The Parity Violating Primordial Curvature Perturbation

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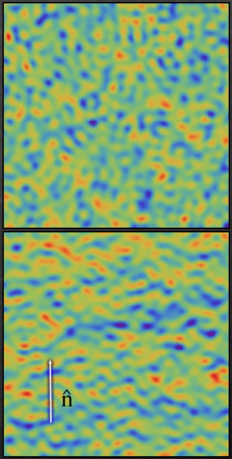
Universidad de Granada

K.Dimopoulos & MK, arXiv:1203.0230 [astro-ph.CO]

Statistical Anisotropy

- Planck satellite - inflationary parameters;
- New observable - statistical anisotropy;
- Anisotropy of the spectrum
  - $\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta^{\rm iso}(k)\left[1 + g_\zeta(k)\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}\right)^2 + \dots\right]$
  - $g_\zeta = 0.29 \pm 0.031$ *Groeneboom, Ackerman, Wehus, Eriksen (2010);*
  - $|g_\zeta| < 0.07$ *Hanson, Lewis, Challinor (2010);*
  - Planck prospects: *Ma, Efstathiou, Challinor (2011);*
    - $\Delta g_\zeta \sim 0.01$ (2σ);
    - $g_\zeta(k) \propto k^n$ to an accuracy $\Delta n \sim 0.3$ (1σ);



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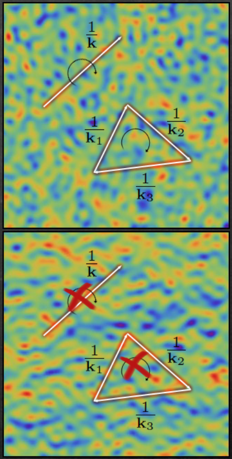

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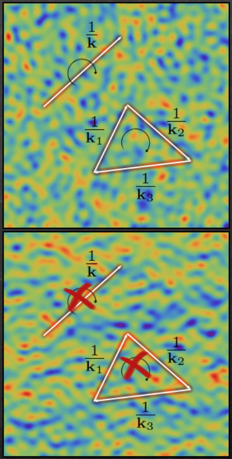

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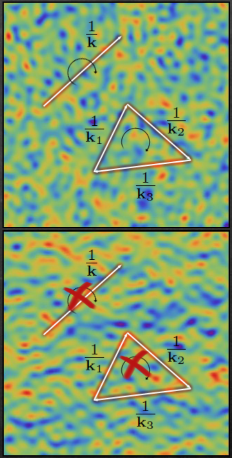

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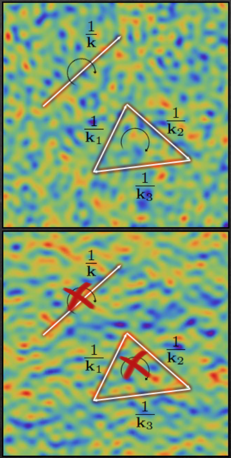
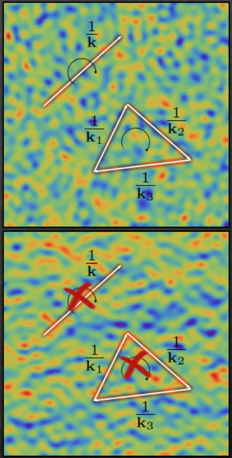


Statistical Anisotropy (Bispectrum)

- Anisotropy of the bispectrum

$$f_{\rm NL} = f_{\rm NL}^{\rm iso}\left(1 + f_{\rm NL}^{\rm aniso}\right)$$

  - No observational constraints yet;
  - Even if $g_\zeta \ll 1$
    - anisotropy in $f_{\rm NL}$ can be dominant, i.e. $f_{\rm NL}^{\rm aniso} \gg 1$; *Dimopoulos, MK, Wagstaff (2010)*
    - Parity violation does not affect the spectrum;



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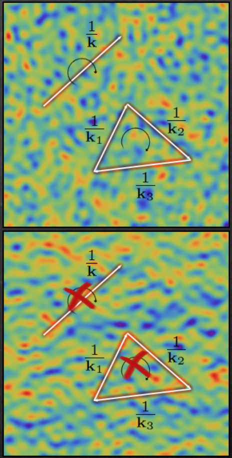
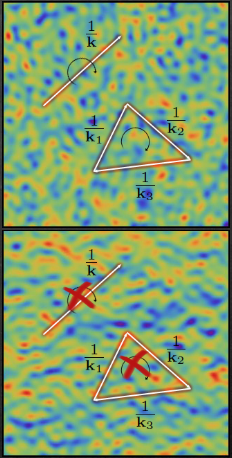


Vector Fields

- Statistical anisotropy from vector fields
    1. Backreaction on the background metric;
       Ackerman, Carroll, Wise (2007);
       Watanabe, Kanno, Soda (2010)
    2. Contribution to ζ from the vector field perturbation;
       Yokoyama & Soda (2008)
       Dimopoulos, MK, Lyth, Rodriguez (2009)
- A new parameter space for model building;
    - possible alleviation of the η problem;
      Dimopoulos, Lazarides, Wagstaff (2012);
    - a source for ζ:
        - heavy vector field; *Dimopoulos, MK, Wagstaff (2010)*
        - non-Abelian vector fields; *MK (2012)*
- Probe of gauge fields at high energies.
- Vector fields can generate parity violating ζ.



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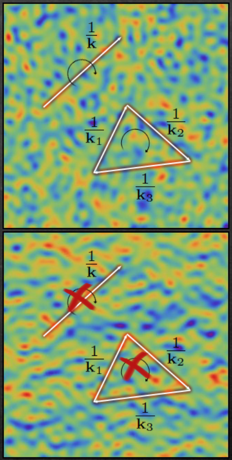

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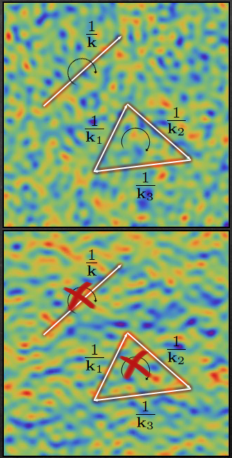

Vector Field Perturbations

- Quantum vector field $\mathbf{W} = \mathbf{A}/a \rightarrow \hat{\mathbf{W}}$:

$$\hat{\mathbf{W}}(t,\mathbf{k}) = \sum_{\lambda} \left[\mathbf{e}_{\lambda}\left(\hat{\mathbf{k}}\right) w_{\lambda}(t,k)\, \hat{a}_{\lambda}(\mathbf{k}) + \mathbf{e}^{*}_{\lambda}\left(\hat{\mathbf{k}}\right) w^{*}_{\lambda}(t,k)\, \hat{a}^{\dagger}_{\lambda}(\mathbf{k}) \right]$$

$$\lambda = \begin{cases} \mathrm{L},\ \mathrm{R} & \text{massless} \\ \mathrm{L},\ \mathrm{R},\ || & \text{massive} \end{cases}$$

- $w_{\lambda}(t,k)$ mode functions:
  - $k/a \gg H$: Bunch-Davies vacuum;
  - $k/a \ll H$: classical perturbation;
- Spectra: $\mathcal{P}_{\mathrm{L}}$, $\mathcal{P}_{\mathrm{R}}$ and $\mathcal{P}_{||}$;
- Anisotropy parameters:

$$p \equiv \frac{2\mathcal{P}_{||}}{\mathcal{P}_{\mathrm{R}}+\mathcal{P}_{\mathrm{L}}} - 1$$

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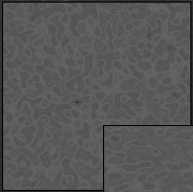

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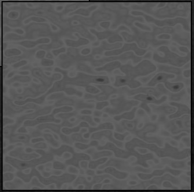

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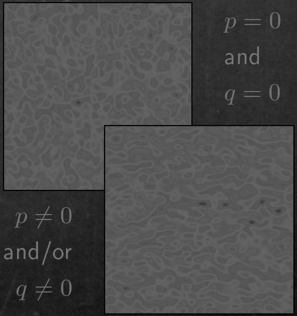

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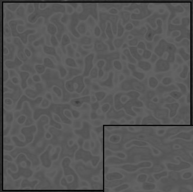

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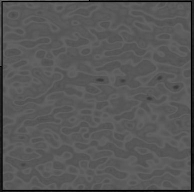

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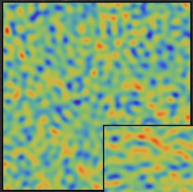

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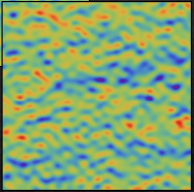

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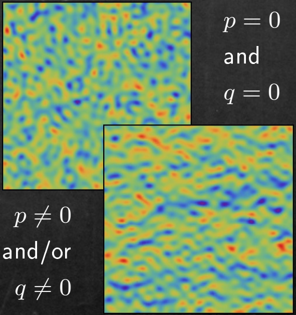

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Anisotropic Spectrum

- δN formula: $\zeta = N_\phi \delta\phi + N^i \delta W_i + \frac{1}{2} N^i N^j \delta W_i \delta W_j + \ldots$
  - $\delta\phi$ - statistically isotropic & gaussian;

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta^{\rm iso}(k)\left[1 + g_\zeta(k)\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{W}}\right)^2\right]$$

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- for $p \gtrsim 1$, $\xi \ll 1$: vector field contribution is subdominant;

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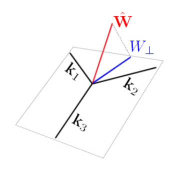
Anisotropic $f_{\rm NL}$

$$\frac{6}{5} f_{\rm NL}^{\rm eql} = \frac{\xi^2}{(1+\xi)^2} \frac{3}{2\hat{\Omega}_W} \left[\left(1+\tfrac{1}{2}q^2\right) + \left(p + \tfrac{1}{8}p^2 - \tfrac{1}{4}q^2\right) W_\perp^2\right]$$

$$\frac{6}{5} f_{\rm NL}^{\rm sqz} = \frac{\xi^2}{(1+\xi)^2} \frac{3}{2\hat{\Omega}_W} \left[1 + pW_\perp^2 + ipqW_\perp\sqrt{1-W_\perp^2}\,\sin\omega\right]$$

$$\frac{6}{5} f_{\rm NL}^{\rm flt} = \frac{\xi^2}{(1+\xi)^2} \frac{3}{2\hat{\Omega}_W} \left[\left(1-\tfrac{3}{5}q^2\right) + \left(2p + p^2 + \tfrac{3}{5}q^2\right) W_\perp^2 \cos^2\varphi\right]$$



1. p and q determines the magnitude and angular modulation of $f_{\rm NL}$;
2. For $p \gg 1$ $f_{\rm NL}$ predominantly anisotropic.
3. Parity violation modulates the shape of $f_{\rm NL}$;
4. Angular modulation is configuration dependent;

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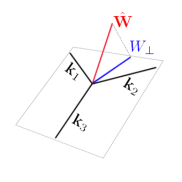

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Parity Violating Vector Field

$$\mathcal{L} = -\frac{1}{4} f\,(t)\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{4} h\,(t)\, F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2} m^2\,(t)\, A_\mu A^\mu$$

- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde{F}_{\mu\nu} = \frac{1}{2}\frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} F_{\rho\sigma}$;
- $f\,(t)$, $h\,(t)$ and $m\,(t)$ are functions of other DoFs or the theory;
- FRW metric: $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2\,(t)\,\mathrm{d}\mathbf{x}^2$;
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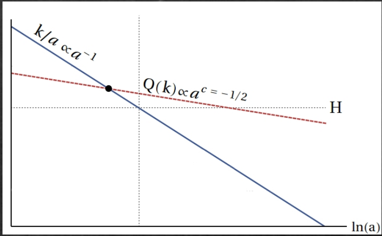

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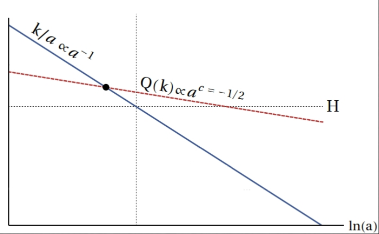

- Bunch-Davies initial conditions
  - $H \ll Q(t_x) \ll m_{\rm PL} \Rightarrow 1 \ll \vartheta \ll m_{\rm PL}/H$
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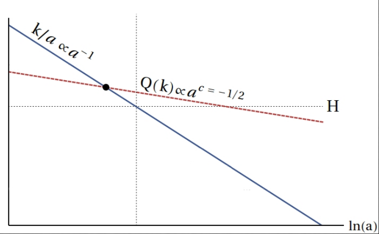

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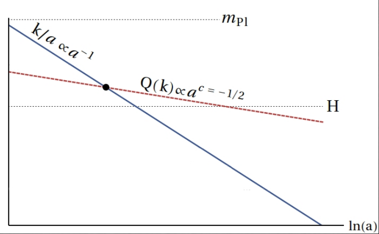

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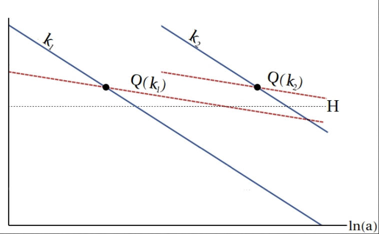

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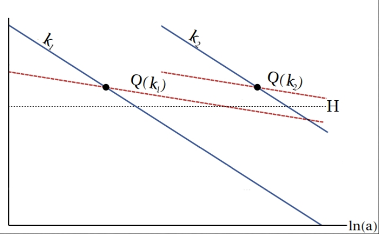

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Vector Curvaton and Non-Gaussianity

- The bound on inflationary scale:

$$H > \sqrt{-g_\zeta}\,10^6\mathrm{GeV} \;\Leftrightarrow\; V^{1/4} > (-g_\zeta)^{1/4}\,10^{12}\mathrm{GeV}$$

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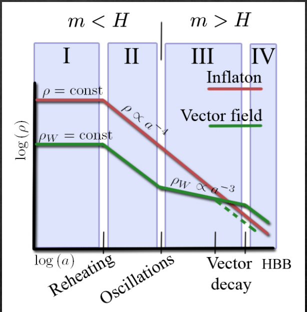

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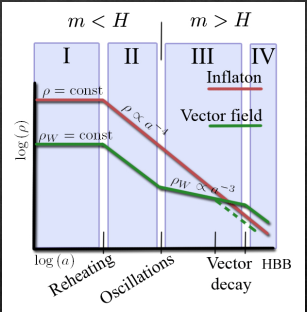

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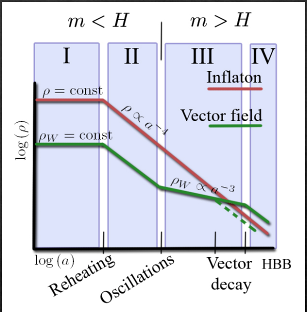

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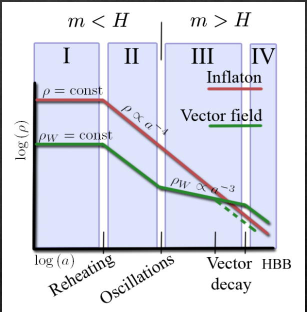
$$\frac{6}{5}f_{\rm NL}^{\rm eql} = 3\frac{g_\zeta^2}{\Omega_W}\left(1 - \frac{3}{4}W_\perp^2\right)$$

$$\frac{6}{5}f_{\rm NL}^{\rm sqz} = 2\frac{g_\zeta^2}{\Omega_W}\left(1 - W_\perp^2 - iqW_\perp\sqrt{1 - W_\perp^2}\sin\omega\right)$$

$$\frac{6}{5}f_{\rm NL}^{\rm flt} = \frac{4}{5}\frac{g_\zeta^2}{\Omega_W}\left(1 - W_\perp^2\cos^2\varphi\right)$$



Summary

- Vector fields can generate or contribute to ζ;
- Generally statistically anisotropic ζ;
- Present bounds:
  - $\mathcal{P}_\zeta = \mathcal{P}_\zeta^{\rm iso}\left[1 + g_\zeta\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}\right)^2\right]$: $g_\zeta < 0.29$ or $|g_\zeta| < 0.07$;
  - Planck: $\Delta g_\zeta \sim 0.01$ and $g_\zeta \sim k^n$, $\Delta n \sim 0.3$;
  - $f_{\rm NL} = f_{\rm NL}^{\rm iso}\left[1 + f_{\rm NL}^{\rm aniso}\right]$: no bounds on $f_{\rm NL}^{\rm aniso}$
- $f_{\rm NL}^{\rm aniso} \gg 1$ can be dominant and observable even for $g_\zeta \ll 1$;

Parity Violating Perturbations

- Vectors with axial couplings can generate parity violating ζ;
- Parity violation affects only higher order correlators;
- An example with the vector curvaton scenario;

Excessive Large Scale Anisotropy

- For massless or light $U(1)$ vector field $\mathbf{W} = (0, 0, W)$:

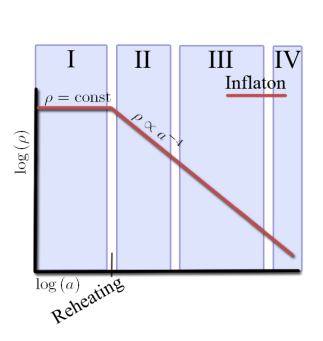
$$T_\mu^\nu = \mathrm{diag}\,(\rho, -p, -p, +p)$$

$$p = \frac{1}{2} f \left(\frac{\dot{A}}{a}\right)^2 - \frac{1}{2} m^2 \left(\frac{A}{a}\right)^2.$$

1. Vector Curvaton Scenario;
2. End-of-Inflation Scenario.

Vector Curvaton

1. Inflation $m/\sqrt{f} < H$: Particle production;
2. Light vector field $m < H$;
   $$\ddot{\mathbf{A}} + H\dot{\mathbf{A}} + m^2\mathbf{A} = 0$$
3. Heavy vector field $m > H$:
   Vector field oscillates and behaves as preasureless, isotropic matter:
   $$\bar{p}_W \approx 0 \;\&\; \rho \propto a^{-3};$$
4. Vector field decay
   4.1 Onset of the Hot Big Bang;
   4.2 Generation of ζ:
   $$\boxed{\zeta = (1 - \Omega_W)\,\zeta_\gamma + \Omega_W \zeta_W} \quad \Omega_W \equiv \frac{\rho_W}{\rho}.$$



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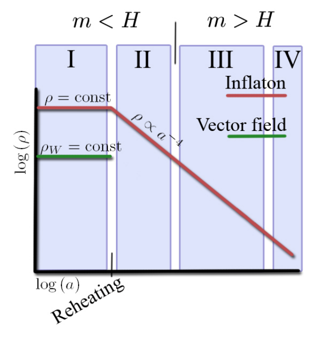

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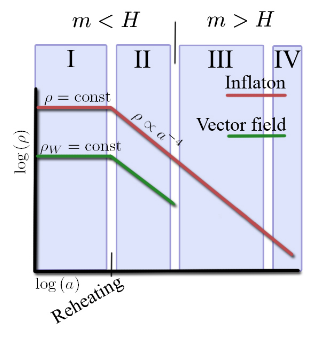

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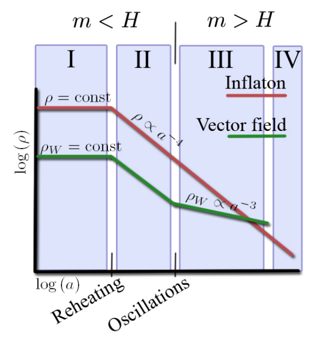

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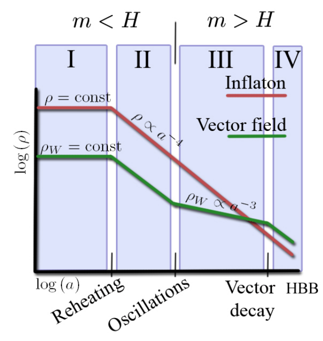

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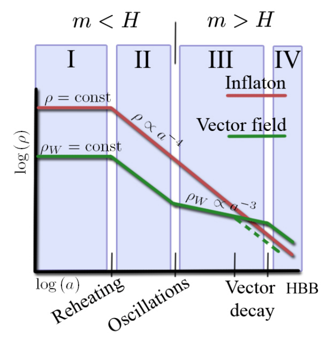

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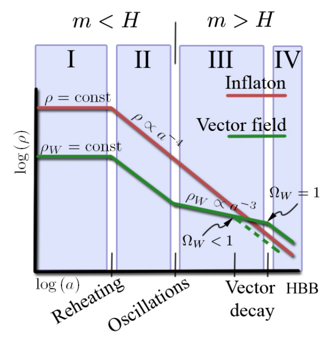

A Note on $f \propto a^{-4}$ Scaling

- If $f(\phi)$ with ϕ being the inflaton, $f \propto a^{-4}$ is an attractor solution
  - For $U(1)$: *Watanabe, Kanno, Soda (2009); Kanno, Soda, Watanabe (2010); Wagstaff, Dimopoulos (2011);*
  - For $SU(2)$: *Murata, Soda (2011);*
- "Short cosmic hair": weak large scale anisotropy;
- Statistical anisotropy due to anisotropic background;
- Here $f(\sigma(t)) \propto a^{-4}$ is assumed.