$\mathbb{Z}_N$ Symmetric Scalar Dark Matter

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$\mathbb{Z}_2$ and Beyond

- Dark matter – likely a WIMP
- Why is dark matter stable?
- Simplest discrete symmetry: parity $\mathbb{Z}_2$
- But also possible: $\mathbb{Z}_3, \mathbb{Z}_4, \ldots$

. . . and Beyond

Semi-Annihilations

For several species x_i of dark matter,

$$x_i\, x_j \rightarrow x_k\, X,$$

where X is a Standard Model particle

- Possible for $\mathbb{Z}_N$, $N > 2$
  (and non-Abelian groups)

d'Eramo & Thaler, 1003.5912,

Hambye et al. 0811.0172, 0907.1007, 0912.4496

Dark Matter Conversion

Annihilation from one kind of dark matter to another

$$x_2 x_2 \to x_1 x_1$$

Liu, Wu & Zhou 1101.4148, Bélanger & Park 1112.4491,

Adulpravitchai, Batell & Pradler 1103.3053

$\mathbb{Z}_N$ Symmetries

Field φ with charge X_φ transforms under $\mathbb{Z}_N$ as

$$\varphi \to e^{i\frac{X_\varphi}{N}2\pi}\varphi$$

- Addition modulo N
- Discrete charges $X = 0, 1, \ldots, N-1$

$\mathbb{Z}_N$ Symmetries

- $SO(10) \supset U(1)_X \to \mathbb{Z}_N$ by a GUT Higgs with $X = N$
- Different assignments of charges can give the same low evergy potential
- Limited number of Lagrangian terms due to renormalisability
- For higher N, the $U(1)_X$ symmetric part plus one or two terms

Scalar Dark Matter

- Scalars – simplest dark matter
- May be seen via the Higgs portal
- New scalars improve vacuum stability of the scalar potential

Scalar Field Content

- Standard Model Higgs H_1
- Inert doublet $H_2 = \begin{pmatrix} -iH^+ \\ \frac{H^0 + iA^0}{\sqrt{2}} \end{pmatrix}$
- Complex singlet S
- $\langle H_1 \rangle = \frac{v}{\sqrt{2}}, \langle H_2 \rangle = \langle S \rangle = 0$

$\mathbb{Z}_2$ Invariant Potential

$$
\begin{aligned}
V_{\mathbb{Z}_2} = {} & \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \mu_S^2|S|^2 + \frac{\mu_S'^2}{2}(S^2 + S^{\dagger 2}) \\
& + \frac{\mu_{SH}}{2}(S^\dagger H_1^\dagger H_2 + S H_2^\dagger H_1) + \frac{\mu_{SH}'}{2}(S H_1^\dagger H_2 + S^\dagger H_2^\dagger H_1) \\
& + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 \\
& + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{\lambda_5}{2}\left[(H_1^\dagger H_2)^2 + (H_2^\dagger H_1)^2\right] \\
& + \lambda_S|S|^4 + \frac{\lambda_S'}{2}(S^4 + S^{\dagger 4}) + \frac{\lambda_S''}{2}|S|^2(S^2 + S^{\dagger 2}) \\
& + \lambda_{S1}|S|^2|H_1|^2 + \frac{\lambda_{S1}'}{2}|H_1|^2(S^2 + S^{\dagger 2}) \\
& + \lambda_{S2}|S|^2|H_2|^2 + \frac{\lambda_{S2}'}{2}|H_2|^2(S^2 + S^{\dagger 2})
\end{aligned}
$$

$\mathbb{Z}_2$ Invariant Potential

- $\mathbb{Z}_2$ invariant potential studied in the $SO(10)$ context

  Kadastik, K.K., Raidal et al. 0903.2475, 0907.1894, …

Constraints on $\mathbb{Z}_N$ Charges

$$X_2 \neq X_1 \qquad \text{no } H_2 \text{ Yukawas}$$

$$X_S > 0 \qquad \text{no } |H_1|^2 S \text{ term}$$

$$-X_\ell + X_1 + X_e = 0 \mod N$$

$$-X_q + X_1 + X_d = 0 \mod N$$

$$-X_q - X_1 + X_u = 0 \mod N$$

Constraints on $\mathbb{Z}_N$ Charges

$X_1 = 0$	SM Yukawas
$X_2 > 0$	no H_2 Yukawas
$X_S > 0$	no $\|H_1\|^2 S$ term

$\mathbb{Z}_3$ Invariant Potential

$$
\begin{aligned}
V_{\mathbb{Z}_3} = & \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \mu_S^2|S|^2 + \frac{\mu_S''}{2}(S^3 + S^{\dagger 3}) \\
& + \frac{\mu_{SH}}{2}(SH_2^\dagger H_1 + S^\dagger H_1^\dagger H_2) + \lambda_1|H_1|^4 \\
& + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\
& + \lambda_S|S|^4 + \lambda_{S1}|S|^2|H_1|^2 + \lambda_{S2}|S|^2|H_2|^2 \\
& + \frac{\lambda_{S12}}{2}(S^2 H_1^\dagger H_2 + S^{\dagger 2} H_2^\dagger H_1)
\end{aligned}
$$

with $X_1 = 0, X_2 = 1, X_S = 1$

- Another potential related to$V_{\mathbb{Z}_3}$ via $S \leftrightarrow S^\dagger$

$\mathbb{Z}_3$ Invariant Potential

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\begin{aligned}
V_{\mathbb{Z}_3} = {} & \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \mu_S^2|S|^2 + \frac{\mu_S''}{2}(S^3 + S^{\dagger 3}) \\
& + \frac{\mu_{SH}}{2}(SH_2^\dagger H_1 + S^\dagger H_1^\dagger H_2) + \lambda_1|H_1|^4 \\
& + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\
& + \lambda_S|S|^4 + \lambda_{S1}|S|^2|H_1|^2 + \lambda_{S2}|S|^2|H_2|^2 \\
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& + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\
& + \lambda_S|S|^4 + \lambda_{S1}|S|^2|H_1|^2 + \lambda_{S2}|S|^2|H_2|^2 \\
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&+ \frac{\mu_{SH}}{2}(SH_2^\dagger H_1 + S^\dagger H_1^\dagger H_2) + \lambda_1|H_1|^4 \\
&+ \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\
&+ \lambda_S|S|^4 + \lambda_{S1}|S|^2|H_1|^2 + \lambda_{S2}|S|^2|H_2|^2 \\
&+ \frac{\lambda_{S12}}{2}(S^2 H_1^\dagger H_2 + S^{\dagger 2} H_2^\dagger H_1)
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&+ \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\
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with $X_1 = 0, X_2 = 1, X_S = 1$

- Another potential related to$V_{\mathbb{Z}_3}$ via $S \leftrightarrow S^\dagger$

$\mathbb{Z}_3$ Mass Eigenstates

$$H_2 = \begin{pmatrix} -iH^+ \\ x_1 \sin\theta + x_2 \cos\theta \end{pmatrix}, \quad S = x_1 \cos\theta - x_2 \sin\theta$$

In the dark sector,

- complex x_1, x_2, and H^+ with $X = 1$
- $2 = -1 \mod 3$, or
- $X = 2$ particles are antiparticles of $X = 1$ particles

$\mathbb{Z}_3$ Mass Eigenstates

- No λ_5 term to split masses of H^0 and A^0
- Dark matter is *singlet*-like x_1 with

$$\theta \leqslant 0.025$$

to avoid constraints from Z coupling

$\mathbb{Z}_3$ Processes

- Annihilation

$$x_1 x_1^* \to XX$$

from $\lambda_{S1}|S|^2|H_1|^2$ term for Higgs

- Semi-annihilation

$$x_1 x_1 \to x_1^* h$$

from $\frac{\mu_S''}{2}(S^3 + S^{\dagger 3})$ and $\lambda_{S1}|S|^2|H_1|^2$ terms

$\mathbb{Z}_3$ Benchmark Point

λ_2	0.1	λ_S	0.2	λ_{S12}	0.1	M_{x_1}	150 GeV
λ_3	0.1	λ_{S1}	0.05	M_h	125 GeV	M_{x_2}	400 GeV
λ_4	0.1	λ_{S2}	0.1	μ_S''	80 GeV	$\sin\theta$	0.025
				μ_{SH}	-40 GeV	M_{H^+}	396 GeV

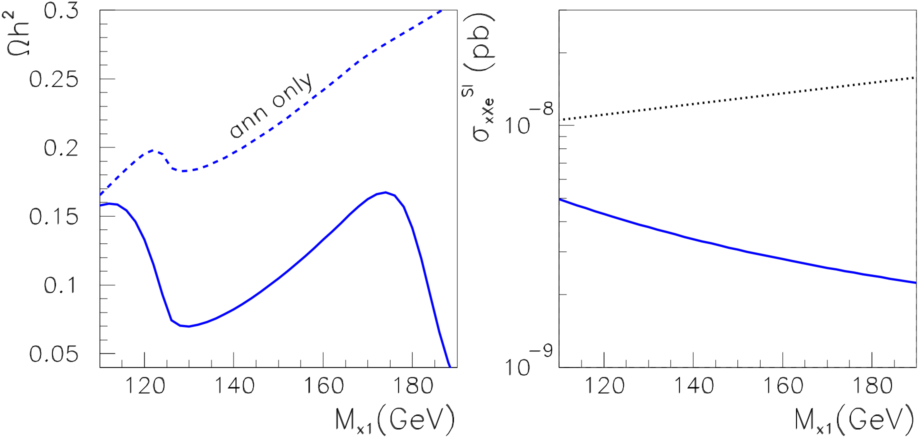
$\mathbb{Z}_3$ Relic Density

Relic density $\Omega h^2 = 0.105$

Contributions to $(\Omega h^2)^{-1}$

$x_1 x_1 \to x_1^* h$	54%
$x_1 x_1^* \to W^+ W^-$	22%
$x_1 x_1^* \to ZZ$	13%
$x_1 x_1^* \to hh$	10%

$\mathbb{Z}_3$ Relic Density & Direct Detection



$\mathbb{Z}_4$ Invariant Potential

- $\mathbb{Z}_4$ symmetry admits five potentials with H_1, H_2, S
- Four of them are special cases of the $\mathbb{Z}_2$ potential
- One has semi-annihilation terms

$\mathbb{Z}_4$ Invariant Potential

$$
\begin{aligned}
V^1_{\mathbb{Z}_4} = {} & \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \mu_S^2|S|^2 \\
& + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 \\
& + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{\lambda_5}{2}\left[(H_1^\dagger H_2)^2 + (H_2^\dagger H_1)^2\right] \\
& + \lambda_S|S|^4 + \frac{\lambda_S'}{2}(S^4 + S^{\dagger 4}) + \lambda_{S1}|S|^2|H_1|^2 \\
& + \lambda_{S2}|S|^2|H_2|^2 + \frac{\lambda_{S12}}{2}(S^2 H_1^\dagger H_2 + S^{\dagger 2} H_2^\dagger H_1) \\
& + \frac{\lambda_{S21}}{2}(S^2 H_2^\dagger H_1 + S^{\dagger 2} H_1^\dagger H_2),
\end{aligned}
$$

with $X_1 = 0, X_2 = 2, X_S = 1$

$\mathbb{Z}_4$ Invariant Potential

$$
\begin{aligned}
V^1_{\mathbb{Z}_4} = {} & \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \mu_S^2|S|^2 \\
& + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 \\
& + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{\lambda_5}{2}\left[(H_1^\dagger H_2)^2 + (H_2^\dagger H_1)^2\right] \\
& + \lambda_S|S|^4 + \frac{\lambda'_S}{2}(S^4 + S^{\dagger 4}) + \lambda_{S1}|S|^2|H_1|^2 \\
& + \lambda_{S2}|S|^2|H_2|^2 + \frac{\lambda_{S12}}{2}(S^2 H_1^\dagger H_2 + S^{\dagger 2} H_2^\dagger H_1) \\
& + \frac{\lambda_{S21}}{2}(S^2 H_2^\dagger H_1 + S^{\dagger 2} H_1^\dagger H_2),
\end{aligned}
$$

with $X_1 = 0, X_2 = 2, X_S = 1$

$\mathbb{Z}_4$ Mass Eigenstates

$$H_2 = \begin{pmatrix} -iH^+ \\ \frac{H^0 + iA^0}{\sqrt{2}} \end{pmatrix}, \quad S$$

In the dark sector,

- complex S with $X_S = 1$
- real H^0, A^0 with $X_{H^0} = X_{A^0} = 2$

$\mathbb{Z}_4$ Mass Eigenstates

$$3 = -1 \mod 4,$$

or

- $X = 3$ particles are antiparticles of $X = 1$ particles
- Lightest particle with $X = 1$, mass M_{x_1} is always stable
- LP with $X = 2$ is stable if $M_{x_2} < 2M_{x_1}$

$\mathbb{Z}_4$ Benchmark Point

λ_2	0.1	λ_{S1}	0.1	λ'_S	0.1	M_{A^0}	341 GeV
λ_3	0.1	λ_{S2}	0.3	μ_S	100 GeV	M_{H^0}	339 GeV
λ_4	0.01	λ_{S12}	0.13	M_h	125 GeV	M_S	350 GeV
λ_S	0.1	λ_{S21}	0.13	M_{H^+}	339.5 GeV		

$\mathbb{Z}_4$ Processes

Annihilation

$$x_1 x_1^* \to XX, \qquad x_2 x_2^* \to XX,$$

dark matter conversion

$$x_2 x_2^* \leftrightarrow x_1 x_1^*$$

Semi-annihilation

$$x_1 x_1 \to x_2 X, \qquad x_1 x_2 \to x_1 X$$

$\mathbb{Z}_4$ Processes

Annihilation

$$x_1 x_1^* \to XX, \qquad x_2 x_2^* \to XX,$$

dark matter conversion

$$x_2 x_2^* \leftrightarrow x_1 x_1^*$$

Semi-annihilation

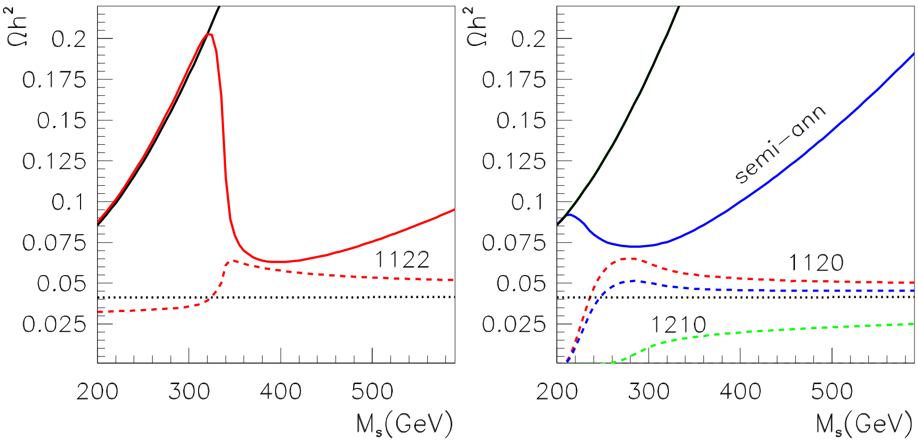
$$x_1 x_1 \to x_2 X, \qquad x_1 x_2 \to x_1 X$$

$\mathbb{Z}_4$ Relic Density

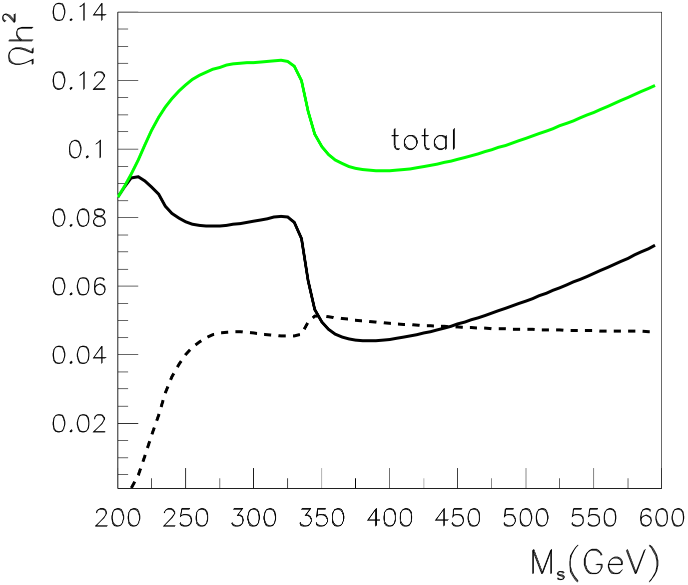
$$\Omega h^2 = \Omega_1 h^2 + \Omega_2 h^2 = 0.1$$

Included terms	$\Omega_1 h^2$	$\Omega_2 h^2$
$\sigma_v^{1100}, \sigma_v^{2200}$	0.24	0.041
$\sigma_v^{1100}, \sigma_v^{2200}, \sigma_v^{1122}$	0.079	0.064
All	0.050	0.051

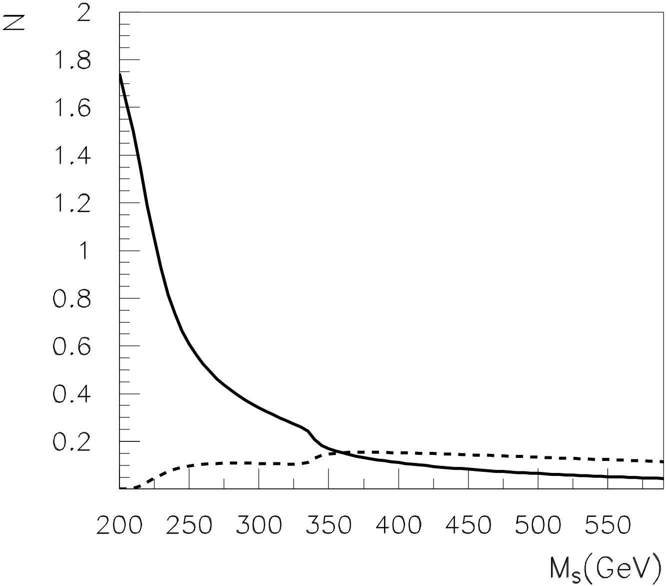
$\mathbb{Z}_4$ Contributions to Relic Density



$\mathbb{Z}_4$ Relic Density



N of Expected Events in XENON100



Conclusions

- Models with H_1, H_2, S have quartic semi-annihilation terms for $\mathbb{Z}_N$, $N > 2$
- Both semi-annihilations

$$x_i x_j \to x_k X$$

  and dark matter conversion

$$x_i x_i \to x_j x_j$$

  important for dark matter phenomenology
- May give correct Ωh^2 despite small σ_{SI} in XENON100 &c.

Backup Slides

$\mathbb{Z}_3$ Conditions for EW Vacuum

Electroweak vacuum with $\langle S \rangle = 0, \langle H_2 \rangle = 0$

$$\lambda_1, \lambda_2, \lambda_S, \lambda_{S1}, \lambda_{S2} > 0$$
$$\lambda_3 + \lambda_4 > 0$$
$$4\lambda_{S1}\lambda_{S2} > \lambda_{S12}^2$$
$$\frac{\mu''^2}{\lambda_S} + \frac{\mu_{SH}^2}{\lambda_3 + \lambda_4} < 4\mu_S^2$$

$\mathbb{Z}_3$ Parameters

$$\mu_S^2 = M_{x_2}^2 \sin^2\theta + M_{x_1}^2 \cos^2\theta - \lambda_{S1}\frac{v^2}{2}$$

$$\mu_2^2 = M_{x_1}^2 \sin^2\theta + M_{x_2}^2 \cos^2\theta - (\lambda_3 + \lambda_4)\frac{v^2}{2}$$

$$\mu_{SH} = -4(M_{x_2}^2 - M_{x_1}^2)\frac{\cos\theta\sin\theta}{\sqrt{2}v}$$

$$M_{H^+}^2 = \mu_2^2 + \lambda_3\frac{v^2}{2}$$

$$\lambda_1 = \frac{1}{2}\frac{M_h^2}{v^2}$$

$\mathbb{Z}_4$ Parameters

$$\mu_S^2 = M_S^2 - \lambda_{S1}\frac{v^2}{2}$$

$$\lambda_5 = \frac{M_{H^0}^2 - M_{A^0}^2}{v^2}$$

$$\mu_2^2 = M_{H^0}^2 - (\lambda_3 + \lambda_4 + \lambda_5)\frac{v^2}{2}$$

$$M_{H^+} = \sqrt{\frac{M_{A^0}^2 + M_{H^0}^2}{2} - \lambda_4\frac{v^2}{2}}$$

$$\lambda_1 = \frac{1}{2}\frac{M_h^2}{v^2}$$

$\mathbb{Z}_4$ Conditions for EW Vacuum

Electroweak vacuum with $\langle S \rangle = 0, \langle H_2 \rangle = 0$:

$$\begin{aligned} \lambda_1, \lambda_2, \lambda_{S1}, \lambda_{S2} &> 0 \\ \lambda_S - |\lambda'_S| &> 0 \\ \lambda_3 + \lambda_4 - |\lambda_5| &> 0 \\ (|\lambda_{S12}| + |\lambda_{S21}|)^2 &< \lambda_{S1}\lambda_{S2} \end{aligned}$$

$\mathbb{Z}_3$ Evolution Equations

$$\frac{dn}{dt} = -\langle v\sigma^{xx^*\to XX}\rangle \left(n^2 - \overline{n}^2\right) - \frac{1}{2}\langle v\sigma^{xx\to x^*X}\rangle \left(n^2 - n\,\overline{n}\right) - 3Hn,$$

$$\sigma_v \equiv \langle v\sigma^{xx^*\to XX}\rangle + \frac{1}{2}\langle v\sigma^{xx\to x^*X}\rangle \text{ and } \alpha = \frac{1}{2}\frac{\sigma_v^{xx\to x^*X}}{\sigma_v}, \tag{1}$$

meaning $0 \leq \alpha \leq 1$

- $\sigma_v^{xx\to x^*X} \equiv \langle v\sigma^{xx\to x^*X}\rangle$

$\mathbb{Z}_3$ Evolution Equations

In terms of the abundance, $Y = n/s$, where s is the entropy density, we obtain

$$\frac{dY}{dt} = -s\sigma_v \left(Y^2 - \alpha Y\overline{Y} - (1-\alpha)\overline{Y}^2 \right)$$

or, using the entropy conservation condition $ds/dt = -3Hs$,

$$3H\frac{dY}{ds} = \sigma_v \left(Y^2 - \alpha Y\overline{Y} - (1-\alpha)\overline{Y}^2 \right).$$

where $\overline{Y} = Y_{\rm eq}$ is the equilibrium abundance.

$\mathbb{Z}_3$

$$f_p = (4\sin^2\theta_W - 1)f_n = -0.075 f_n.$$

$$\sigma_{xN}^{\mathrm{SI}} = \frac{2}{\pi}\left(\frac{M_N M_{x_1}}{M_N + M_{x_1}}\right)^2 \left(\frac{[Zf_p + (A-Z)f_n]^2}{A^2} + \frac{[Z\bar{f}_p + (A-Z)\bar{f}_n]^2}{A^2}\right)$$

T Evolution, Close M_S, M_{H^0}