Electroweak and Sommerfeld corrections
to the Wino dark matter annihilation

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Based on:

- work in progress with **Ilias Cholis, Maryam Tavakoli** and **Piero Ullio**
- **AH, Roberto Iengo, Piero Ullio; JHEP 1103 (2011) 069 [arXiv:1010.2172]**
Introduction

Strong evidence for particle dark matter, with

$$\Omega_{\text{obs}}h^2 = 0.1123 \pm 0.0035$$

obtained within $\Lambda$CDM model from WMAP7 + BAO + $h$

Thermal relic density of dark matter particles:

$$\Omega h^2 \approx 0.1 \left( \frac{3 \cdot 10^{-26} \text{cm}^3 \text{s}^{-1}}{\langle \sigma_A V \rangle_{T_{\text{f.o.}}} } \right)$$

$\langle \sigma_A V \rangle_{T_{\text{f.o.}}}$ annihilation cross section at the about freeze-out temperature
Indirect detection depends on the **annihilation cross section**, but for low velocity WIMPs in DM halos

\[ \text{flux} \sim n^2 \langle \sigma_A v \rangle_{\text{DM halo}}, \]

i.e. essentially in \( v \to 0 \) limit
Indirect detection depends on the **annihilation cross section**, but for low velocity WIMPs in DM halos

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\text{flux} \sim n^2 \langle \sigma_A v \rangle_{\text{DM halo}},
\]

i.e. essentially in \( v \rightarrow 0 \) limit

After the annihilation, the final states decay and/or fragmentate and produce showers of softer stable states \( \gamma, e^+, \bar{p}, \nu, \bar{d} \)

\( \rightarrow \) those propagate down to Earth
Electroweak corrections

Tree level annihilation + Monte Carlo
shower/hadronization/fragmentation
code (e.g. PYTHIA)
Electroweak corrections

One-loop level annihilation + Monte Carlo shower/hadronization/fragmentation code (e.g. PYTHIA)
Importance of EW corrections for DM

- corrections (large in some cases) to the $\langle \sigma v \rangle$
- softer SM particles spectra at DM annihilation
- all stable SM particles in the final spectrum, even if not present in the annihilation channel
- additional new spectral features: bumps and sharp cutoffs

Rich literature in recent years about this topic: Boudjema, Kechelriess, Serpico, Ciafaloni, Ciafaloni, Comelli, Urbano, de Simone, Strumia, Cirelli, Bergstrom, Bringmann, Eriksson, Gustafsson, Dent, Weiler, ...

Ciafaloni et al. 1202.0692
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Ciafaloni et al. 1009.0224
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Bringmann et al. 0710.3169
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**Wino dark matter**

In the MSSM the neutralino is a combination of gauginos \((\tilde{B}, \tilde{W}^3)\) and higgsinos \((\tilde{h}^0_1, \tilde{h}^0_2)\):

\[
\tilde{\chi}_i^0 = N_{i1} \tilde{B} + N_{i2} \tilde{W}^3 + N_{i3} \tilde{h}^0_1 + N_{i4} \tilde{h}^0_2
\]

If \(N_{i2} \gg N_{i1}, N_{i3}, N_{i4}\) then neutralino is **Wino-like** and

- is nearly degenerated in mass with the lightest chargino
  \[
m_{\chi^\pm} - m_{\chi^0} \approx 170 \text{ MeV}
  \]
- is in an adjoint of \(SU(2)\)
- if \(m_{\chi^0} > m_W\) has very efficient annihilation channel into \(W^+W^-\)
  \[
  \Rightarrow \text{typically too small thermal relic density, at tree level:}
  \]
  \[
  \Omega_{\text{DM}} h^2 \approx 0.11 \Rightarrow m_{\chi^0} \approx 2.2 \text{ TeV}
  \]

... but then, large corrections!
Why corrections are large?

Typically, one expects that EW one-loop corrections are at most a few %. At TeV scale, however, soft/collinear Bremsstrahlung gauge bosons are enhanced by large (Sudakov) logarithms:

\[ \alpha_2 \log \frac{m^2}{m_W^2}, \quad \alpha_2 \left( \log \frac{m^2}{m_W^2} \right)^2 \]

\[ m = 1 \text{ TeV}, \quad \alpha_2 \approx \frac{1}{30} \Rightarrow \approx 0.17 \approx 0.86 \]

When \( m \gg m_W \) this resembles IR divergence of QED or QCD

\[ \rightarrow \text{Bloch-Nordsieck violation [Ciafaloni, Ciafaloni, Comelli, ’00]} \]

Bloch-Nordsieck: in QED the inclusive cross-section IR Logs cancel

Kinoshita-Lee-Nauenberg: generalized to SM, but only when summed over initial non-abelian charge
Sommerfeld enhancement (effect) is a non-relativistic effect changing the cross section due to the wave function distortion by a long range potential.

Conditions for significant enhancement:

- **slow incoming particles**

\[
\frac{m_\chi v^2}{\alpha^2 m_\chi} \lesssim \frac{1}{\alpha m_\chi}
\]

- **long range force**

\[
\frac{1}{m_\phi} \lesssim \frac{1}{\alpha m_\chi}
\]
Sommerfeld effect in the MSSM

In the MSSM:

- Dark matter $\rightarrow$ lightest neutralino $\chi_1^0$
- possible intermediate bosons: $\gamma, W^\pm, Z^0, h_1^0, h_2^0, H^+$

$\Rightarrow O\left(\alpha \frac{m}{m_W}\right)$ correction

It would seem that to have a large effect

$$\frac{1}{m_W} \gtrsim \frac{1}{\alpha m_\chi} \quad \Rightarrow \quad m_\chi \gtrsim 2.3 \text{ TeV}$$

Moreover, if $\delta m = m_{\chi^+} - m_\chi$ is too large then the effect is suppressed.
Sommerfeld effect in the MSSM

... but

- as soon as one can produce nearly on-shell $\chi^+$, i.e. when $\mathcal{E} \approx 2\delta m$:

\[ \chi^0 \rightarrow \chi^0, \chi^+ \rightarrow \chi^+ \chi^0 \]

- for relic density also co-annihilations are important $\rightarrow$ one needs to compute Sommerfeld effect also for incoming $\chi^+ \chi^-, \chi^+ \chi_1^0, ...$

Wino-like $\chi^0$ has $\delta m \ll m_{\chi^0}$ $\Rightarrow$ Sommerfeld effect has to be included
Sommerfeld enhancement without dark force

→ for the pure wino or pure higgsino in MSSM [Hisano et al. ’03, ’05]
→ for the Minimal Dark Matter model [Strumia et al. ’07]

Effect not so big as in models with dark force, but still important and much less speculative!

DarkSE: a numerical package for DarkSUSY computing relic density with Sommerfeld effect for a general MSSM setup [AH, 1102.4295]
Most of the computations in DM literature are done at tree level → clearly not enough for TeV scale

To take the radiative corrections into account one often take the value of $g$ at the scale of DM mass $m$ and simply use RGE with one- or two-loop $\beta$-function

This is not fully correct way to proceed:

1. RGE holds in deep Euclidean regime: when external lines are on-shell not only UV but also IR large Logs occur ⇒ threshold corrections

2. RGE is appropriate when there is one single large scale $\mu^2$: in computation of the Sommerfeld effect, there are two: DM mass $m$ and the momentum transfer $\mathcal{O}(m_W)$
Since $\chi^0$ is:

- a Majorana fermion
- non-relativistic, with essentially $v \rightarrow 0$
- in adjoint of $SU(2)$ and neutral under $U(1)$

therefore:

1. the only interaction is through vertex $\chi^0 \chi^\pm W^\mp$
2. the initial $\chi^0 \chi^0$ state is spin singlet

The radiative amplitude corrections can be written as:

$$A = A_{\text{tree}} \left( 1 + \frac{g^2}{(4\pi)^2} C_i(m) \right)$$
One-loop $\chi^0\chi^0 \rightarrow W^+W^-$ results
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**One-loop $\chi^0 \chi^0 \rightarrow W^+ W^-$ results**

![Graphs showing the results of the one-loop corrections for Wino Dark Matter decay](image)
One-loop $\chi^0 \chi^0 \rightarrow W^+ W^-$ results

\[ \chi^0 \chi^0 \rightarrow W^+ W^- \]
One-loop $\chi^+\chi^- \rightarrow W^+W^-$ annihilation

Recall that the Sommerfeld effect:

$\chi^0\chi^0 \rightarrow \chi^+\chi^- \rightarrow \chi^0\chi^0 \rightarrow ... \rightarrow \text{SM}$

To be consistent one needs also to compute one-loop corrections to $
\chi^+\chi^- \rightarrow W^+W^- \text{ annihilation}$

Then the Sommerfeld enhanced amplitude:

$$A_{\chi^0\chi^0 \rightarrow W^+W^-}^{SE} = s_0 A_{\chi^0\chi^0 \rightarrow W^+W^-} + s_\pm A_{\chi^+\chi^- \rightarrow W^+W^-}$$

where $s_0$ and $s_\pm$ are (complex) Sommerfeld factors
One-loop $\chi^+ \chi^- \rightarrow W^+ W^-$ results
Cross-section results

The total results for the $\sigma v$ vs. DM mass $m$:

- tree level result $\sim 1/m^2$
Cross-section results

The total results for the $\sigma v$ vs. DM mass $m$:

- tree level result $\sim 1/m^2$
- when $g$ at the scale $m$ is used with SM running
Cross-section results

The total results for the $\sigma v$ vs. DM mass $m$:

- tree level result $\sim 1/m^2$
- when $g$ at the scale $m$ is used with SM running
- full $O(g^6)$ result (with one-loop Sommerfeld correction)
The total results for the $\sigma v$ vs. DM mass $m$:

- tree level result $\sim 1/m^2$
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- tree level with re-summed Sommerfeld effect
Cross-section results

The total results for the $\sigma v$ vs. DM mass $m$:

- tree level result $\sim 1/m^2$
- when $g$ at the scale $m$ is used with SM running
- full $O(g^6)$ result (with one-loop Sommerfeld correction)
- tree level with re-summed Sommerfeld effect
- **full** $O(g^6)$ result with re-summed Sommerfeld effect
Cross-section results

The total results for the $\sigma v$ vs. DM mass $m$:

- tree level result $\sim 1/m^2$
- when $g$ at the scale $m$ is used with SM running
- full $O(g^6)$ result (with one-loop Sommerfeld correction)
- tree level with re-summed Sommerfeld effect
- full $O(g^6)$ result with re-summed Sommerfeld effect
- what if $g$ at the scale $m$ is used for the Sommerfeld effect
One-loop $\chi^+ \chi^-$ to neutral gauge bosons

Analogically, due to Sommerfeld enhancement, additional annihilation channels:

$$\chi^0 \chi^0 \rightarrow \chi^+ \chi^- \rightarrow ZZ, Z\gamma, \gamma\gamma$$
Cross-section for $\chi^0 \chi^0 \rightarrow ZZ, Z\gamma, \gamma\gamma$

At the leading order (LO) the annihilation into $ZZ, Z\gamma$ or $\gamma\gamma$ occurs at $\mathcal{O}(g^8) \rightarrow$ dotted lines

Sommerfeld effect is suppressing in the low $m$ region (since one-loop corrections are negative) but gives strong enhancement near the resonance
How one can experimentally test the **heavy Wino DM scenario**?

- **Direct Detection** → too heavy: sensitivity drops at a TeV scale ⇒ NO (or at least not now, possibly in next generation, e.g. DARWIN)
- **LHC** → again too heavy ⇒ NO
- **Indirect Detection** ⇒ YES?

Two interesting questions:

1. Is the **thermal Wino** still allowed and if yes, can it be probed in near future?
2. Can Wino explain **CR anomalies**? [e.g. Grajek et al. ’08; Kane et al. ’09]
Thermal Wino scenario

\[ \bar{p} \] flux

Propagation parameters:

- \( \delta = 0.5 \)
- \( z_d = 4 \text{ kpc} \)
- \( r_d = 20 \text{ kpc} \)
- \( d\nu_{c}/dz = 0 \)
- \( D_0 = 2.49 \times 10^{28} \text{ cm}^2/\text{s} \)
- \( \eta = -0.363 \)
- \( \nu_A = 19.5 \text{ km/s} \)

Best fit from [Cholis et al.; 1106.5073]
Thermal Wino scenario

$p$ flux

$d$ flux

PAMELA '10

$\Omega_{DM} = 0.1$

$\langle \sigma v \rangle = 9.61 \times 10^{-25}$ cm$^3$/s

$\Phi_p$ [GeV$^{-1}$ m$^{-2}$ s$^{-1}$]

$\Phi_d$ [GeV$^{-1}$ m$^{-2}$ s$^{-1}$]

$E$ [GeV]

$E$ [GeV/n]
Thermal Wino scenario

\( e^+ \) fraction

\( e^+ + e^- \) flux

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\( e^+ \) fraction

- CAPRICE '94
- HEAT '94
- PAMELA '10
- FERMI '11

\( m_{DM} = 2700 \text{ GeV} \)
\( \Omega_{DM} = 0.1 \)
\( \langle \sigma v \rangle = 9.61 \times 10^{-25} \text{ cm}^3/\text{s} \)

---

\( e^+ + e^- \) flux

- MAGIC
- HESS
- ATIC
- FERMI '11

\( m_{DM} = 2700 \text{ GeV} \)
\( \Omega_{DM} = 0.1 \)
\( \langle \sigma v \rangle = 9.61 \times 10^{-25} \text{ cm}^3/\text{s} \)
Thermal Wino scenario

\( \gamma \) spectrum

\( \nu_\mu \) spectrum
Thermal Wino scenario

\( \gamma \) spectrum

\( \nu_\mu \) spectrum

\( m_{DM} = 2700 \text{ GeV} \)
\( \Omega_{DM} = 0.1 \)
Tree level
PPPC4DMID

\( m_{DM} = 3000 \text{ GeV} \)
\( \Omega_{DM} = 0.103 \)
Tree level
PPPC4DMID
Thermal Wino scenario

$\gamma$ spectrum

$\nu_\mu$ spectrum
Thermal Wino scenario

\[ \gamma \text{ spectrum} \]

\[ \nu_\mu \text{ spectrum} \]

\( m_{DM} = 2700 \, \text{GeV} \)
\( \Omega_{DM} = 0.1 \)
\( \langle \sigma v \rangle = 9.61 \times 10^{-25} \, \text{cm}^3/\text{s} \)

Tree level
PPC4DMID
One-loop level
One-loop + Sommerfeld effect

\( m_{DM} = 3000 \, \text{GeV} \)
\( \Omega_{DM} = 0.103 \)
\( \langle \sigma v \rangle = 2.85 \times 10^{-25} \, \text{cm}^3/\text{s} \)

Tree level
PPC4DMID
One-loop level
One-loop + Sommerfeld effect
Thermal Wino scenario

\( \gamma \) diff. cross-section

\( \nu_\mu \) diff. cross-section
Can it explain CR anomalies?

\[ \bar{p} \text{ flux} \]

The strategy:
- look for max. cross-section allowed by \( \bar{p} \) data ⇒ resonance
- is it sufficient to solve \( e^+ / e^- \) puzzle?
- check if it satisfies constraints from \( \bar{d}, \nu \)s and \( \gamma \)
Can it explain CR anomalies?

$ar{p}$ flux

$ar{d}$ flux

$\bar{p}$ flux

$\bar{d}$ flux

$E [\text{GeV}]$

$E [\text{GeV/n}]$

$\Omega_{DM} = 0.062$

$\langle \sigma v \rangle = 5.71 \times 10^{-23} \text{ cm}^3 / \text{s}$

$\Phi_p [\text{MeV}^{-1} \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1}]$

$\Phi_d [\text{GeV}^{-1} \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1}]$
Can it explain CR anomalies?

**$e^+$ fraction**

**$e^+ + e^-$ flux**
Can it explain CR anomalies?

\( \gamma \) diff. cross-section

\( \nu_\mu \) diff. cross-section
Conclusions

1. Electroweak corrections cannot be neglected in the computation of heavy DM annihilation processes

2. Full $\mathcal{O}(g^6)$ computation needed to correlate some of the spectral features (like lines or bumps) with the diffuse spectrum

3. In all cases when Sommerfeld effect can occur it must be included and we provide a method how to do that in a consistent way

4. Taking simply the $\beta$-function and using RGE without threshold corrections is incorrect way to proceed

5. Thermal Wino DM can be most easily found/excluded in $\gamma$ rays, antideuterons and (maybe) neutrinos

6. Resonant case disfavoured by data $\Rightarrow$ Wino DM does not solve the CR puzzle