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# Neutrino mass models and sizable $\theta_{13}$

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# Prologue

- ▶ remarkable results from neutrino oscillation experiments
- ▶ tri-bimaximal lepton mixing (until recently)
- ▶ family symmetries like  $A_4$  and  $S_4$
- ▶ origin of the Klein symmetry in the neutrino sector
- ▶ strategies of implementing a sizable reactor angle  $\theta_{13}$  (post T2K)
  - tri-bimaximal mixing plus corrections (from extra ingredient)
  - new family symmetries → Feruglio's talk
  - non-standard vacuum configurations → Spinrath's talk

# A brief history of neutrino mixing

- ▶ atmospheric neutrinos
  - $\nu_\mu / \bar{\nu}_\mu$  disappear – Super-Kamiokande (1998)
- ▶ accelerator neutrinos
  - $\nu_\mu$  disappear – K2K (2002), MINOS (2006)
  - $\nu_\mu$  converted to  $\nu_\tau$  – OPERA (2010)
  - $\nu_\mu$  converted to  $\nu_e$  – T2K, MINOS (2011)
- ▶ solar neutrinos
  - $\nu_e$  disappear – Chlorine (1998), Gallium (1999 - 2009), Super-Kamiokande (2002), Borexino (2008)
  - $\nu_e$  converted to  $(\nu_\mu + \nu_\tau)$  – SNO (2002)
- ▶ reactor neutrinos
  - $\bar{\nu}_e$  disappear – Double Chooz (2011), Daya Bay, RENO (2012)
  - $\bar{\nu}_e$  disappear – KamLAND (2002)

# 2011/2012 story of non-zero $\theta_{13}$

T2K [arXiv:1106.2822]

- $\theta_{13} \neq 0$  disfavored at  $\sim 2.5\sigma$

MINOS [arXiv:1108.0015]

- $\theta_{13} \neq 0$  disfavored at  $\sim 1.6\sigma$

Double Chooz [arXiv:1112.6353]

- $\theta_{13} \neq 0$  disfavored at  $\sim 2\sigma$

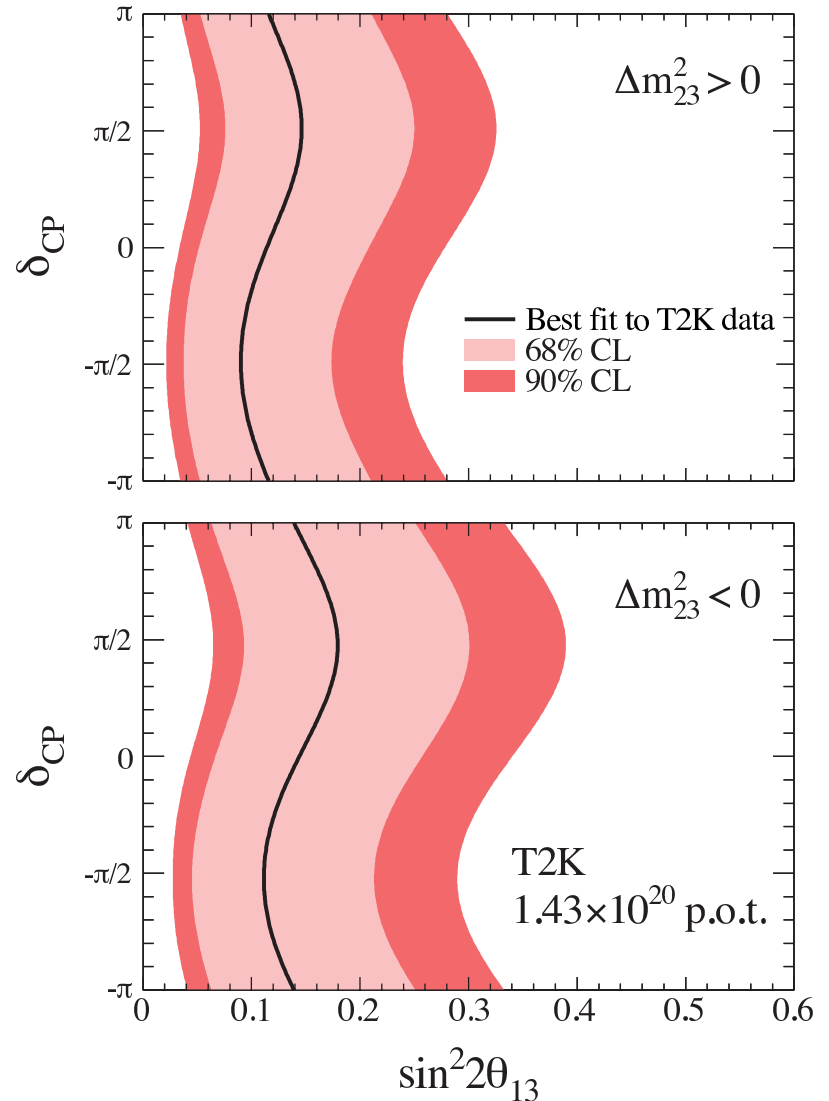
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Daya Bay [arXiv:1203.1669]

- $\theta_{13} \neq 0$  disfavored at  $\sim 5.2\sigma$
- $7.9^\circ \lesssim \theta_{13} \lesssim 9.6^\circ$

RENO [arXiv:1204.0626]

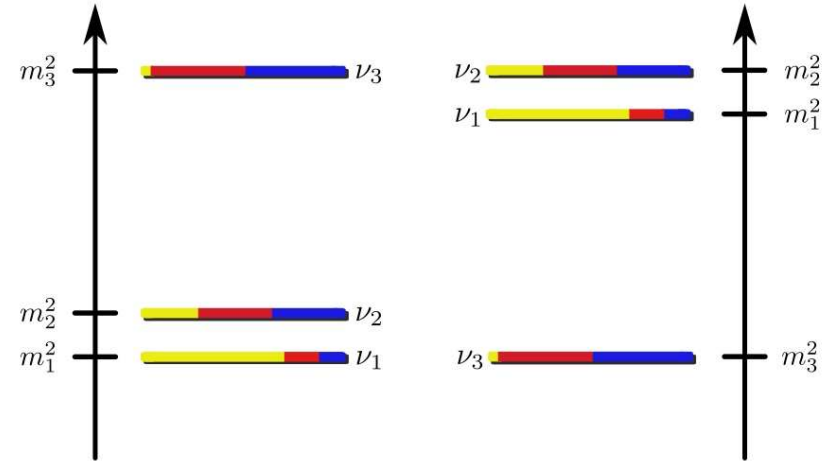
- $\theta_{13} \neq 0$  disfavored at  $\sim 4.9\sigma$
- $8.7^\circ \lesssim \theta_{13} \lesssim 10.8^\circ$



# Three neutrino flavor mixing

(in diagonal charged lepton basis)

$$\begin{array}{c} \text{flavor} \\ \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \end{array} = \begin{array}{c} \text{PMNS mixing} \\ \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \end{array} \begin{array}{c} \text{mass} \\ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \end{array}$$



$$U_{\text{PMNS}} = \begin{array}{c} \text{atmospheric} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \end{array} \begin{array}{c} \text{reactor + Dirac} \\ \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \end{array} \begin{array}{c} \text{solar} \\ \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \begin{array}{c} \text{Majorana} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{\frac{\alpha_3}{2}} \end{pmatrix} \end{array}$$

# Tri-bimaximal lepton mixing vs. global neutrino fits

$$U_{\text{PMNS}} \approx U_{\text{TB}} \equiv \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$



$$\Rightarrow \left\{ \begin{array}{l} \text{PMNS-angles} \quad \text{tri-bimax.} \quad 1\sigma \text{ exp.} \quad 1\sigma \text{ exp.} \\ \hline \sin^2 \theta_{12} : \quad \frac{1}{3} \quad 0.303 - 0.335 \quad 0.291 - 0.325 \\ \sin^2 \theta_{23} : \quad \frac{1}{2} \quad 0.44 - 0.58 \quad 0.37 - 0.44 \\ \sin^2 \theta_{13} : \quad 0 \quad 0.022 - 0.030 \quad 0.021 - 0.028 \end{array} \right.$$

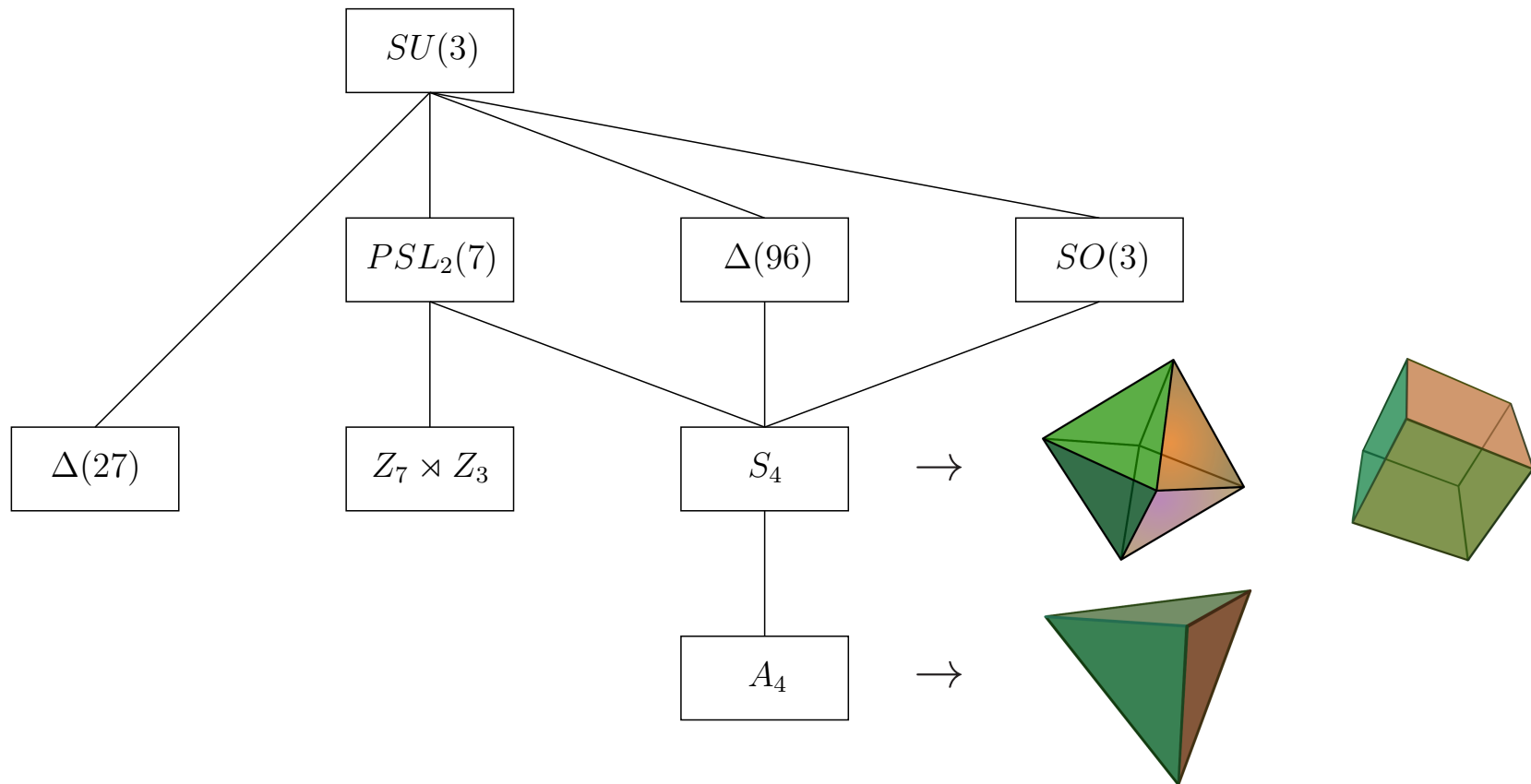
Tortola et al. (2012)

Fogli et al. (2012)

- TB mixing fits relatively well  $\rightarrow$  family symmetry, e.g.  $A_4, S_4$
- how to accommodate sizable  $\theta_{13} \sim 8^\circ - 10^\circ$ ?

# Non-Abelian family symmetries

- unify three families in multiplets of family symmetry
- group should have three-dimensional representations



# Symmetries of the mass matrices

charged leptons  $M_\ell = \text{diag}(m_e, m_\mu, m_\tau)$

symmetric under diagonal phase transformation  $h$

$$\boxed{M_\ell = h^T M_\ell h^*} \quad \text{e.g. } h = \text{diag}\left(1, e^{\frac{4\pi i}{3}}, e^{\frac{2\pi i}{3}}\right)$$

neutrinos

$$M_\nu = U_{\text{PMNS}} \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U_{\text{PMNS}}^T$$

symmetry of  $M_\nu$  depends on  $U_{\text{PMNS}}$

$$\boxed{M_\nu = k^T M_\nu k} \quad k = U_{\text{PMNS}}^* \text{diag}(\pm 1, \pm 1, \pm 1) U_{\text{PMNS}}^T$$

require  $\det k = 1$

four different  $k \rightarrow$  generate  $Z_2 \times Z_2$  symmetry group

Klein symmetry  $\mathcal{K} = \{1, k_1, k_2, k_3\}$

for  $U_{\text{PMNS}} = U_{\text{TB}}$ :

$$k_1 = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad k_2 = - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad k_3 = k_1 k_2$$



# Origin of the Klein symmetry

## ► direct models

- Klein symmetry  $\mathcal{K} \subset$  family symmetry  $\mathcal{G}$
- flavon fields  $\phi$  break  $\mathcal{G}$  down to  $\mathcal{K}$  in neutrino sector
- for TB mixing  $(k_1, k_2, h)$  generate  $S_4$

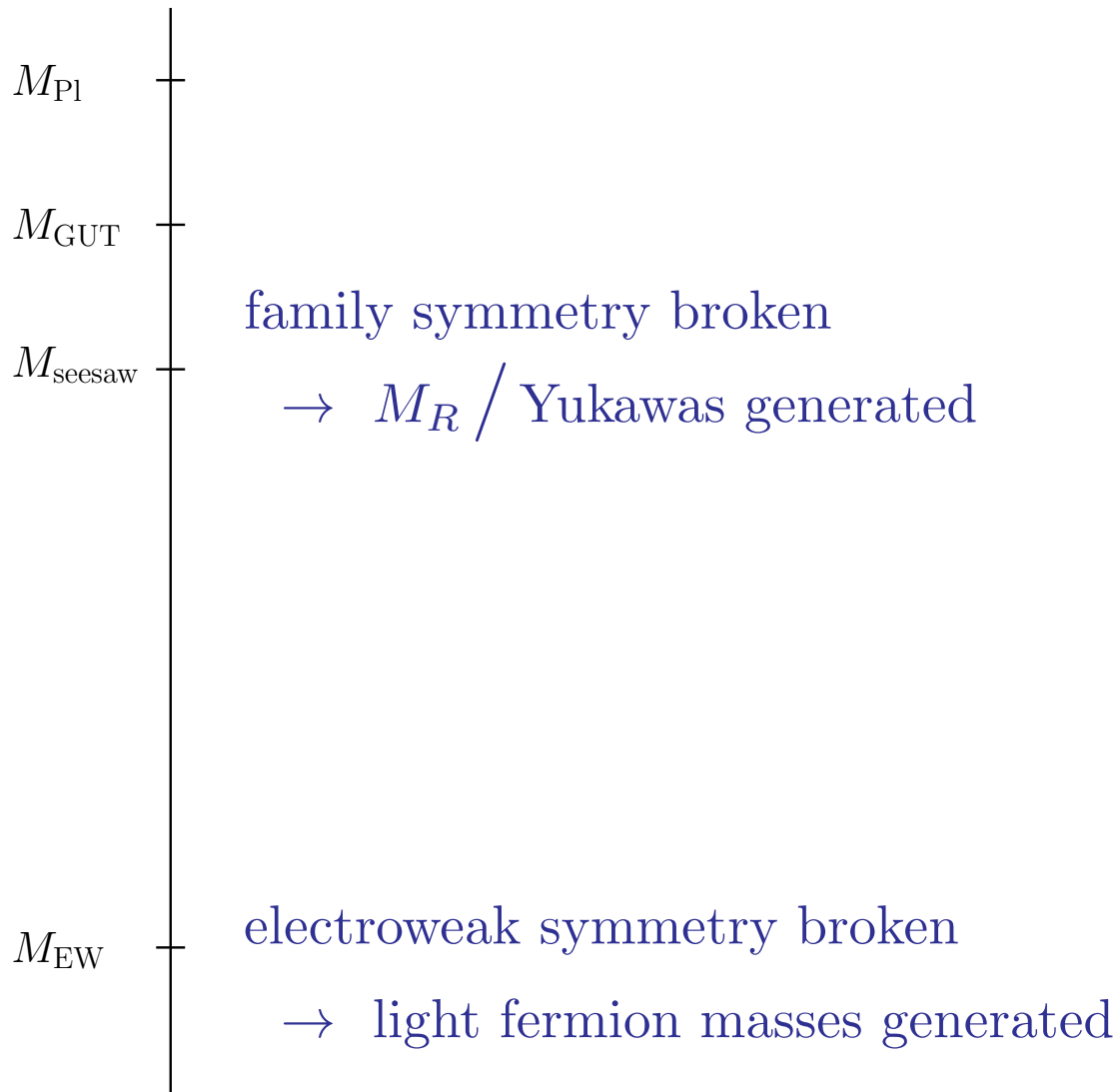
## ► indirect models

- Klein symmetry  $\mathcal{K}$  not necessarily  $\subset$  family symmetry  $\mathcal{G}$
- $\mathcal{G}$  responsible for generating particular flavon VEV configurations
- for TB mixing – from e.g.  $\Delta(27), Z_7 \rtimes Z_3$

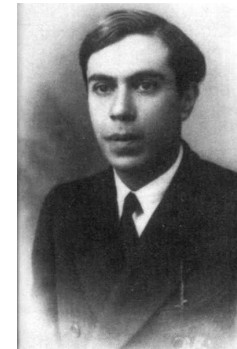
$$\langle \phi_1 \rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \quad \langle \phi_2 \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \langle \phi_3 \rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \mathcal{L}_\nu \sim \nu (\phi_1 \phi_1^T + \phi_2 \phi_2^T + \phi_3 \phi_3^T) \nu H H$$

# Typical model setup



ingredients:



Majorana  $\nu_L$



seesaw

# Implementing sizable $\theta_{13}$

## direct models

TB plus corrections

→ also Merlo's talk

other family symmetries

with non-standard  $\mathcal{K}$

→ Feruglio's talk

## indirect models

TB plus corrections

non-standard flavon

VEV configurations

→ Spinrath's talk

TB plus non-diagonal charged leptons

# Charged lepton corrections to TB mixing

- charged lepton mass matrix might not be diagonal (GUTs)
- $U_{\text{PMNS}} = V_{\ell_L} V_{\nu_L}^\dagger$  and  $V_{\nu_L}^\dagger = U_{\text{TB}}$

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & \hat{s}_{23} \\ 0 & -\hat{s}_{23}^* & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \hat{s}_{13} \\ 0 & 1 & 0 \\ -\hat{s}_{13}^* & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & \hat{s}_{12} & 0 \\ -\hat{s}_{12}^* & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{12} e^{i\delta_{12}} \approx \frac{1}{\sqrt{3}} \left( e^{i\delta_{12}^\nu} - \theta_{12}^\ell e^{i\delta_{12}^\ell} + \theta_{13}^\ell e^{i(\delta_{13}^\ell - \delta_{23}^\nu)} \right)$$

$$s_{23} e^{i\delta_{23}} \approx \frac{1}{\sqrt{2}} \left( e^{i\delta_{23}^\nu} - \theta_{23}^\ell e^{i\delta_{23}^\ell} \right)$$

$$s_{13} e^{i\delta_{13}} \approx \frac{1}{\sqrt{2}} \left( -\theta_{12}^\ell e^{i(\delta_{12}^\ell + \delta_{23}^\nu)} - \theta_{13}^\ell e^{i\delta_{13}^\ell} \right)$$

$$c_{ij} = \cos \theta_{ij}$$

$$\hat{s}_{ij} = \sin \theta_{ij} e^{-i\delta_{ij}}$$

- $\theta_{12}^\ell \sim \theta_C \sim 0.22 \rightarrow \theta_{13} \sim 9^\circ$
- not (easily) compatible with Georgi-Jarlskog relations  $\rightarrow$  Ross' talk

TB plus new TB breaking flavon

# An $S_4$ model of leptons

matter	$L$	$\tau^c$	$\mu^c$	$e^c$	$N^c$	$H_u$	$H_d$
$S_4$	<b>3</b>	<b>1'</b>	<b>1</b>	<b>1</b>	<b>3</b>	<b>1</b>	<b>1</b>
$Z_3^\nu$	1	2	2	2	2	0	0
$Z_3^\ell$	0	2	1	0	0	0	0

King, Luhn (2011)

$$\langle \varphi_\ell \rangle = \begin{pmatrix} 0 \\ v_\ell \\ 0 \end{pmatrix} \quad \langle \eta_\mu \rangle = \begin{pmatrix} 0 \\ w_\mu \end{pmatrix}$$

$$\langle \eta_e \rangle = \begin{pmatrix} w_e \\ 0 \end{pmatrix}$$

flavons	$\varphi_\ell$	$\eta_\mu$	$\eta_e$	$\varphi_\nu$	$\eta_\nu$	$\xi_\nu$	$\zeta_\nu$
$S_4$	<b>3'</b>	<b>2</b>	<b>2</b>	<b>3'</b>	<b>2</b>	<b>1</b>	<b>1'</b>
$Z_3^\nu$	0	0	0	2	2	2	0
$Z_3^\ell$	1	1	2	0	0	0	0

$$\langle \varphi_\nu \rangle = v_\nu \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \langle \eta_\nu \rangle = w_\nu \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \langle \xi_\nu \rangle = u_\nu \quad \langle \zeta_\nu \rangle = z_\nu$$

# Charged lepton sector

$$W_\ell \sim \left[ \frac{1}{M} (L\varphi_\ell)_{1'} \tau^c + \frac{1}{M^2} (L\varphi_\ell)_2 \eta_\mu \mu^c + \frac{1}{M^2} (L\varphi_\ell)_2 \eta_e e^c \right] H_d$$

- $Z_3^\ell$  controls pairing of flavons with right-handed charged fermions
- different  $S_4$  contractions of  $(L\varphi_\ell)$  pick out different  $L_i$  components

$$(L\varphi_\ell)_{1'} = L_1\varphi_{\ell 1} + L_2\varphi_{\ell 3} + L_3\varphi_{\ell 2} \rightarrow L_3$$

$$(L\varphi_\ell)_2 = \begin{pmatrix} L_1\varphi_{\ell 3} + L_2\varphi_{\ell 2} + L_3\varphi_{\ell 1} \\ L_1\varphi_{\ell 2} + L_2\varphi_{\ell 1} + L_3\varphi_{\ell 3} \end{pmatrix} \rightarrow \begin{pmatrix} L_2 \\ L_1 \end{pmatrix}$$

- mass matrix **diagonal by construction**
- $m_\tau$  heavier than  $m_\mu$  and  $m_e$
- hierarchy between  $m_\mu$  and  $m_e$  due to hierarchy of VEVs  $w_\mu$  and  $w_e$
- just a toy model of charged lepton sector (with GUTs off-diagonals)



# Neutrino sector

$$W_\nu \sim LN^c H_u + (\varphi_\nu + \eta_\nu + \xi_\nu) N^c N^c + \frac{1}{M} \zeta_\nu \eta_\nu N^c N^c$$

- trivial Dirac neutrino Yukawa ✓
- neutrino mixing governed by heavy right-handed neutrinos
- $S_4$  multiplication rule ( $N^c \sim \mathbf{3}$ )

$$\mathbf{3} \otimes \mathbf{3} = (\mathbf{3}' + \mathbf{2} + \mathbf{1})_s$$

- three TB conserving flavons  $\varphi_\nu$   $\eta_\nu$   $\xi_\nu$
- $\zeta_\nu$  flavon is neutral except for  $S_4$  ( $\zeta_\nu \sim \mathbf{1}'$ )

$$\mathbf{1}' \otimes (\mathbf{3} \otimes \mathbf{3}) = (\mathbf{3} + \mathbf{2} + \mathbf{1}')_s$$

- only one extra term involving  $\zeta_\nu$
- this **breaks TB** structure (at higher order) ...

# Breaking of the TB Klein symmetry $\mathcal{K}$

Dirac term  $LN^c H_u$  respects  $\mathcal{K} \subset S_4$

Majorana terms  $(\varphi_\nu + \eta_\nu + \xi_\nu + \frac{1}{M}\zeta_\nu\eta_\nu) N^c N^c$  respect  $k_1$  but **break  $k_2$**

$S_4$ irrep	$k_1$	$k_2$	alignment
$\mathbf{3}'$	$\frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\langle \varphi_\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\langle \eta_\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\mathbf{1}$	1	1	$\langle \xi_\nu \rangle \propto 1$
$\mathbf{1}'$	1	<b>-1</b>	$\langle \zeta_\nu \rangle \propto 1$

# Resulting mixing

$$M_R = \frac{M_1+M_3}{6} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \frac{2M_2+M_3-M_1}{6} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \frac{M_1+M_2-M_3}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$+ \Delta \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad \leftarrow \text{small TB breaking term}$$

$$\Rightarrow U_{\text{PMNS}} \approx \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \end{pmatrix}$$

$$\text{Re } \alpha \approx -\sqrt{3} \cdot \left[ \text{Re} \left( \frac{\Delta}{M_1 - M_3} \right) + \text{Im} \left( \frac{\Delta}{M_1 - M_3} \right) \frac{\text{Im} \left( \frac{M_1+M_3}{M_1-M_3} \right)}{\text{Re} \left( \frac{M_1+M_3}{M_1-M_3} \right)} \right]$$

$$\text{Im } \alpha \approx \sqrt{3} \cdot \frac{\text{Im} \left( \frac{\Delta}{M_1 - M_3} \right)}{\text{Re} \left( \frac{M_1+M_3}{M_1-M_3} \right)}$$

# Trimaximal neutrino mixing

- second column of  $U_{\text{PMNS}} \propto (1, 1, 1)^T$
- one could have guessed this special structure
  - (i)  $(1, 1, 1)^T$  is an eigenvector of  $M_R$
  - (ii)  $k_1$  generator of TB Klein symmetry  $\mathcal{K}$  unbroken
- such a TB breaking affects  $\theta_{13}$  and  $\theta_{23}$  – but not  $\theta_{12}$
- get correlations between deviation parameters  $r, a, s$

$$\sin \theta_{13} = \frac{1}{\sqrt{2}} r \quad \sin \theta_{23} = \frac{1}{\sqrt{2}} (1 + a) \quad \sin \theta_{12} = \frac{1}{\sqrt{3}} (1 + s)$$

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$$r \cos \delta \approx -\frac{2}{\sqrt{3}} \operatorname{Re} \alpha \quad a \approx \frac{1}{\sqrt{3}} \operatorname{Re} \alpha \quad \delta \approx \pi + \arg \alpha$$

→ testable sum rules

$a \approx -\frac{1}{2} r \cos \delta$	$s \approx 0$
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Revisiting a GUT model with TB neutrino mixing

# An $S_4 \times SU(5)$ model

matter	$T_3$	$T$	$F$	$N^c$	$H_5$	$H_{\bar{5}}$	$H_{\overline{45}}$
$SU(5)$	<b>10</b>	<b>10</b>	$\bar{\mathbf{5}}$	<b>1</b>	<b>5</b>	$\bar{\mathbf{5}}$	$\overline{\mathbf{45}}$
$S_4$	<b>1</b>	<b>2</b>	<b>3</b>	<b>3</b>	<b>1</b>	<b>1</b>	<b>1</b>
$U(1)$	0	5	4	-4	0	0	1

Hagedorn, King, Luhn (2012)

$$\langle \Phi_2^u \rangle \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\langle \tilde{\Phi}_2^u \rangle \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

flavons	$\Phi_2^u$	$\tilde{\Phi}_2^u$	$\Phi_3^d$	$\tilde{\Phi}_3^d$	$\Phi_2^d$	$\Phi_{3'}^\nu$	$\Phi_2^\nu$	$\Phi_1^\nu$	$\eta$
$S_4$	<b>2</b>	<b>2</b>	<b>3</b>	<b>3</b>	<b>2</b>	<b>3'</b>	<b>2</b>	<b>1</b>	<b>1</b>
$U(1)$	-10	0	-4	-11	1	8	8	8	7

$$\langle \Phi_3^d \rangle \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\langle \tilde{\Phi}_3^d \rangle \propto \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$\langle \Phi_2^d \rangle \propto \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\langle \Phi_{3'}^\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\langle \Phi_2^\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

# Charged fermions

up sector  $\mathbf{10} \mathbf{10} \mathbf{5}_H$

$$M_u \sim \begin{pmatrix} \lambda^8 & 0 & 0 \\ 0 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_u$$

- diagonal up quark matrix
- renormalizable top Yukawa

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down sector  $\bar{\mathbf{5}} \mathbf{10} \bar{\mathbf{5}}_H$  &  $\bar{\mathbf{5}} \mathbf{10} \bar{\mathbf{45}}_H$

$$M_d \sim \begin{pmatrix} 0 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ 0 & 0 & \lambda^2 \end{pmatrix} v_d$$

$$M_\ell \sim \begin{pmatrix} 0 & \lambda^5 & 0 \\ \lambda^5 & \mathbf{3} \lambda^4 & 0 \\ \lambda^5 & \mathbf{3} \lambda^4 & \lambda^2 \end{pmatrix} v_d$$

- Georgi-Jarlskog relations  $m_b \sim m_\tau$ ,  $m_s \sim \frac{1}{3} m_\mu$ ,  $m_d \sim \mathbf{3} m_e$
- Gatto-Sartori-Tonin relation  $M_{12}^d = M_{21}^d \rightarrow \theta_{12}^d \sim \sqrt{\frac{m_d}{m_s}} \sim \lambda$
- charged lepton mixing  $\theta_{12}^\ell \sim \frac{1}{3} \lambda \sim 4^\circ$

# Neutrino sector and PMNS mixing

$$W_\nu \sim FN^c H_5 + (\Phi_{3'}^\nu + \Phi_2^\nu + \Phi_1^\nu) N^c N^c + \frac{1}{M} \eta \Phi_2^d N^c N^c$$

- new flavon  $\eta$  does not break TB symmetry
- effective doublet  $\langle \eta \Phi_2^d \rangle \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  respects  $k_1$  but breaks  $k_2$
- neutrino sector has trimaximal structure

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- charged lepton corrections modify previous sum rules
  - additional phases enter  $\rightarrow$  sum rule bounds

$$|a| \lesssim \frac{1}{2} \left( r + \frac{\theta_C}{3} \right) |\cos \delta| \qquad |s| \lesssim \frac{\theta_C}{3}$$



# Conclusion

- ▶ experimental measurement of  $\theta_{13} \sim 8^\circ - 10^\circ$
- ▶ review role of family symmetries
- ▶ implementing non-zero  $\theta_{13}$  via corrections to TB mixing
  - from non-diagonal charged leptons
  - $S_4$  model of leptons with TB breaking flavon
  - $S_4 \times SU(5)$  model
- ▶ sum rules/sum rule bounds for mixing angles

Thank you