Generalized Higgs inflation models

based on: KK, T. Kobayashi, M.Yamaguchi & J. Yokoyama, PRD85 (2011) 043503

KK, T. Kobayashi, T. Takahashi, M. Yamaguchi & J. Yokoyama, arXiv:1203.4059

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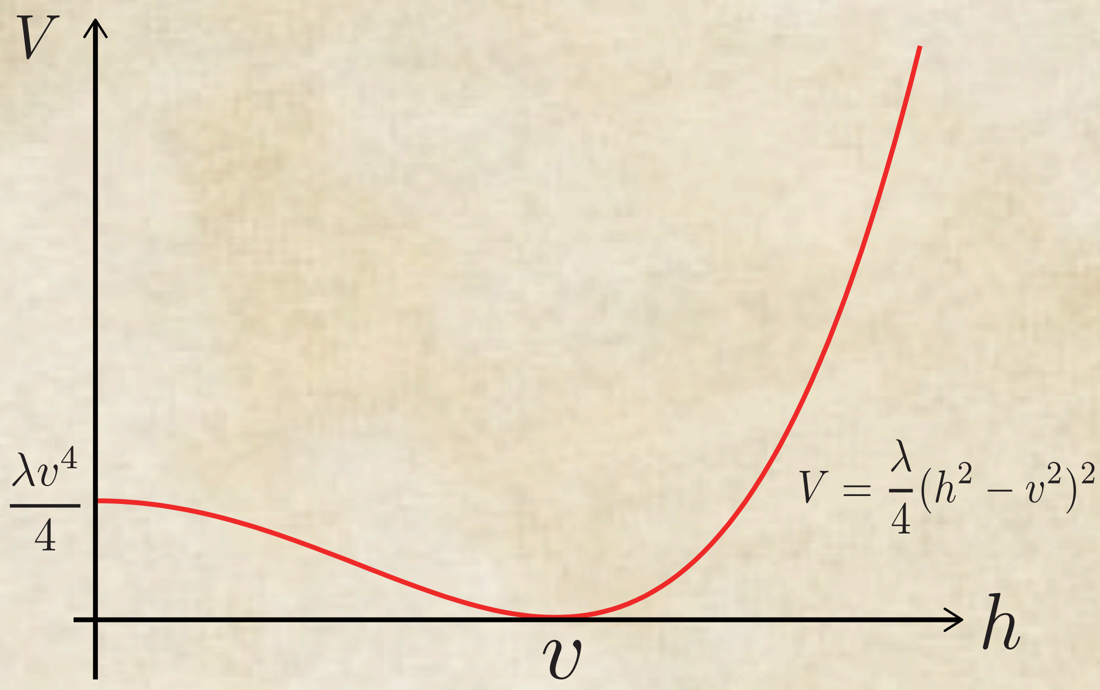
PLANCK2012 @ Warsaw, 30/5/2012

What is the origin of inflation?

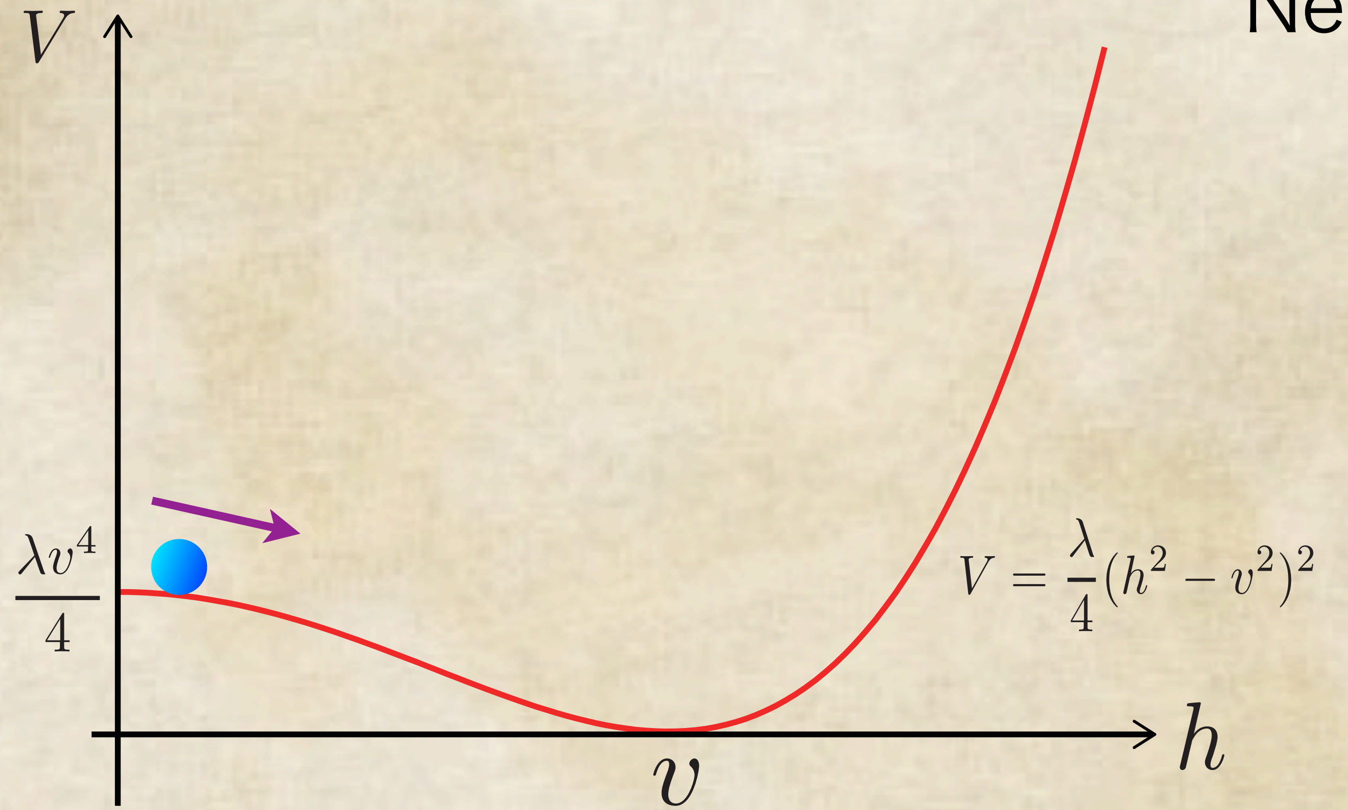
We often say, “Standard Model of particle physics (SM) does not have any candidates for inflaton and hence we need physics beyond the SM.”



Is this true? There is a scalar field in the SM, the Higgs field.
Isn’t it possible for the Higgs field to be the inflaton?

This question becomes important because we are now close to the discovery of the SM Higgs field.

Why is it considered that the Higgs field cannot be inflaton?



New inflation? ('82, Linde)

: impossible because the potential is too steep to realize accelerating expansion of the Universe.

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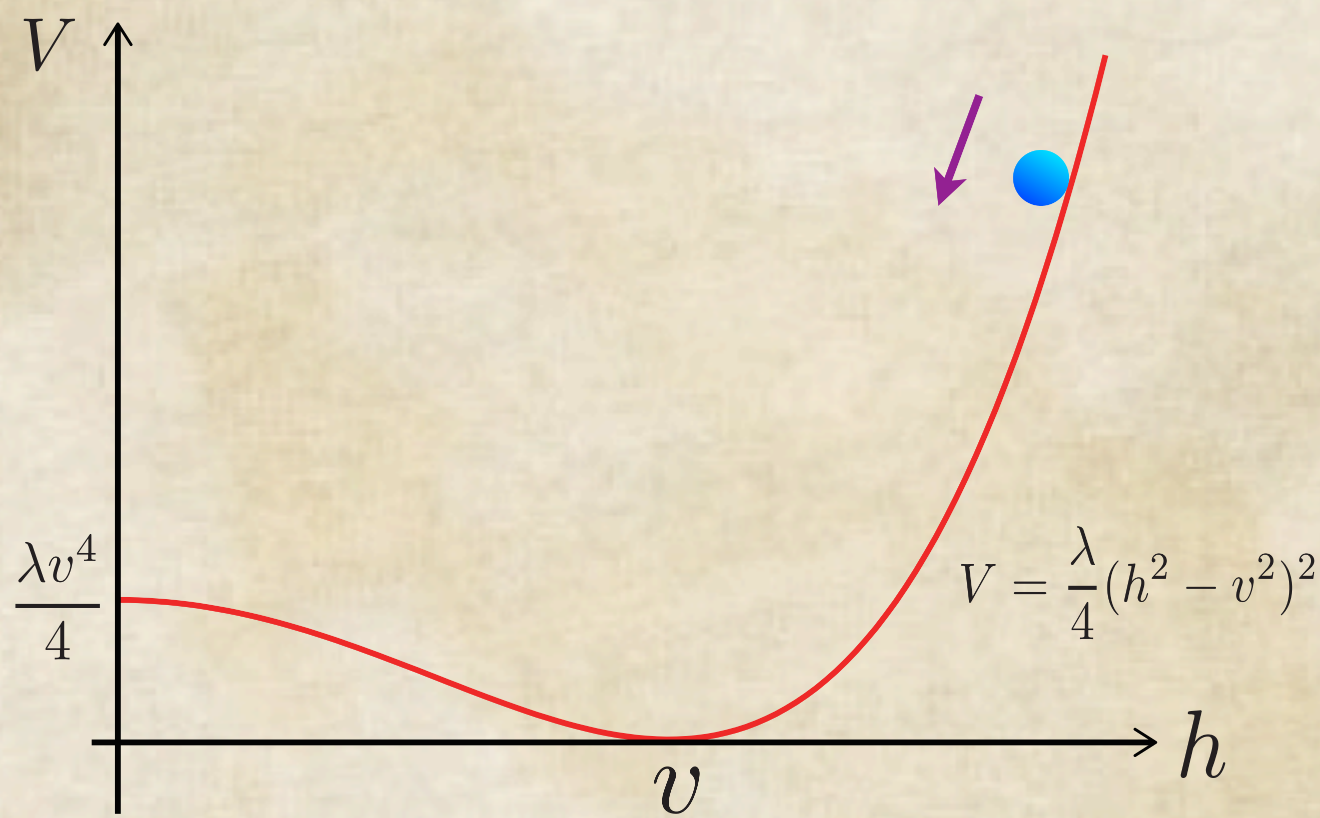


Chaotic inflation? ('83, Linde)

: possible to realize accelerating expansion of the Universe, but the primordial density perturbation becomes too large.

$$\boxed{\mathcal{P}_{\mathcal{R}} \sim 10^3 \lambda \quad \text{for} \quad V = \frac{\lambda}{4}\phi^4}$$

$\lambda_{\rm Higgs} \sim \mathcal{O}(1)$ is inconsistent with the observation $\mathcal{P}_{\mathcal{R}}^{\rm obs} \simeq 2.4 \times 10^{-9}$ (WMAP('11))

However, these conclusions are based on a theory with the canonical kinetic term and minimal coupling to gravity.

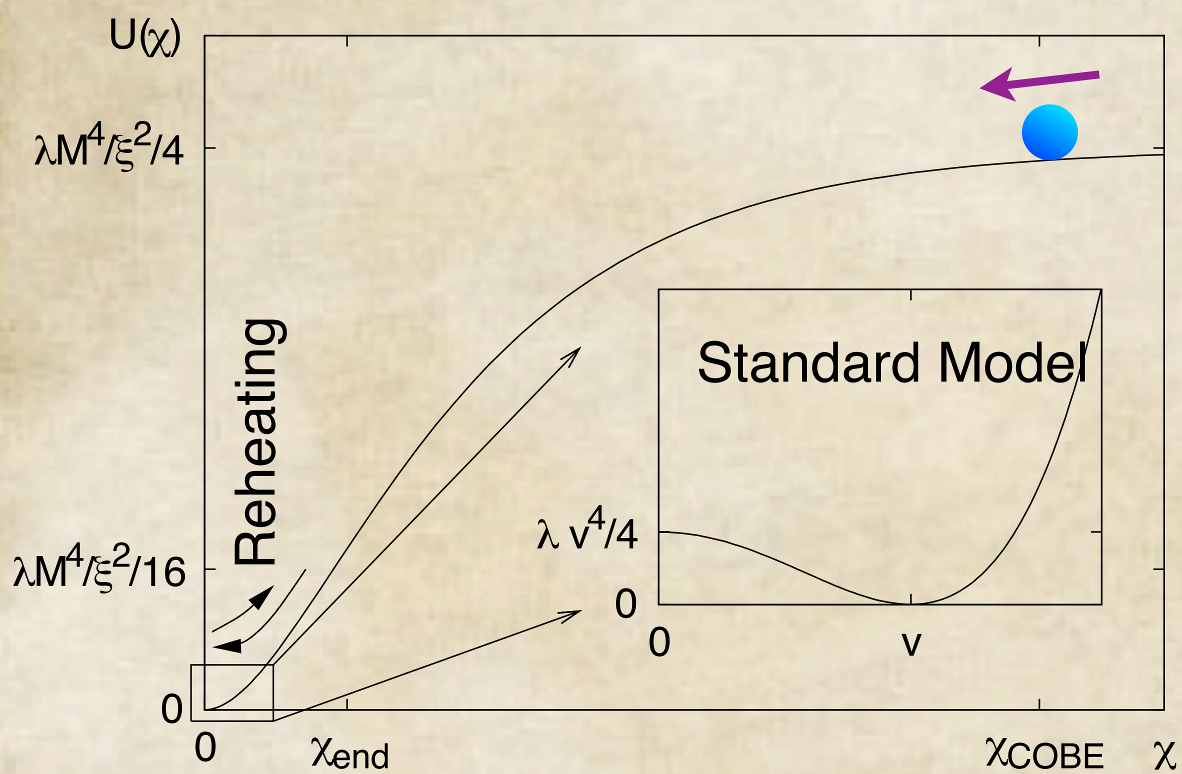
$$\boxed{S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu h\partial_\nu h - \frac{\lambda}{4}(h^2 - v^2)^2\right]}$$

There would arise possibilities that the SM Higgs field is the inflaton if we loose the above assumptions.

The first possibility is to consider nonminimal coupling to gravity.

('84, Spokoiny; '89 Futamase & Maeda)

$$\boxed{\Delta S = \int d^4x\sqrt{-g}\left[\frac{-\xi h^2}{2}R\right]}$$



For $|\xi|h^2 \gg 1$, the effective Planck mass becomes large

$$M_{\rm pl}^{\rm eff\,2} = M_{\rm pl}^2 + |\xi|h^2$$

and hence the primordial density fluctuation is suppressed.

('08, Bezrukov & Shaposhnikov)

This can be also understood by considering the effective potential in the Einstein frame.

Nonminimal coupling to gravity is <u>the only possibility</u> to realize the SM Higgs inflation up to renormalizable level.

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However, since the Einstein gravity is not renormalizable, we do not have to require renormalizability to inflation model as long as it is valid up to inflationary scale.

New Higgs inflation ('10, Germani & Kehagias),
running kinetic inflation ('10, Nakayama & Takahashi),
Higgs G-inflation ('11, Kamada, Kobayashi, Yamaguchi, & Yokoyama)

Noncanonical kinetic terms make Higgs inflation models possible.
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In this motivation, we require that the basic field equations should be expressed by up to 2nd order derivative terms in order to avoid ghost instability.

This would be the most general possibility of the SM Higgs inflation, which should be listed in preparation for the forthcoming discovery of the SM Higgs at LHC.

The most general theory of gravity and scalar fields with up to 2nd order derivatives is described by so-called “generalized Galileons”

('74, Horndeski; '09, '10, '11, Deffayet+;)

$$\mathcal{L}_2 = K(\phi, X),$$
$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi,$$
$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$
$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].$$

$$S = \int d^4x\sqrt{-g}\sum_{i=2}^{5}\mathcal{L}_i$$

$$X \equiv -\frac{1}{2}(\nabla\phi)^2, \quad G_{iX} \equiv \frac{\partial G_i}{\partial X}$$

This is the covariantization of field equations with Galilean shift symmetry (Galileon ('09, Nicolis+)) in flat space: $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$

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Inflation model with generalized Galileons is called **"G-inflation"**.

('10, Kobayashi, Yamaguchi, Yokoyama)

This means that all the single field inflation models with 2nd order field equations are described by only 4 functions;

$$\boxed{\begin{aligned} K(\phi, X) =& -V(\phi) + \mathcal{K}(\phi)X + \cdots\,, \\ G_i(\phi, X) =& g_i(\phi) + h_i(\phi)X + \cdots\,. \\ & \qquad\qquad\qquad\qquad i = 3, 4, 5 \end{aligned}}$$

Hereafter we neglect all the higher order terms in X.

This expansion contains 5 independent functions, $\boxed{\mathcal{K}, g_4, h_3, h_4, h_5}$

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Einstein-Hilbert action $\frac{M_{\rm pl}^2}{2} R \Leftrightarrow \; g_4 = \frac{M_{\rm pl}^2}{2}$

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(Original) Higgs inflation ('08, Bezrukov & Shaposhnikov)	$-\frac{\xi h^2}{2}R \Leftrightarrow g_4=-\frac{\xi h^2}{2}$
New Higgs inflation ('08, Germani & Kehagias)	$\frac{1}{2\mu^2}G^{\mu\nu}\partial_\mu h\partial_\nu h \Leftrightarrow h_5=\frac{1}{2\mu^2}$
running kinetic inflation ('10, Nakayama & Takahashi)	$-\frac{\kappa}{2}h^2\partial_\mu h\partial^\mu h \Leftrightarrow \mathcal{K}=\kappa h^2$
Higgs G-inflation ('11, Kamada+)	$\frac{h}{2M^4}\partial_\mu h\partial^\mu h\Box h \Leftrightarrow h_3=\frac{h}{M^4}$

Then, we find the last possibility of the SM Higgs inflation,

$$h_5 = \frac{h}{m^6} \quad \Rightarrow \quad \frac{h}{2m^6}\partial_\rho h \partial^\rho h G_{\mu\nu} \nabla^\mu \nabla^\nu h - \frac{h}{m^6}\left[(\Box h)^3 - 3(\Box h)(\nabla^\mu\nabla^\nu h)^2 + 2(\nabla^\mu\nabla^\nu h)^3\right]$$

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"running Einstein inflation"

This completes the possibility of single field inflation model without ghost instability, including the SM Higgs inflation.

Field equations

... Friedmann equation and the scalar field equation are extended.

Under the homogeneous and isotropic background,

$$ds^2 = -dt^2 + a(t)^2 dx^2, \quad \phi(x,t) = \phi(t)$$

gravitational field equation

$$\sum_{i=2}^{5} \mathcal{E}_i = 0,$$

$$\mathcal{E}_2 = 2XK_X - K,$$

$$\mathcal{E}_3 = 6X\dot{\phi}HG_{3X} - 2XG_{3\phi},$$

$$\mathcal{E}_4 = -6H^2G_4 + 24H^2X(G_{4X} + XG_{4XX}) - 12HX\dot{\phi}G_{4\phi X} - 6H\dot{\phi}G_{4\phi},$$

$$\mathcal{E}_5 = 2H^3X\dot{\phi}(5G_{5X} + 2XG_{5XX}) - 6H^2X(3G_{5\phi} + 2XG_{5\phi X}),$$

scalar field equation

$$\frac{1}{a^3}\frac{\mathrm{d}}{\mathrm{d}t}(a^3 J) = P_\phi,$$

$$J = \dot{\phi}K_X + 6HXG_{3X} - 2\dot{\phi}G_{3\phi} + 6H^2\dot{\phi}(G_{4X} + 2XG_{4XX}) - 12HXG_{4\phi X} + 2H^3X(3G_{5X} + 2XG_{5XX}) - 6H^2\dot{\phi}(G_{5\phi} + XG_{5\phi X}),$$

$$P_\phi = K_\phi - 2X\left(G_{3\phi\phi} + \ddot{\phi}G_{3\phi X}\right) + 6\left(2H^2 + \dot{H}\right)G_{4\phi} + 6H\left(\dot{X} + 2HX\right)G_{4\phi X} - 6H^2XG_{5\phi\phi} + 2H^3X\dot{\phi}G_{5\phi X}. - 6H^2\dot{\phi}(G_{5\phi} + XG_{5\phi X}),$$

Note: canonical & minimal coupling:

$$K = X - V(\phi), \quad G_4 = \frac{M_{\rm pl}^2}{2}$$

Potential-driven slow-roll inflation

Requiring slow-roll conditions:

$$\boxed{\epsilon := -\frac{\dot H}{H^2} \ll 1, \quad \eta := -\frac{\ddot\phi}{H\dot\phi} \ll 1, \quad \delta := \frac{\dot g_4}{H g_4} \ll 1,, \quad \alpha_2 := \frac{\dot{\mathcal{K}}}{H\mathcal{K}} \ll 1, \quad \alpha_i := \frac{\dot h_i}{H h_i} \ll 1 \;\; (i = 3,4,5).}$$

and potential domination: $\boxed{V \gg \dot\phi J}$

Slow-roll equations:

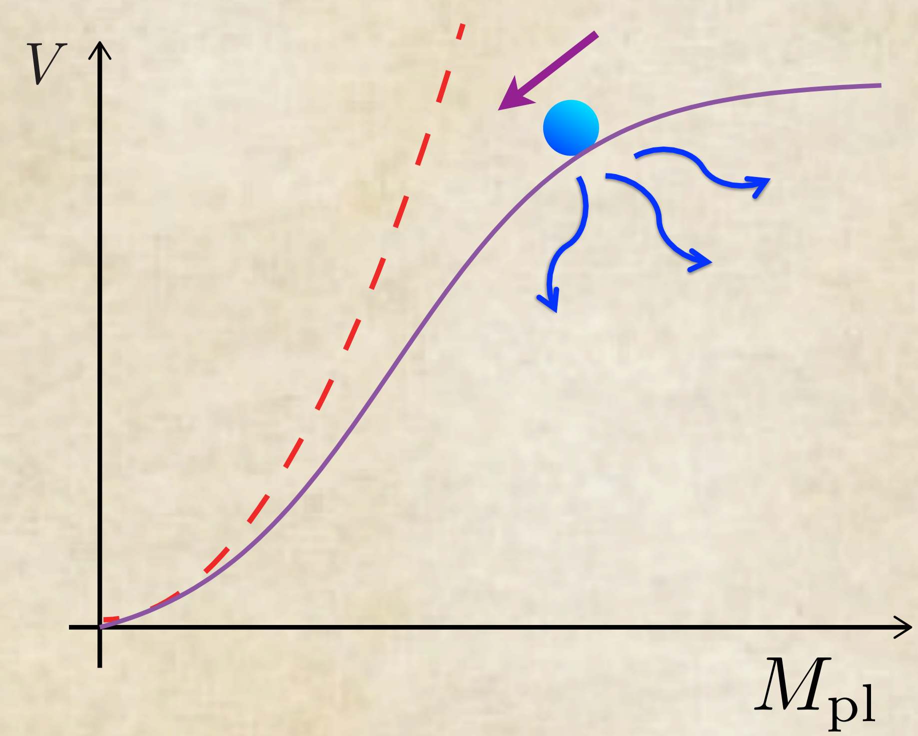
$$\boxed{6 g_4 H^2 \simeq V, \quad 3HJ \simeq -V' + 12 H^2 g_4'}$$

Large effective Planck mass;

Flatten the potential

$$\boxed{J \simeq \mathcal{K}\dot\phi + 3h_3 H\dot\phi^2 + 6h_4 H^2\dot\phi + 3h_5 H^3\dot\phi^2}$$

Additional friction term



sub-Planckian inflation becomes possible!!!

Information from observation:
Cosmological perturbations

... needed to determine/constrain the parameters

Amplitude of the powerspectrum of primordial density perturbation

$$\mathcal{P}_{\mathcal{R}}^{\rm obs} \simeq 2.4 \times 10^{-9}$$

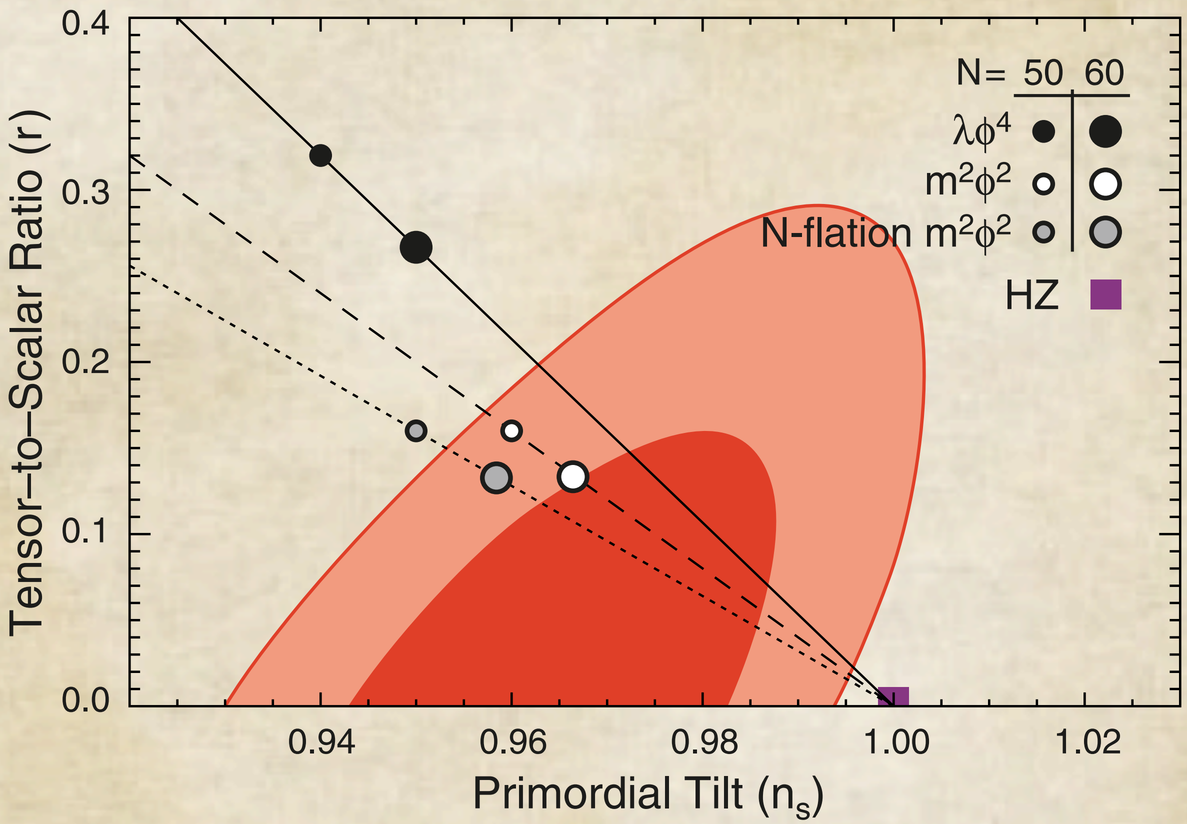
Spectral tilt of the powerspectrum of primordial density perturbation

$$n_s = \frac{d\ln \mathcal{P}_{\mathcal{R}}}{d\ln k}$$

and amplitude of tensor perturbation, parameterized by the scalar-to-tensor ratio,

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}}$$

are constrained by WMAP observations.



'11, Komatsu+

Primordial perturbations in potential-driven Generalized G-inflation

scalar perturbation:

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{2}\frac{\mathcal{G}_S^{1/2}}{\mathcal{F}_S^{3/2}}\frac{H^2}{4\pi^2}\bigg|_{c_s k\eta=-1}$$

tensor perturbation:

$$\mathcal{P}_T = 8\frac{\mathcal{G}_T^{1/2}}{\mathcal{F}_T^{3/2}}\frac{H^2}{4\pi^2}\bigg|_{c_T k\eta=-1}$$

Evaluated at sound horizon

$$\mathcal{F}_T \simeq \mathcal{G}_T \simeq 2g_4,$$

$$\mathcal{F}_S \simeq \frac{g_4}{3}(2\epsilon+\delta)\left(4-\frac{u}{W}\right),$$

$$\mathcal{G}_S \simeq g_4(2\epsilon+\delta)\left(2-\frac{u}{W}\right),$$

$$u(\phi) := \mathcal{K} + \frac{h_4 V}{g_4}, v(\phi) := h_3 + \frac{h_5 V}{6g}.$$

$$U'(\phi) := g_4^2\left(\frac{V}{g^2}\right)',$$

$$W(\phi) := \frac{1}{2}\left(u+\sqrt{u^2-4U'v}\right).$$

By using this formulation, we can distinguish the SM Higgs inflation models (or their combination) from the cosmological observations.

For the SM Higgs inflation,

$$\xi \simeq 10^4, \quad \mu \simeq 10^{11}\text{GeV}, \quad M \simeq 10^{13}\text{GeV}, \quad \text{and so on}$$

Moreover, we find the consistency relation,

$$r = -8n_T$$

for g_4(original Higgs inflation), h_4 (New Higgs inflation), and $\mathcal{K}$ (running kinetic inflation) domination.

$$r = -\frac{32\sqrt{6}}{9}n_T$$

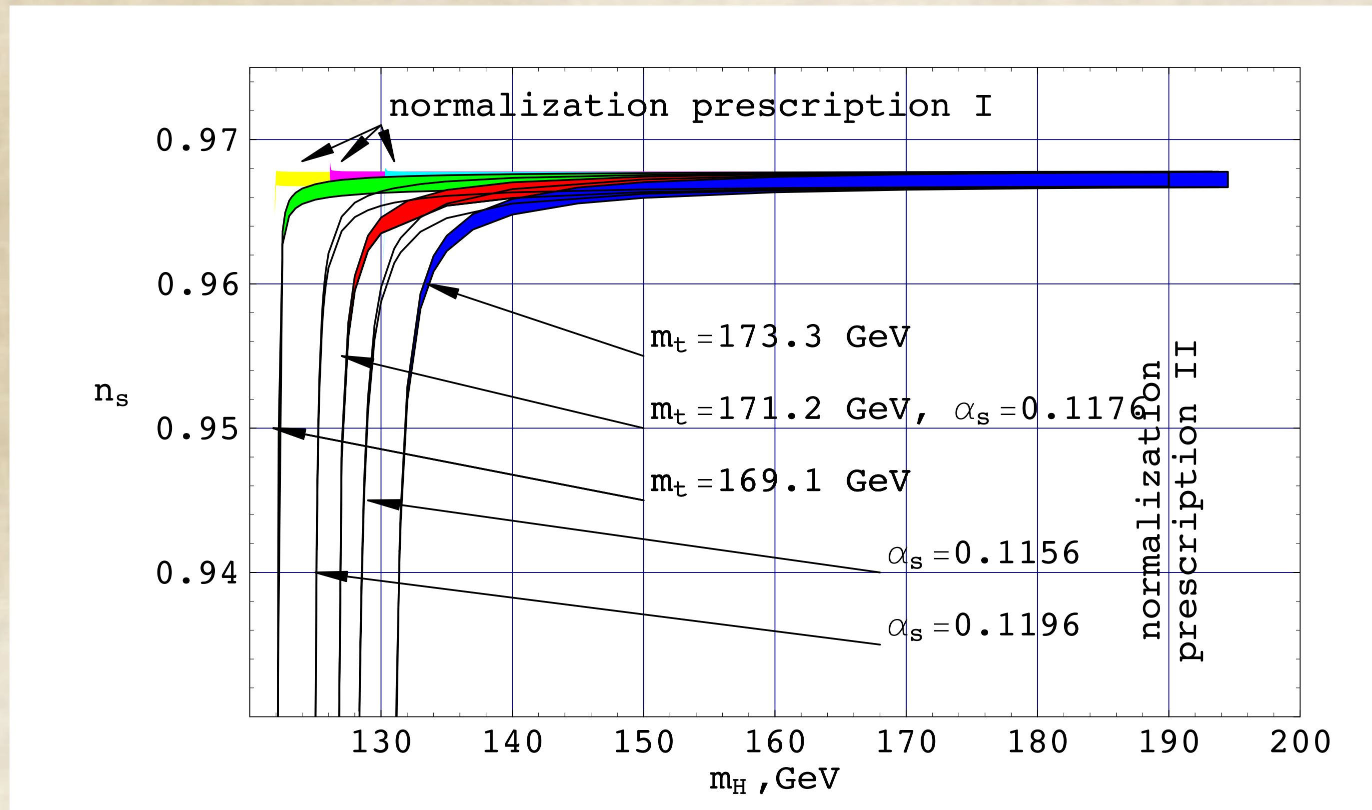
for h_3 (Higgs G-inflation), and h_5 (running Einstein inflation) domination.

This also would help model discrimination.

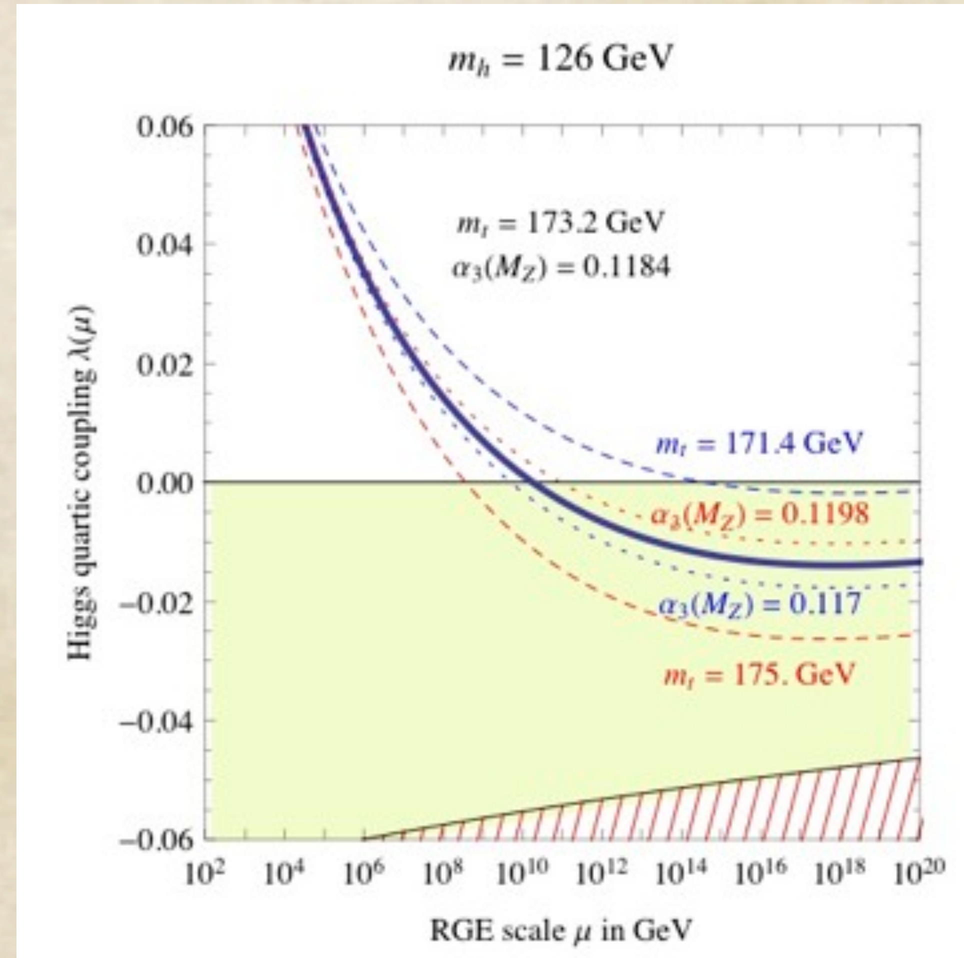
$n_T \equiv \dfrac{d\ln\mathcal{P}_T}{d\ln k}$ is the spectral tilt of tensor perturbation.

However...

it is known that quantum correction to the Higgs potential is important, especially for the 125 GeV Higgs.



('09, Bezrukov & Shaposhnikov)



('12, Elias-Miro+)

This is because the quartic coupling becomes negative around 10^{10} GeV for the 125 GeV Higgs.

⇒Evaluation of the quantum correction to generalized Higgs inflation is our future study.

Conclusion & Discussion

✓ The possibility that the SM Higgs field is the inflaton still remains if we consider nonminimal coupling to gravity or noncanonical kinetic terms.

✓ We find that all the Higgs inflation models proposed thus far can be understood in terms of potential-driven generalized G-inflation.

✓ We find another class of single field inflation model, "running Einstein inflation".

✓ Quantum correction to the Higgs potential should be evaluated.

We can tell whether the SM Higgs can be inflaton or not in the near future !!!

Backup slides

metric perturbations:

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \gamma_{ij}\left(\mathrm{d}x^i + N^i\mathrm{d}t\right)\left(\mathrm{d}x^j + N^j\mathrm{d}t\right),$$
$$N = 1 + \delta n, \quad N_i = \partial_i\chi,$$
$$\gamma_{ij} = a^2(t)e^{2\zeta}\left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj}\right).$$

$\delta n, \quad \chi$ can be removed by constraint equations.

Quartic action for tensor perturbations:

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla} h_{ij})^2\right],$$

where

$$\mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right],$$

$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right].$$

spectral tilt of tensor perturbation is evaluated as,

$$n_T = 3 - 2\nu_T.$$

$$f_T := \frac{\dot{\mathcal{F}}_T}{H\mathcal{F}_T}, \quad g_T := \frac{\dot{\mathcal{G}}_T}{H\mathcal{G}_T}$$

$$\nu_T := \frac{3 - \epsilon + g_T}{2 - 2\epsilon - f_T + g_T}.$$

Quartic action for scalar perturbations:

$$S_S^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_S\dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\vec{\nabla}\zeta)^2\right]$$

where

$$\mathcal{F}_S := \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T,$$

$$\mathcal{G}_S := \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T,$$

$$\begin{aligned}\Sigma :=\ & XK_X + 2X^2K_{XX} + 12H\dot{\phi}XG_{3X} \\ & + 6H\dot{\phi}X^2G_{3XX} - 2XG_{3\phi} - 2X^2G_{3\phi X} - 6H^2G_4 \\ & + 6\Big[H^2\left(7XG_{4X} + 16X^2G_{4XX} + 4X^3G_{4XXX}\right) \\ & - H\dot{\phi}\left(G_{4\phi} + 5XG_{4\phi X} + 2X^2G_{4\phi XX}\right)\Big] \\ & + 30H^3\dot{\phi}XG_{5X} + 26H^3\dot{\phi}X^2G_{5XX} \\ & + 4H^3\dot{\phi}X^3G_{5XXX} - 6H^2X\big(6G_{5\phi} \\ & + 9XG_{5\phi X} + 2X^2G_{5\phi XX}\big),\end{aligned}$$

$$\begin{aligned}\Theta :=\ & -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} \\ & - 8HX^2G_{4XX} + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X} \\ & - H^2\dot{\phi}\left(5XG_{5X} + 2X^2G_{5XX}\right) \\ & + 2HX\left(3G_{5\phi} + 2XG_{5\phi X}\right).\end{aligned}$$

spectral tilt:

$$n_s - 1 = 3 - 2\nu_S.$$

$$f_S := \frac{\dot{\mathcal{F}}_S}{H\mathcal{F}_S}, \quad g_S := \frac{\dot{\mathcal{G}}_S}{H\mathcal{G}_S},$$

$$\nu_S := \frac{3 - \epsilon + g_S}{2 - 2\epsilon - f_S + g_S}.$$

Bispectrum:

$$B_\zeta = \frac{(2\pi)^4 \mathcal{P}_\zeta^2}{4k_1^3k_2^3k_3^3}\left[6\mathcal{C}_1\frac{(k_1k_2k_3)^2}{K^3} + \frac{\mathcal{C}_2}{K}\left(2\sum_{i>j}k_i^2k_j^2 - \frac{1}{K}\sum_{i\neq j}k_i^2k_j^3\right) + \mathcal{C}_3\left(\sum_i k_i^3 + \frac{4}{K}\sum_{i>j}k_i^2k_j^2 - \frac{2}{K^2}\sum_{i\neq j}k_i^2k_j^3\right)\right.$$
$$\left. + \frac{\mathcal{C}_4}{K}\left(\sum_i k_i^4 - 2\sum_{i>j}k_i^2k_j^2\right)\left(1 + \frac{1}{K^2}\sum_{i>j}k_ik_j + \frac{3k_1k_2k_3}{K^3}\right)\right],$$

$$K := k_1 + k_2 + k_3$$

$$\begin{aligned}
\mathcal{C}_1 &= \frac{1}{c_s^2} - 1 + \frac{2(2-c_s^2)}{\mathcal{F}_S}\frac{\dot\phi X}{H}v \\
&\simeq \frac{1}{c_s^2} - 1 + 2(2-c_s^2)\frac{1-u/W}{4-u/W}, \\
\mathcal{C}_2 &= 3\left(1 - \frac{1}{c_s^2}\right), \\
\mathcal{C}_3 &= \frac{1}{2}\left(\frac{1}{c_s^2} - 1\right), \\
\mathcal{C}_4 &= -\frac{1}{c_s^2\mathcal{F}_S}\frac{\dot\phi X}{H}v \simeq -\frac{1}{c_s^2}\frac{1-u/W}{4-u/W},
\end{aligned}$$

Nongaussianity

$$f_{\rm NL}^{\rm eqil} = \frac{5}{81}\left(\mathcal{C}_1 + 6\mathcal{C}_2 + \frac{51}{2}\mathcal{C}_3 - \frac{13}{2}\mathcal{C}_4\right)$$
$$\simeq \frac{5}{243}\frac{(1-u/W)^2\,(99-43u/W)}{(4-u/W)^2\,(2-u/W)}.$$

$$\Rightarrow \quad f_{\rm NL}^{\rm equil} \approx \frac{5}{81}c_s^2 \gg 1.$$

for $u \approx -6H\dot{\phi}v$

For the specific choice of parameters,
large nongaussianity can be generated even in the SM Higgs inflation.