

(Non-)Universal Anomalies and Discrete Symmetries from the Heterotic String

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PRD **85** [arXiv:1203.5789] and work in progress

- MSSM superpotential contains (potentially) bad terms:

$$W_{\text{bad}} \supset \mu H_u H_d + QLd^c + u^c d^c d^c + LL e^c \\ + QQQ L + u^c u^c d^c e^c + \dots$$

- Forbid/constrain these operators by (discrete) symmetries, (matter parity, proton hexality, \mathbb{Z}_4^R, \dots) [many people here]
- In string constructions, discrete symmetries can arise as remnants of gauge and internal Lorentz symmetries

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- Forbid/constrain these operators by (discrete) symmetries, (matter parity, proton hexality, \mathbb{Z}_4^R, \dots) [many people here]
- In string constructions, discrete symmetries can arise as remnants of gauge and internal Lorentz symmetries
- Discuss anomalies in such models (bottom-up) and the appearance of discrete symmetries in the $E_8 \times E_8$ heterotic string
- Simple example: Consider \mathbb{Z}_3 orbifold, its blowup and GLSM realisation

- ① Green–Schwarz Mechanism and Universality
- ② Heterotic Models
- ③ Remnant Discrete Symmetries
- ④ Conclusion

Anomalies

- determined by chiral spectrum
- given in terms of anomaly polynomial I_6 (via descent equations) [Wess, Zumino '71; Stora '84; Alvarez-Gaumé, Ginsparg '84]
- insensitive to continuous deformations

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Require:

- $U(1)_A^2 - U(1)_Y$ anomaly should vanish
- for \mathbb{Z}_N symmetries, only $G^2 - \mathbb{Z}_N$ meaningful [Ibanez, Ross '91; Banks, Dine '92; Araki et al. '08]

[Green, Schwarz '84]

Cancel transformation of measure with variation of action – requires

a) factorisation of anomaly polynomial, $I_6 = X_4 Y_2$, i.e. $Y_2 = F_2 = dA_1$

b) axion field a with shift gauge transformation

$$\delta a = -Y_0^{(1)} = -\lambda$$

[Green, Schwarz '84]

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- c) axion coupling to X_4

$$S_{\text{GS}} = \int a X_4 + \dots \quad \Rightarrow \quad \delta S_{\text{GS}} = - \int I_4^{(1)}$$

- Needs shift and coupling
- Dualise a to two-form B_2 : exchange $Y_2 \leftrightarrow X_4$, shift \leftrightarrow coupling
- Generalisation: sum of factorised anomalies $I_6 = \sum_a Y^a X^a$ is cancelled by set of axions

Green–Schwarz Axion

$Y_2 = dA_A$ is field strength of the “anomalous $U(1)$ ”, axion transforms with a shift

$$\implies S_{a,\text{kin}} = \int \frac{1}{2} |da + A_A|^2$$

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- Axion kinetic term gives Stueckelberg mass term for $U(1)_A \Rightarrow$ anomalous $U(1)$ s massive, but remain as (perturbative) selection rules
- X_4 contains field strengths of other gauge group factors G_i , weighted with (arbitrary) anomaly coefficients:

$$X_4 = A_{\text{grav-}U(1)} \text{tr} R^2 + \sum_i A_{G_i^2-U(1)_A} \text{tr} F_i^2$$

- **Special case:** If anomaly is cancelled by Kalb–Ramond $b_2 \Rightarrow X_4$ is reduction of 10D X_4^{uni} , and universal axion a_0 couples universally to all gauge groups

Take MSSM with additional $U(1)_X$, charges q_Q, \dots, q_{H_d}
 \rightsquigarrow Anomaly coefficients generically not universal

Impose e.g.

- allowed Yukawa couplings,
- $U(1)_X$ is flavour-blind and commutes with $SU(5)$ (but assume doublet-triplet splitting, i.e. no Higgs triplets),
- may or may not be an R symmetry (i.e. $R = 0$ or $R = 1$)

$$A_{SU(3)^2-U(1)_X} = 3(3q_{10} + q_{\bar{5}}) - 6,$$

$$A_{SU(2)^2-U(1)_X} = 2(3q_{10} + q_{\bar{5}}) - 6,$$

$$A_{U(1)_Y^2-U(1)_X} = 2(3q_{10} + q_{\bar{5}}) - 9.$$

Universality from Underlying GUT?

- Unbroken GUT: Anomaly coefficients universal (duh)
- After GUT breaking: Depends on mechanism!
- 4D: Breaking by VEVs removes vector-like states (under unbroken group)
 \rightsquigarrow no change to anomalies
- VEVs are continuous deformations

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- VEVs are continuous deformations
- Higher-dimensional models: 4D chiral spectrum formed by zero modes of internal Dirac operators
- Number of zero modes depends on discrete choices: boundary conditions, fluxes \rightsquigarrow change in anomalies

Example: Broken $SU(5)$

Anomalies get contributions from matter, gauginos and Higgses:

	Matter	gauginos		Higgses	
	$\bar{5}, 10$	SM	X, Y	3's	2's
$A_5 = \frac{3}{2} (3q_{10} + q_{\bar{5}} - 4R)$		+5R		+ C_H	
$A_3 = \frac{3}{2} (3q_{10} + q_{\bar{5}} - 4R)$		+3R	+2R	+ C_H	
$A_2 = \frac{3}{2} (3q_{10} + q_{\bar{5}} - 4R)$		+2R	+3R		+ C_H
$A_1 = \frac{3}{2} (3q_{10} + q_{\bar{5}} - 4R)$			+5R	+ $\frac{2}{5}C_H$	+ $\frac{3}{5}C_H$

$$(C_H = \frac{1}{2} (q_{H_u} + q_{H_d} - 2R))$$

- For unbroken $SU(5)$, $A_3 = A_2 = A_1 = A_5$

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- $SU(5)$ breaking removes X, Y gauginos \rightsquigarrow non-universality for R symmetries
- doublet-triplet splitting \rightsquigarrow generic non-universality

More Possibilities, e.g. for $G_{SM} \times \mathbb{Z}_N$

Operator				
$(LH_u)^2$				
$H_u H_d$				
LH_u				
$10\bar{5}\bar{5}$				
$101010\bar{5}$				
$101010H_d$				
$LH_u H_d H_u$				

More Possibilities, e.g. for $G_{SM} \times \mathbb{Z}_N$

Operator	SM Yukawas	Weinberg Op.	
$(LH_u)^2$	$4R - 4q_{10} + 2q_{\bar{5}}$	$2R$	
$H_u H_d$	$4R - 3q_{10} - q_{\bar{5}}$	$5R - 5q_{10} + k\frac{N}{2}$	
LH_u	$2R - 2q_{10} + q_{\bar{5}}$	$R + k\frac{N}{2}$	
$10\bar{5}\bar{5}$	$q_{10} + 2q_{\bar{5}}$	$-2R + 5q_{10}$	
$10\ 10\ 10\bar{5}$	$3q_{10} + q_{\bar{5}}$	$-R + 5q_{10} + k\frac{N}{2}$	
$10\ 10\ 10H_d$	$4R + 2q_{10} - q_{\bar{5}}$	$3R + k\frac{N}{2}$	
$LH_u H_d H_u$	$6R - 5q_{10}$	$6R - 5q_{10}$	

More Possibilities, e.g. for $G_{SM} \times \mathbb{Z}_N$

Operator	SM Yukawas	Weinberg Op.	\mathbb{Z}_6^X
$(LH_u)^2$	$4R - 4q_{10} + 2q_{\bar{5}}$	$2R$	0
$H_u H_d$	$4R - 3q_{10} - q_{\bar{5}}$	$5R - 5q_{10} + k\frac{N}{2}$	4
LH_u	$2R - 2q_{10} + q_{\bar{5}}$	$R + k\frac{N}{2}$	3
$10\bar{5}\bar{5}$	$q_{10} + 2q_{\bar{5}}$	$-2R + 5q_{10}$	5
$10\ 10\ 10\bar{5}$	$3q_{10} + q_{\bar{5}}$	$-R + 5q_{10} + k\frac{N}{2}$	2
$10\ 10\ 10H_d$	$4R + 2q_{10} - q_{\bar{5}}$	$3R + k\frac{N}{2}$	3
$LH_u H_d H_u$	$6R - 5q_{10}$	$6R - 5q_{10}$	1

\rightsquigarrow forbid all bad terms e.g. by non- R \mathbb{Z}_6^X with

$$q_{10} = 1, \quad q_{\bar{5}} = 5, \quad q_{H_u} = 4, \quad q_{H_d} = 0$$

\mathbb{Z}_2 matter parity as subgroup

More Possibilities, e.g. for $G_{SM} \times \mathbb{Z}_N$

Operator	SM Yukawas	Weinberg Op.	\mathbb{Z}_6^X	$SO(10)$
$(LH_u)^2$	$4R - 4q_{10} + 2q_{\bar{5}}$	$2R$	0	$2R$
$H_u H_d$	$4R - 3q_{10} - q_{\bar{5}}$	$5R - 5q_{10} + k\frac{N}{2}$	4	0
LH_u	$2R - 2q_{10} + q_{\bar{5}}$	$R + k\frac{N}{2}$	3	R
$10\bar{5}\bar{5}$	$q_{10} + 2q_{\bar{5}}$	$-2R + 5q_{10}$	5	$3R$
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$10\ 10\ 10H_d$	$4R + 2q_{10} - q_{\bar{5}}$	$3R + k\frac{N}{2}$	3	$3R$
$LH_u H_d H_u$	$6R - 5q_{10}$	$6R - 5q_{10}$	1	R

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Requiring $SO(10)$ relations for matter, we need $R = 1$

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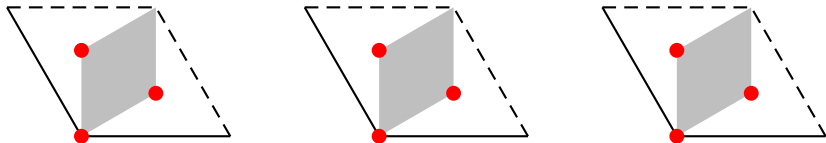
Overview of Heterotic GS Mechanisms

In heterotic models, all axions arise from B_2 in 10D with transformation
Distinguish Orbifolds and smooth Calabi–Yaus with vector bundles:

Orbifolds	Calabi–Yau X with Gauge Bundle
single two-form $b_2 \leftrightarrow$ universal axion a_0	$B_2 = b_2 + \beta_r E_r$, $E_r \in H^2(X) \rightsquigarrow$ additional axions β_r
Universality: a_0 couples to reduction of X_4^{uni}	Couplings $\beta_r X_4^r$ depend on gauge background and curvature – some remnants of universality
(at most) one anomalous $U(1)$ (determined by shift of a_0)	Number of anomalous $U(1)$ s given by rank of bundle

T^6/\mathbb{Z}_3 Orbifold

For illustration, consider simple T^6/\mathbb{Z}_3 orbifold model



$V = \frac{1}{3} (1, 1, -2, 0^5) (0^8)$, no Wilson lines
 \Rightarrow standard embedding, 27 equivalent fixed points

$$\begin{array}{ll} \text{Gauge group} & E_6 \times SU(3) [\times E_8] \\ \text{spectrum} & 3 (\mathbf{27}, \bar{\mathbf{3}}) + 27 [(\mathbf{27}, \mathbf{1}) + 3 (\mathbf{1}, \mathbf{3})] \end{array}$$

In particular, no anomalous $U(1)$, hence universal axion does not shift under gauge transformations, and no FI term has to be cancelled

[Groot Nibbelink et al. 07-09]

Idea: VEVs for twisted states (blow-up modes) corresponds to smoothing out singularities – connection to smooth Calabi–Yau with bundles

In particular: VEVs for twisted non-oscillator states **(27, 1)**

↔ line bundles (i.e. Abelian fluxes)

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In particular: VEVs for twisted non-oscillator states **(27, 1)**

\leftrightarrow line bundles (i.e. Abelian fluxes)

Procedure:

- 1 Replace fixed points by exceptional divisors E_r (\mathbb{P}^2 s)
- 2 Turn on Abelian gauge flux along the exceptional divisors,

$$\mathcal{F} = V_r^I H_I E_r, \quad H_I: \text{Cartan generators of } E_8$$

(automatically (1, 1)-form)

Note: Line bundles don't reduce rank, axion shifts do:

$$B_2 = b_2 - \beta_r E_r, \quad \delta B_2 = -\text{tr } \lambda \mathcal{F} \quad \Rightarrow \quad \delta \beta_r = \text{tr } \lambda V_r$$

Bundle Vectors

Bundle has to satisfy DUY eqns. (analogue of D -term equation)

$$0 = \frac{1}{2} \int_X J \wedge J \wedge \mathcal{F} = \sum_r \text{vol}(E_r) V_r, \quad \text{with all } \text{vol}(E_r) > 0$$

Take three bundle vectors from p_{sh} of twisted **27** which sum to zero, distribute among exceptional divisors

\rightsquigarrow flux quantisation and Bianchi Identity fulfilled automatically
DUY eqns. easily satisfied for arbitrary large volumes

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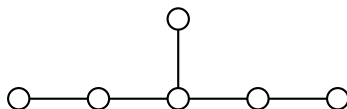
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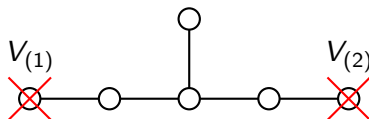
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\Rightarrow gauge group $SO(8) \times U(1)_A \times U(1)_B \times SU(3)$

Massless spectrum depends on distribution (given by (k, p, q) with $k + p + q = 27$)

Blowup Anomalies

Blowup: two $U(1)$'s, different spectrum

\Rightarrow anomaly polynomial $I_6 = \int_X I_{12}$ with backgrounds inserted, or from triangle diagrams

$$\begin{aligned} \Rightarrow I_6 \sim & F_A^3 \cdot \left(\frac{k-6}{12} \right) + F_A F_B^2 \cdot \left(\frac{k-18}{4} \right) \\ & + F_A \left[\text{tr } F_{SU(3)}^2 + \text{tr } F_{SO(8)}^2 + \frac{7}{48} \text{tr } R^2 \right] \cdot \left(\frac{k-9}{2} \right) \\ & + F_B \left[\frac{1}{8} F_B^2 + \frac{1}{48} F_A^2 + \text{tr } F_{SU(3)}^2 + \text{tr } F_{SO(8)}^2 + \frac{7}{48} \text{tr } R^2 \right] \cdot \left(\frac{p-q}{2} \right) \end{aligned}$$

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- For $p = q$, $U(1)_B$ is anomalous, while $U(1)_A$ is always anomalous
- Remnant universality: Coefficients of non-Abelian groups from one E_8 are equal, and proportional to gravitational anomaly (only true if one E_8 unbroken)

Axion Shifts and massive $U(1)_B$

Axions β_r shift under $U(1)_{A,B}$ – universal axion does not!
 $\Rightarrow U(1)_A$ and $U(1)_B$ always massive, even if one of them is omalous:

$$\int_X H_3 \wedge *H_3 = A_\mu^I A^{\mu J} M_{IJ}^2 + \dots, \quad M_{IJ}^2 = V_r^I V_s^J \cdot \int_X E_r \wedge *_6 E_s$$

Mass matrix is positive definite and always rank-two (and depends on the Kähler parameters)

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\rightarrow Stueckelberg mass possible without anomaly (but not *vice versa*)

Note: Still a coupling of the universal axion to X_4 , as required by supersymmetry

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Remnant non- R symmetries: discrete subgroup of $U(1)_A \times U(1)_B$ which leaves VEVs invariant

Blow-up modes:

$$\mathbf{1}_{4,0}, \quad \mathbf{1}_{-2,-2}, \quad \mathbf{1}_{-2,2}$$

\Rightarrow discrete remnant $\mathbb{Z}_4 \times \mathbb{Z}_4$, generated by

$$T_{\pm} : \phi_{(q_A, q_B)} \longrightarrow \exp\left\{ \frac{2\pi i}{4} (q_A \pm q_B) \right\} \phi_{(q_A, q_B)}$$

However: Charges of all massless fields are even under both \mathbb{Z}_4 s
 \rightsquigarrow only $\mathbb{Z}_2 \times \mathbb{Z}_2$ realised on massless spectrum

Both \mathbb{Z}_2 factors are omalous

- R symmetries do not commute with SUSY \leftrightarrow θ transforms, and different components of SUSY multiplets have different charges
- Only defined up to mixing with non- R symmetries
- For $\mathcal{N} = 1$ SUSY, only one $\theta \rightsquigarrow$ only one $U(1)$ or \mathbb{Z}_N R symmetry – otherwise can redefine generators such that only one acts on θ
- Usual convention: θ has charge 1 \Rightarrow Superpotential W has charge 2 (\curvearrowright \mathbb{Z}_2 doesn't really count as an R symmetry)

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- In compactifications, internal Lorentz transformations treat spinors and scalars differently \rightsquigarrow can lead to R symmetries in 4D
- Orbifolds are special points in moduli space, so expect more symmetries – in particular, for general smooth spaces, expect no R symmetry in general

R Symmetries from Orbifolds

R transformations from sublattice rotations act as

$$\mathcal{R} : \Phi \longrightarrow e^{2\pi i \nu R} \Phi$$

where (for Z_3 orbifolds) $\nu = \left(\frac{1}{3}, 0, 0 \right)$, $R = q_{\text{sh}} - \Delta N$

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Symmetry conventions somewhat tricky:

- For bosons, both ν and R quantised in units of $\frac{1}{3}$, so \mathbb{Z}_9 symmetry (i.e. $\mathcal{R}^9 = \mathbb{1}$)
- For fermions, $R^f = R^b - \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, so θ has charge $\frac{1}{6}$ (i.e. \mathbb{Z}_6 “R symmetry”)
- Hence: \mathbb{Z}_{18} symmetry with charges for

$$(\text{bosons, fermions, } \theta) = \frac{1}{18} (2n, 2n - 3, 3)$$

- Can redefine charges such that θ has charge 1 and superpotential has charge 2 mod 6, but then fields have non-integer charges

Our blow-up modes have

$$R = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right)$$

Seek unbroken combinations of the three sublattice rotations and $U(1)_{A,B}$:

$$\begin{aligned} \mathbf{1}_{4,0} &\longrightarrow \mathcal{R}_1^p \mathcal{R}_2^q \mathcal{R}_3^r T_A T_B \mathbf{1}_{4,0} = \mathbf{1}_{4,0}, \\ \mathbf{1}_{-2,-2} &\longrightarrow \mathcal{R}_1^p \mathcal{R}_2^q \mathcal{R}_3^r T_A T_B \mathbf{1}_{-2,-2} = \mathbf{1}_{-2,-2}, \\ \mathbf{1}_{-2,2} &\longrightarrow \mathcal{R}_1^p \mathcal{R}_2^q \mathcal{R}_3^r T_A T_B \mathbf{1}_{-2,2} = \mathbf{1}_{-2,2} \end{aligned}$$

However, this implies $p + q + r = 3 \Rightarrow$ only a \mathbb{Z}_2 R symmetry survives in blow-up

Model: GLSM Description

[Witten '93; Groot Nibbelink '10; Blaszczyk et al. '11]

Algebraically, describe the orbifold by $(\mathbb{P}^2[3] \text{ is a } T^2)$

$$\frac{\mathbb{P}^2[3] \times \mathbb{P}^2[3] \times \mathbb{P}^2[3]}{\mathbb{Z}_3}$$

Blowup (crepant resolution) in $(0, 2)$ GLSM description:

- 2D supersymmetric field theory with $U(1)$ gauge symmetries, fields \sim coordinates
- Geometry given by F and D term equations, GLSM FI terms become CY Kähler parameters
- In IR, flows to worldsheet description
- To resolve singularities, introduce extra coordinates (exceptional divisors) and $U(1)$ s
- Gauge bundle given by “chiral-Fermi” superfields Λ_I with charges determined by the bundle vectors

R Symmetries in the GLSM

Set of F and D terms fixes geometry.

\exists discrete transformations of the fields which leave F and D terms invariant

\rightsquigarrow R symmetries if holomorphic three-form Ω transforms [Witten 85]

$$\Omega \sim \eta^T \Gamma_{ijk} \eta dz^i dz^j dz^k \quad \Rightarrow \quad Q_R(\Omega) = Q_R(W) = 2$$

Different types of R symmetries:

- Phases $z \rightarrow e^{2\pi i/3} z$: always possible (but see next slide)
 $\rightsquigarrow \mathbb{Z}_6$ R symmetries
- Permutations of fields: Only possible for special values of Kähler parameters – corresponds to groupwise exchange of exceptional divisors

R Symmetry breaking in GLSM

\mathbb{P}^2 coordinates $z_{i\alpha}$ only appear as $z_{i\alpha}^3$ or $|z_{i\alpha}|^2$

\Rightarrow unbreakable \mathbb{Z}_3 rotations?

(Presumably) broken by marginal deformations of Kähler potential terms in presence of gauge bundles (correspond to massless charged matter, ϕ_{4d} 4D modes)

$$\int d^2\theta^+ \phi_{4d}(x^\mu) N(z) \underbrace{\Lambda \bar{\Lambda}}_{\text{gauge bundle fields}}$$

Fits with orbifold: Bundle corresponds to blowup

Presence of deformations controlled by Kähler (FI) parameters

\Rightarrow Generically, no R symmetry in blow-up (all FI terms large), but enhanced at certain loci of parameter space

- ① Green–Schwarz Mechanism and Universality
- ② Heterotic Models
- ③ Remnant Discrete Symmetries
- ④ Conclusion

Conclusions

- Discussed (discrete) symmetries in 4D low-energy theories from the heterotic string
- Anomalies are generically not universal: Not required for anomaly cancellation, not generic from unification
- orbifold anomalous $U(1)$ is the exception because it is cancelled by the universal axion
- For blowups of heterotic orbifolds, many axions possible

Conclusions

- Discussed (discrete) symmetries in 4D low-energy theories from the heterotic string
- Anomalies are generically not universal: Not required for anomaly cancellation, not generic from unification
- orbifold anomalous $U(1)$ is the exception because it is cancelled by the universal axion
- For blowups of heterotic orbifolds, many axions possible
- Line bundles do reduce the rank via the axion shift – also anomalous $U(1)$ s can become massive
- Blow-ups can leave gauged discrete subgroups unbroken – important for phenomenology
- On orbifold, R symmetries exist but are broken by the blow-up

- Found nice agreement between orbifold and blow-up picture, up to some subtleties
- “Geometry part” of GLSM generically has many “unbreakable” R -like symmetries – seem to be broken by the gauge bundle, but better understanding of their breaking required
- Linked to determination of charged massless spectrum
- Non-generic type of R symmetries: Exchange symmetries, appearing for certain loci in Kähler moduli space, e.g. exchange of exceptional divisors if their volumes are equal
- Study these symmetries for more realistic models, including Wilson lines etc.