Cosmology challenges brane scenarios in AdS_5

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From the Planck Scale to the Electroweak Scale

Outline

beyond General Relativity

- ongoing search for a unified description of
  - gravity
  - gauge interactions of the Standard Model
  - ↪ *string theories* amongst the most promising proposals
- *low-energy effective action* in string theories (only massless modes)
  - depending on the string theory type: includes terms for various particles
  - *dilaton* (ϕ): scalar field accompanying gravity (common for all string theories)
  - at the leading order (when restricted to gravity and the dilaton)
  - ↪ Einstein's gravity coupled to the dilaton
  - in a spacetime with additional spatial dimensions
  - ↪ i.e. an extended, higher-dimensional theory of gravity: *dilaton gravity*
- dilaton gravity in a 5D brane scenario
  - Standard Model localized on a 4D brane embedded in a 5D spacetime

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scalar-tensor theories of gravity & conformal frames

- *dilaton gravity*: a scalar-tensor theory of gravity
  - ↪ can be formulated in various conformally-related frames
    - $g_{\mu\nu}$ & $\widetilde{g}_{\mu\nu}$ related by a Weyl (conformal) transformation: $g_{\mu\nu} = \Omega(x)^2\, \widetilde{g}_{\mu\nu}$
  - gravitational Lagrangians differ e.g. in the coefficient of the Ricci scalar
    - ↪ (generically) scalar field dependent coefficients
    - Einstein frame: $\mathcal{L} = \frac{1}{2\kappa}\, \mathcal{R} + \cdots$

      (natural in standard gravity & cosmology; coefficient: a constant)
    - Jordan frame: e.g. $\mathcal{L} = \frac{1}{16\pi} \phi\, \mathcal{R} + \cdots$

      ("traditional" frame in scalar-tensor theories of gravity; coefficient: a polynomial function of the scalar field)
    - string frame: e.g. $\mathcal{L} = e^{-\phi}\, \frac{\alpha_1}{2} \mathcal{R} + \cdots$

      (natural in string theories; coefficient: an exponential function of the dilaton)

non-minimal matter-dilaton coupling

- if a matter term $\mathcal{L}_m$ is included into the Lagrangian in one frame
  - ⥀ conformal transformation to another frame will change its coefficient
  - ↪ if constant in one frame, it will become dilaton dependent in others
- which conformal frame is the *natural physical* frame?
  - ⥀ no clear consensus in the literature
  - ↪ in which frame the matter-dilaton coupling should be minimal?
- thus: a general *non-minimal coupling* $f(\phi)\,\mathcal{L}_m$
  - ⥀ of the dilaton
  - ↪ to the matter content of the universe $\mathcal{L}_m$

(localized on the brane)

the aim of the game

- framework:
  - dilaton gravity in a 5D brane scenario
  - non-minimal matter-dilaton coupling $f(\phi)\,\mathcal{L}_m$

    ($\mathcal{L}_m$: matter content of the universe localized on the brane)

- take assumptions crucial to many models in the modern literature:
  - *bulk*: exact anti de Sitter type spacetime (AdS_5)

    (vital for many higher-dimensional scenarios, one of the simplest types of spacetimes)
  - *brane*: matter content of the universe (as in cosmological considerations) described by an inhomogeneous perfect fluid

- and answer the question:

can the observed large-scale structure of the universe

exist on the brane in an AdS_5 bulk?

i.e.: is a “sufficiently” inhomogeneous perfect fluid (and thus the large-scale structure) permitted on the brane?

Outline

dilaton gravity at the brane with general matter-dilaton coupling

- dilaton gravity in a 5D brane scenario: (Einstein frame)

$$\mathcal{L} = \frac{\alpha_1}{2}\left[\mathcal{R} - \frac{2}{3}\nabla^\sigma\partial^{(5)}_\sigma\phi - \frac{1}{3}(\partial^{(5)}\phi)^2\right] - V(\phi) + \left[f(\phi)\mathcal{L}_m + \lambda(\phi)\right]\delta_B$$

  - $\left[\mathcal{R} - \frac{2}{3}\nabla^\sigma\partial^{(5)}_\sigma\phi - \frac{1}{3}(\partial^{(5)}\phi)^2\right]$: 5D dilaton gravity ($\alpha_1$: constant)
  ($V(\phi)$: scalar potential in the bulk)
  - $\mathcal{L}_m$: (brane localized) matter content of the universe
  $\lambda(\phi)$: 'cosmological constant'-type term on the brane
    ↪ position of the co-dimension 1 brane: Dirac delta type distribution δ_B
  - $f(\phi)\,\mathcal{L}_m$: (non-minimal) coupling of the dilaton ϕ to brane localized matter $\mathcal{L}_m$

- how does the gravity look like on the brane? (i.e. for us)
  ↪ its effective 4D description has to be established

derivation of the effective gravitational equations at the brane

- induced (projected) brane metric: $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ (covariant approach)
  - n^μ: vector field orthonormal to the brane at its position
  - $g_{\mu\nu}$: $\mathcal{R}_{\mu\nu}{}^{\rho\sigma}$ & ∇_μ vs $h_{\mu\nu}$: $R_{\mu\nu}{}^{\rho\sigma}$ & D_μ
- assume a $\mathbb{Z}_2$ symmetry for the bulk (with its fixed point at the brane's position)
  - ↪ usually imposed 'automatically'
  - crucial for the existence of the effective gravitational equations at the brane
- in the absence of the bulk $\mathbb{Z}_2$ symmetry
  only consistency condition (on the brane sources): $D_\lambda \left(f(\phi) \tau_\mu^\lambda \right) = f(\phi)\, \tau_\phi (\partial_\mu \phi)$

  (on the brane: "generalized" covariant conservation of the energy-momentum tensor)

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at the brane: effective Einstein-like equation

- consequently, the *effective* Einstein-like equation at the brane reads

$$R_{\mu\nu} - \frac{1}{2}\, h_{\mu\nu} R = 8\pi \overline{G}(\phi)\, \tau_{\mu\nu} - h_{\mu\nu} \overline{\Lambda}(\phi) + \frac{f^2(\phi)}{4\alpha_1^2} \pi_{\mu\nu} - E_{\mu\nu}$$
$$+ \frac{2}{9}\, (\partial_\mu \phi)(\partial_\nu \phi) - \frac{5}{36}\, h_{\mu\nu}\, (\partial\phi)^2$$

  - $\overline{G}(\phi) = \frac{-1}{48\pi\alpha_1^2} f(\phi)\lambda(\phi)$ (effective brane Newton's constant)
  - $\tau_{\mu\nu} = h_{\mu\nu}\, \mathcal{L}_m - 2\, \frac{\delta \mathcal{L}_m}{\delta h^{\mu\nu}}, \tau_\phi = \frac{f'(\phi)}{f(\phi)}\, \mathcal{L}_m + \frac{\delta \mathcal{L}_m}{\delta \phi}$ (brane localized sources)
  - $\overline{\Lambda}(\phi) = \frac{1}{2\alpha_1} V - \frac{f^2}{4\alpha_1^2} \left[\frac{3}{4}\tau_\phi^2 + \frac{3\lambda'}{2f}\tau_\phi - \frac{\lambda^2}{3f^2} + \frac{3\lambda'^2}{4f^2}\right]$ (eff. brane cosmol. const.)

- three types of 'corrections' to the standard Einstein equation
  - terms quadratic in the brane energy-momentum tensor:
    $\pi_{\mu\nu} = -\tau_{\mu\rho}\tau^\rho_\nu + \frac{1}{3}\, \tau\, \tau_{\mu\nu} + \frac{1}{2}\, h_{\mu\nu}\tau^\sigma_\rho \tau^\rho_\sigma - \frac{1}{6}\, h_{\mu\nu}\, \tau^2$ (typical of brane gravity theories)
  - kinetic terms for the dilaton (typical of scalar-tensor theories of gravity)
  - explicit bulk's influence *on the brane gravity*: $E_{\mu\nu} = n^\alpha h^\beta_\mu n^\gamma h^\delta_\nu\, \mathcal{C}_{\alpha\beta\gamma\delta}$
    (bulk Weyl tensor projected on the brane: a single term, but generically non-vanishing)

  ↪ to describe gravity induced on the brane: solution of the e.o.m.'s for the bulk gravity necessary

Outline

at the brane:

constraint on the spatial derivative of the matter energy density

- OR: "sufficiently" inhomogeneous perfect fluid on the brane in AdS_5 bulk?
- calculus ingredients:
  - effective gravitational (Einstein-like) equation at the brane:

$$R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R = 8\pi \overline{G}(\phi)\, \tau_{\mu\nu} - h_{\mu\nu} \overline{\Lambda}(\phi) + \frac{f^2(\phi)}{4\alpha_1^2} \pi_{\mu\nu} - E_{\mu\nu} + \frac{2}{9} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{5}{36} h_{\mu\nu} (\partial\phi)^2$$

  - consistency condition (on the brane sources): $D_\lambda \left(f(\phi) \tau^\lambda_\mu \right) = f(\phi)\, \tau_\phi (\partial_\mu \phi)$
  - 4D Bianchi identity: $D^\nu \left(R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R \right) = 0$
- assumptions:
  - bulk: exact anti de Sitter type spacetime: $AdS_5 \rightarrow E_{\mu\nu} = 0$
    (no bulk influence on the brane gravity)
  - brane (matter content of the universe): perfect fluid $\rightarrow \tau_{\mu\nu} = \rho_m\, t_\mu t_\nu + p_m\, \gamma_{\mu\nu}$
    ($\gamma_{\mu\nu}$: 3d spatial metric, ρ_m: (dark) matter & radiation)

on the brane: late universe

- thus, spatial derivative of the matter energy density on the brane reads

$$\rho_{m,i} = -\left(\frac{f'}{f}\rho_m - \frac{\lambda'}{f}\right)\phi_{,i} + \frac{\alpha_1^2}{3f^2(\rho_m+p_m)}\left[D^\nu\partial_i\phi - \dot{\phi}^{-1}\phi_{,i}\,D^\nu\partial_t\phi\right](\partial_\nu\phi)$$

↪ imposes a strict condition on the matter content of the universe

(potentially: a strong constraint on how inhomogeneous the matter distribution can be - and thus on the cosmological large-scale structure as is observed today)

- (at least) late universe: terms $\mathcal{O}((\partial\phi)(D\partial\phi))$ can be neglected, as
  - $\dot{\phi}_0 \lesssim 2.4\,H_0 \simeq 1.8\,(10^{10}\text{ yr})^{-1}$

    (derived: model-independent bound set by current observational data)
  - $|\ddot{\phi}_0| \ll \dot{\phi}_0^2$ can be assumed / expected

    (otherwise: currently observed $\phi_0 \approx \mathrm{const}$ would be yet another coincidence problem)
  - typical models: $|\phi_{,i}| \lesssim c_1|\dot{\phi}|$ ($c_1 > 0$ and of order 1)

    (any initial inhomogeneities of the dilaton washed out by inflation)
- hereafter: $\lambda \neq \lambda(\phi)$ ('cosmological constant'-type term in the energy-momentum tensor on the brane)

  (only a contribution to the effective brane cosmological constant $\overline{\Lambda}(\phi)$)

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late universe: spatial derivative of the energy density - quantify it!

- hence for the late universe we obtain

$$\rho_{m0\,,i} \simeq -\frac{f'}{f}\,\rho_{m0}\,\phi_{0\,,i}$$

highly constrained: how *inhomogeneous* the matter energy density can be

for the common assumptions of AdS_5 bulk and perfect fluid on the brane

  - inhomogeneous perfect fluid ($\rho_{m\,,i} \neq 0$) on the brane? *only if:*
    matter content of the universe coupled non-minimally ($f' \neq 0$) to the *dilaton*
  - if dilaton spatially homogeneous: no inhomogeneous matter energy density
    $\hookrightarrow$ already: $\dot{\phi}_0 \lesssim 2.4\, H_0 \simeq 1.8\,(10^{10}\ \text{yr})^{-1}$

let's quantify the implications of the constraint on $\rho_{m0\,,i}$:

- current observational limits: $|\dot{\overline{G}}_0/\overline{G}_0| < (10^{11}\,\mathrm{yr})^{-1}$ ($\overline{G} = \overline{G}(\phi)$!)
  (searches for time variation of the Newton's constant:
  pulsar timing, solar system, stellar, cosmological constraints)

  $\hookrightarrow$ $\left|\frac{f'}{f}\,\phi_{0\,,i}\right| \lesssim 3.3\,c_1\,(10^5\,\mathrm{Mpc})^{-1}$ (for $|\phi_{,i}| \lesssim c_1|\dot{\phi}|$) *resulting in*

$$|\rho_{m0\,,i}| \lesssim 3.3\,c_1\,\rho_{m0}\,(10^5\,\mathrm{Mpc})^{-1}$$

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cosmological large-scale structure (LSS) data

- LSS of the universe: spatial distribution of galaxies, their groups and clusters
- galaxy distribution: probed by galaxy redshift surveys

(addressed: content and statistical properties of the LSS)

(e.g. Sloan Digital Sky Survey (SDSS))

↪ characterized statistically through the two-point correlation function $\xi(x)$

(excess number of galaxy pairs separated by x relative to that expected for a random distribution)

⟳ density contrast: $\delta(\vec{x},t) \equiv \frac{\rho(\vec{x},t)}{\rho_{av}(t)} - 1$ (ρ_{av}: mean density)

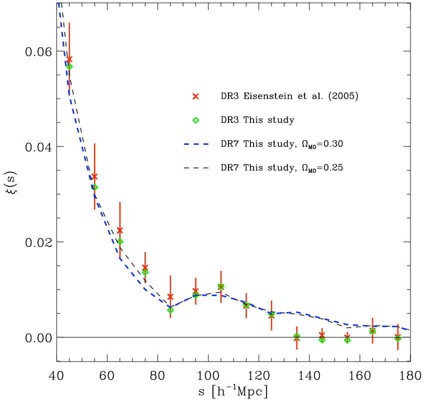
↪ two-point correlation function: $\xi(x) = \langle \delta_1 \delta_2 \rangle$

- ⟳ $\delta_i = \delta(\vec{x}_i)$, $x \equiv |\vec{x}_1 - \vec{x}_2|$
- ⟳ $\langle \delta_1 \delta_2 \rangle$: “averaged” over all pairs separated by x

cosmological large-scale structure (LSS) data

- an estimation of the two-point correlation function of galaxies (example)

[SDSS: 0908.2598]



- however: data usually presented in the form of the power spectrum of the perturbation field (i.e. Fourier transformation of the two-point correlation function)

confrontation with LSS data

- time to *compare*
  - upper limit on $\rho_{m0,i}$ as *predicted* by the model (dilaton gravity, brane, AdS$_5$)

    (constraint on the allowed values of the spatial derivative of the matter energy density on the brane)
  - ↪ with $\rho_{m0,i}$ estimation from the *observational data* on the LSS of the universe
- approximations: (*aim*: just an estimation - allowing to compare orders of magnitude)
  - for the spatial derivative: $\rho_{m,i} \simeq \frac{\rho_m(x_1) - \rho_m(x_2)}{|x_1 - x_2|}$
  - LSS surveys probe the overall baryonic matter distribution
  - spatial distributions of baryonic and dark matter similar

    (typical of most dark matter models)

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confrontation with LSS data

$\hookrightarrow$ and the outcome is . . .

$$\rho_{m0,i}\left[\text{model's prediction - upper limit}\right] \ll \rho_{m0,i}\left[\text{LSS data estimation}\right]$$

(within the entire range of measured scales)

i.e. brane scenario of dilaton gravity with AdS$_5$ bulk

(or any other spacetime with vanishing Weyl tensor)

(and matter content of the universe described by a perfect fluid)

means NO large scale structure as is *observed today*

Outline

conclusions

- dilaton gravity studied in a 5D brane scenario
  - non-minimal coupling $f(\phi)\mathcal{L}_m$ of dilaton to matter content of the universe localized on the brane
  - ↪ derived: effective gravitational equations at the brane
- *can large-scale structure of the universe exist on the brane*? (*inhomogeneous* matter content of the universe)
  - ↪ investigated for AdS_5 bulk & perfect fluid on the brane (matter content of the universe)
  - employed: effective gravitational eqs. at the brane, 4D Bianchi identity (only!)

⇝ spatial derivative of the brane matter energy density *strongly constrained*

- non-minimal dilaton-matter coupling essential (in the Einstein frame!)
- result quantified with current limits from the searches for time variation of the Newton's constant
- and confronted with the observational data on LSS from galaxy surveys

⇝ within the entire range of measured scales

NO large-scale structure as is observed today

dilaton gravity brane scenario ruled out? of course NOT: for exact AdS_5 bulk (or $E_{\mu\nu} = 0$) only!