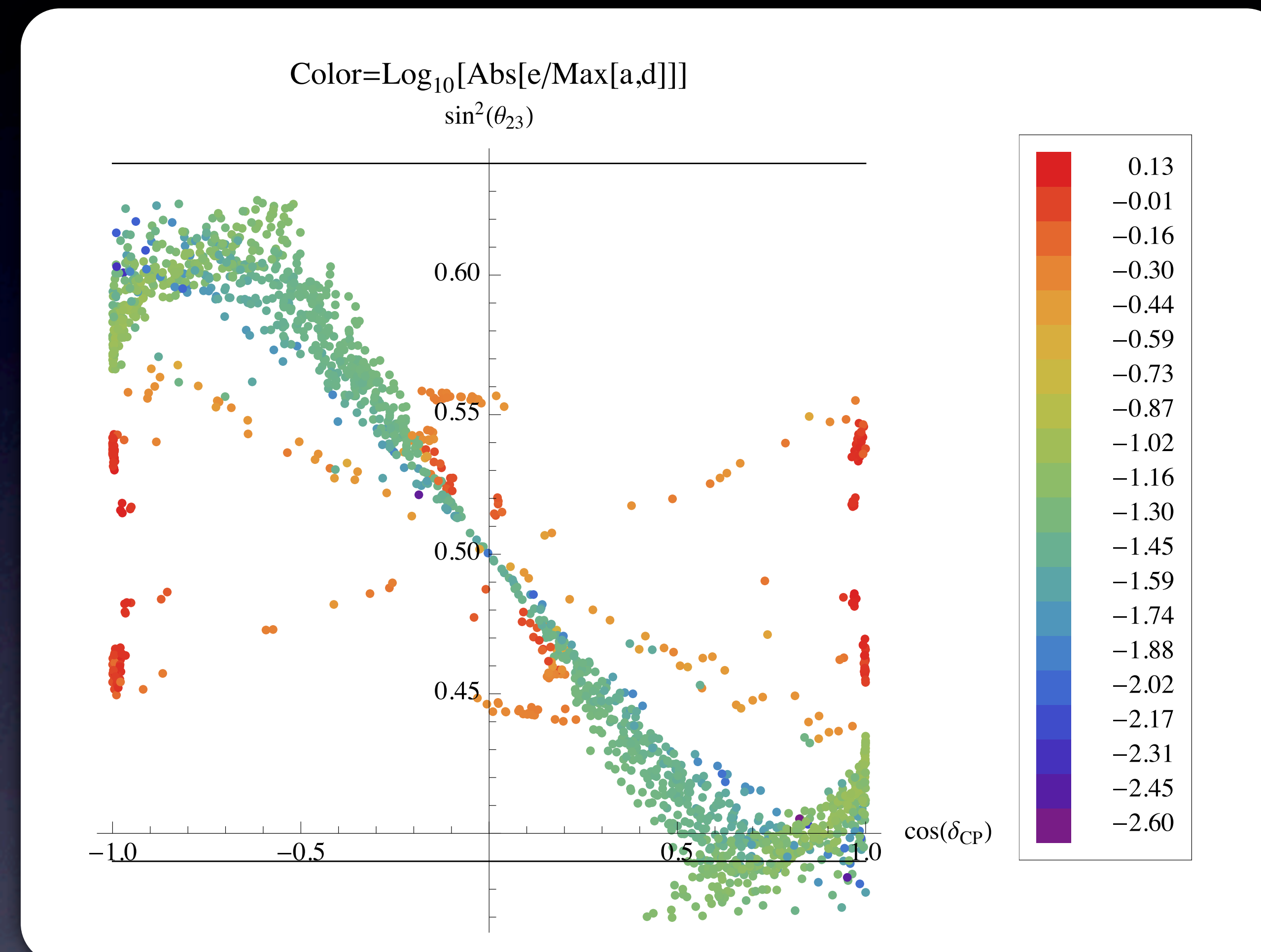
$$\hat{a} = \frac{\sqrt{3}}{18}\, h_1^2\, \lambda_3\, \lambda_1 v^2\, (ac + bd)\, M_S\, I\,(M_1, M_1, M_3, M_S)$$
$$\hat{d} = \frac{\sqrt{3}}{36}\, h_1 h_2\, \lambda_4\, \lambda_1 v^2\, (bc - ad)\, M_S\, I(M_1, M_2, M_3, M_S)$$
$$\hat{b} = \frac{1}{18}\, h_2^2\, \lambda_4\, \lambda_2 v^2\, (bc - ad)\, M_S\, I(M_2, M_2, M_3, M_S)$$
$$\hat{e} = \frac{1}{36}\, h_1 h_2\, \lambda_3\, \lambda_2 v^2\, (ac + bd)\, M_S\, I(M_1, M_2, M_3, M_S)$$

- TBM for λ_2=0, non-zero Θ_{13} can be generated

[MH, M. Lindner, M. Schmidt, in preparation]

Flavour Breaking at the EW Scale-Mixing





- TBM for b=e=0
- e induces 13 rotation $U_{PMNS} = U_{TMB} U_{13}$
- b induces 12 rotation $U_{PMNS} = U_{TMB} U_{12}$
- $\sin^2\Theta_{13} \approx .03$ needs $e/a \approx .1$

$$m_\nu = \begin{pmatrix} \hat{a} & \hat{e}\, e^{i\alpha_\lambda} & \hat{e}\, e^{i\alpha_\lambda} \\ . & \hat{a} + \hat{b}\, e^{i\alpha_\lambda} & \hat{d} + \hat{e}\, e^{i\alpha_\lambda} \\ . & . & \hat{a} \end{pmatrix}$$

Flavour Breaking at the EW Scale-Dark Matter

- model has a dark matter candiate as there is an accidental symmetry

$$\eta_i \to -\eta_i \qquad S \to -S$$

at the renormalizable level, and a remnant symmetry of the Z_4 part of the flavour symmetry $\quad \mathcal{A}: L \to -L \qquad e^c \to -e^c \qquad \eta_i \to -\eta_i$

decay is mediated by operators $\eta_i \mathcal{O}_{SM}^{\mathcal{A}=-1} \langle \mathcal{O}_{\phi_k \phi_l} \rangle$, where O_{SM} must be an EW doublet and Lorentz singlet. The insertion of at least two Φs is needed to form a flavour singlet. The lowest dimensional operators are given by:

$$\begin{array}{lllll} L u^c d^c d^c & \bar{L} \bar{d}^c \bar{d}^c \bar{d}^c & L \bar{Q} \bar{Q} d^c & \bar{e}^c \bar{Q} d^c d^c & \\ \chi^\dagger L Q Q Q & \chi^\dagger e^c u^c u^c d^c & \chi^\dagger \bar{L} \bar{Q} u^c d^c & \chi^\dagger \bar{e}^c Q Q \bar{u}^c & \chi^\dagger L Q \bar{u}^c \bar{d}^c \,. \end{array}$$

B+L=0

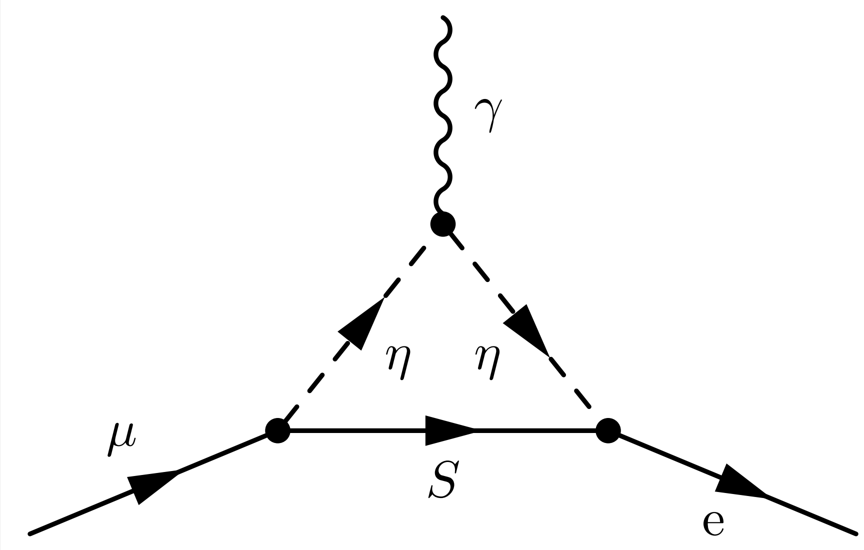
B-L=0

- the lifetime induced by the dim. 6 SM operators $\frac{\eta_i \mathcal{O}_{SM}^{\mathcal{A}=-1}}{\Lambda_B^3} \frac{\langle \phi_k \phi_l \rangle}{\Lambda_F^2}$ is given by $\Gamma^{-1} \sim \frac{8\pi \Lambda_B^6}{m_\eta^7} \left(\frac{\Lambda_F^2}{\langle \phi_k \phi_l \rangle} \right)^2$
- the bound of $\Gamma^{-1} \gtrsim 10^{26} s$ translates into $\left(\Lambda_B^3 \Lambda_F^2\right)^{1/5} \gtrsim 6 \cdot 10^7 \mathrm{GeV} \left(\frac{m_\eta}{1\,\mathrm{TeV}} \right)^{7/10} \left(\frac{\langle \phi_k \phi_l \rangle}{(100\,\mathrm{GeV})^2} \right)^{1/5} .$
- DM abundance produced in same way as normal Inert Dark Matter
- the fermionic DM candidate S is stabilized in an analogous way, if it is the lightest

[MH, M. Lindner, M. Schmidt, in preparation]

Flavour Breaking at the EW Scale-LFV&Higgs

- in models with radiative neutrino mass, lepton flavour violating processes put constraints



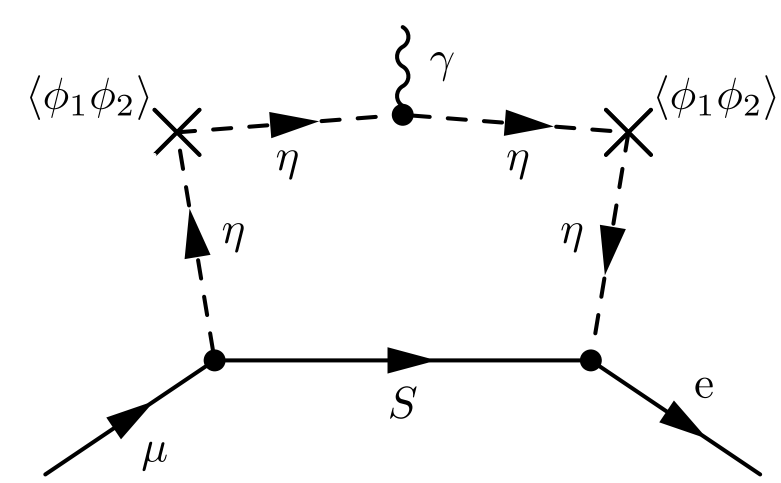
$$Br(\mu \to e\gamma) = \frac{3\alpha}{64\pi(G_F m_0^2)^2} C^4$$

$$C^2 = \left| \sum_{i,J} h_{\mu iJ} h^*_{eiJ} F_2(M_i^2/m_J^2) \right| \quad \text{and} \quad F_2(t) = \frac{1 - 6t + 3t^2 + 2t^3 - 6t^2 \ln t}{6(1-t)^4}.$$

$Br(\mu \to e\gamma) < 1.2 \cdot 10^{-11}$ implies $C^4 \sim 1.5 \cdot 10^{-8}$ for $M_S = M_0 = 100\,\mathrm{GeV}$.

- flavour symmetry gives an additional suppression

$$L\sigma_{\mu\nu} F^{\mu\nu} e^c \tilde{H}/M^2 \sim (\underline{\mathbf{3}}_{\mathbf{1}}, 1)$$



- needs additional mass insertions, natural suppression

$$C^4 \sim \left(\frac{\delta M_\eta^2}{M_\eta^2}\right)^4 \sim 10^{-8}$$

- LFVs mediated by 4 fermi interactions loop suppressed & selection rule $\Delta L_e \Delta L_\mu \Delta L_\tau = \pm 2$

Flavour Breaking at the EW Scale-LFV&Higgs

- in the charged lepton sector, the VEV (1,1,1) leaves the Z3 subgroup generated by T invariant

$$\rho(T) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

- go to a basis where T is diagonal $\Omega_e^\dagger \rho(T) \Omega_e = \mathrm{diag}(1, \omega^2 \omega)$

$$\Omega_T \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

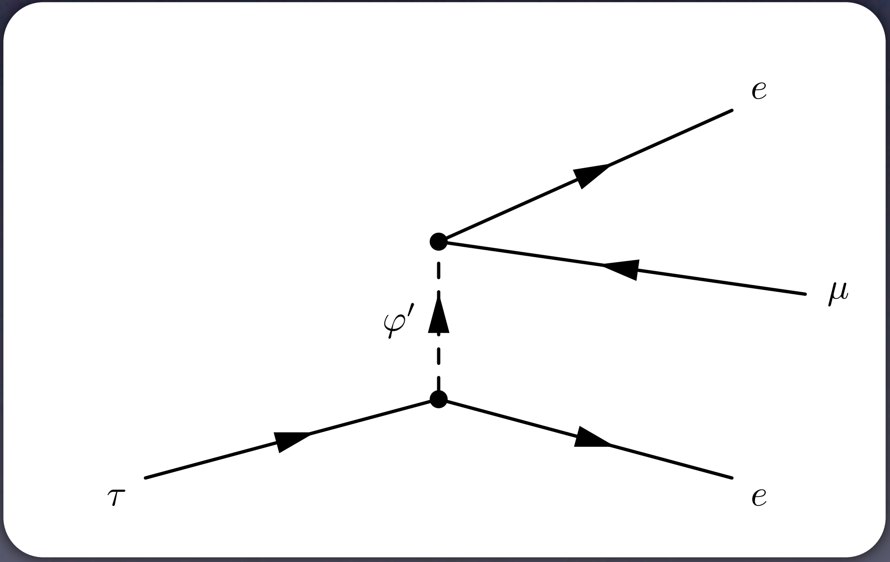
$$(L_e, L_\mu, L_\tau)^T = \Omega_T L \sim (1, \omega^2, \omega)^T \quad (\varphi, \varphi', \varphi'')^T = \Omega_T \chi \sim (1, \omega^2, \omega)^T \quad (e^c, \mu^c, \tau^c)^T \sim (1, \omega, \omega^2)^T$$

- only phi gets a VEV and plays the role of the SM Higgs $\tilde{\varphi}\,(y_e L_e e^c + y_\mu L_\mu \mu^c + y_\tau L_\tau \tau^c)$

- the other two Higgs fields are inert and have flavour off-diagonal couplings

$$\tilde{\varphi}'\,(y_e L_\tau e^c + y_\mu L_e \mu^c + y_\tau L_\mu \tau^c) \qquad \tilde{\varphi}''\,(y_e L_\mu e^c + y_\mu L_\tau \mu^c + y_\tau L_e \tau^c)$$

- this generates LFV 4f operators (with the selection rule $\Delta L_e \Delta L_\mu \Delta L_\tau = \pm 2$)



- the most constraining process is

$$Br(\tau^- \to \mu^+ e^- e^+) \sim 10^{-7} \left(\frac{2\,\mathrm{GeV}}{M^4_{\varphi', \varphi''}} \right)$$

- Higgs admixture of φ, Φ singlets, in the limit H= φ
tree-level branching ratios same as SM, loop-processes altered

- in the quark sector, no mixing at LO, Cabibbo angle has to be generated by cross-talk to neutrino sector flavons

Mathematica Package Discrete

We have developed a Mathematica Package that can be used to facilitate model building using discrete groups. It has the features:

- has access to groups catalogue of GAP, which contains all groups one would ever want to use

Initialization

```
In[8]:= Needs["Discrete`ModelBuildingTools`"];
```

```
In[11]:= Group = MBloadGAPGroup["AlternatingGroup(4)"];
starting GAP generating AlternatingGroup(4)...
...finished

StructureDescription:A4

Size of Group:12

Number of irreps: 4
```

Dimensions of irreps:

1	2	3	4
1	1	1	3

Character Table:

1	1	1	1
1	1	$e^{-\frac{2 i \pi}{3}}$	$e^{\frac{2 i \pi}{3}}$
1	1	$e^{\frac{2 i \pi}{3}}$	$e^{-\frac{2 i \pi}{3}}$
3	-1	0	0

Mathematica Package Discrete

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```
In[193]:= χ = MBgetRepVector[Group, 4, χc]
          L = MBgetRepVector[Group, 4, Lc]
```

Out[193]= $\{\{\},\{\},\{\},\{\{\chi c1,\chi c2,\chi c3\}\}\}$

Out[194]= $\{\{\},\{\},\{\},\{\{Lc1,Lc2,Lc3\}\}\}$

```
In[195]:= MBmultiply[Group, χ, L]
```

Out[195]=

$$\Big\{\Big\{\Big\{\frac{Lc1\,\chi c1+Lc2\,\chi c2+Lc3\,\chi c3}{\sqrt{3}}\Big\}\Big\},$$
$$\Big\{\Big\{\frac{1}{6}\left(2\sqrt{3}\,Lc1\,\chi c1-\left(3\,i+\sqrt{3}\right)Lc2\,\chi c2-\left(-3\,i+\sqrt{3}\right)Lc3\,\chi c3\right)\Big\}\Big\},$$
$$\Big\{\Big\{\frac{1}{6}\left(2\sqrt{3}\,Lc1\,\chi c1-\left(-3\,i+\sqrt{3}\right)Lc2\,\chi c2-\left(3\,i+\sqrt{3}\right)Lc3\,\chi c3\right)\Big\}\Big\},$$
$$\{\{Lc3\,\chi c2,\,Lc1\,\chi c3,\,Lc2\,\chi c1\},\{Lc2\,\chi c3,\,Lc3\,\chi c1,\,Lc1\,\chi c2\}\}\Big\}$$

```
In[197]:= MBmultiply[Group, {χ, χ, χ, L, L}][[1]]
```

Out[197]=

$$\Big\{\Big\{\left(Lc1^2+Lc2^2+Lc3^2\right)\chi c1\,\chi c2\,\chi c3\Big\},$$
$$\Big\{\frac{1}{3}\left(Lc2\,Lc3\,\chi c1+Lc1\,Lc3\,\chi c2+Lc1\,Lc2\,\chi c3\right)\left(\chi c1^2+\chi c2^2+\chi c3^2\right)\Big\},$$
$$\Big\{\frac{Lc1\,Lc3\,\chi c2\,\chi c3^2+Lc2\,\chi c1\left(Lc3\,\chi c2^2+Lc1\,\chi c1\,\chi c3\right)}{\sqrt{3}}\Big\},$$
$$\Big\{\frac{Lc2\,Lc3\,\chi c1\,\chi c3^2+Lc1\,\chi c2\left(Lc3\,\chi c1^2+Lc2\,\chi c2\,\chi c3\right)}{\sqrt{3}}\Big\},$$
$$\Big\{\frac{1}{6\sqrt{3}}\Big(Lc1\,Lc3\,\chi c2\left(-\left(-3\,i+\sqrt{3}\right)\chi c1^2+2\sqrt{3}\,\chi c2^2-\left(3\,i+\sqrt{3}\right)\chi c3^2\right)+$$
$$Lc2\left(Lc1\,\chi c3\left(-\left(3\,i+\sqrt{3}\right)\chi c1^2-\left(-3\,i+\sqrt{3}\right)\chi c2^2+2\sqrt{3}\,\chi c3^2\right)+\right.$$
$$\left.Lc3\,\chi c1\left(2\sqrt{3}\,\chi c1^2-\left(3\,i+\sqrt{3}\right)\chi c2^2-\left(-3\,i+\sqrt{3}\right)\chi c3^2\right)\right)\Big)\Big\},$$

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- reduce set covariants to a smaller set of independent covariants
- calculate flavon potentials

```
In[200]:= MBgetFlavonPotential[Group, χ, 4, λ]
          2
          3
          4
```

$$\text{Out[200]=}\ \lambda 3n1\,\chi c1\,\chi c2\,\chi c3 + \frac{\lambda 2n1\left(\chi c1^2 + \chi c2^2 + \chi c3^2\right)}{\sqrt{3}} + \lambda 4n1\left(\chi c1^4 + \chi c2^4 + \chi c3^4\right) + \frac{1}{3}\lambda 4n2\left(\chi c2^2\,\chi c3^2 + \chi c1^2\left(\chi c2^2 + \chi c3^2\right)\right)$$

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- available at http://projects.hepforge.org/discrete/

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Thank you for your attention!