The Curvaton Unification: Non-linear formalism for the generalized curvaton mechanism with/without inevitable modulation

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pre-Summary

For the curvaton mechanism

We consider the evolution of the curvature perturbations caused by the Slow-rolling curvaton.

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For the modulation

"Modulation" is usually discussed for the "Moduli perturbation", but any kind perturbation can cause creation of δN when the scaling of the density changes. In this sense, we are considering "modulation without moduli".

This "peculiar modulation" is described using the non-linear formalism and unified with the curvaton mechanism.

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"Usual curvaton" uses TWO components;

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Multi-component Universe

Just after inflation, there will be many components in the Universe (Multi-component Universe), whose components scale like Matter, Radiation, slow-rolling, and so forth

So why not generalize the curvaton mechanism so that we can use it when **Slow-rolling** is significant.

For the slow-rolling component, we already have

Papers

1. **Inflating** curvaton, [arXiv:1110.2951]
   K. Dimopoulos, K. Kohri, D. H. Lyth and T. Matsuda.
2. **Hybrid curvaton**, [arXiv:1201.6037]
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2. Non-Gaussianity is not trivial.(We will show this calculation.)
3. Unconventional (Moduli-less) Modulation. (Will be discussed later)
4. Secondary inflation is not always needed for the mechanism. Critical !

Obviously, the story must be discriminated from the double inflation.

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2) Must be generalized for any w_i;

$$\dot{\rho}_i = -3H(1 + w_i)\rho_i,$$

3) The transition (end of slow-roll, decay, etc..)

does not always coincide with uniform density.

$\rightarrow$ Additional δN creation at the boundary is possible.

This is the "modulation" we are thinking about in this talk.

Usually, the "modulation" is caused by an additional "moduli", but in this talk the source is the usual isocurvature perturbation of the curvaton.

OK, let us see the details using the Non-linear formulation =>

Non-linear formulation

NL-formulation is useful for the higher order calculation.

A General proof of the conservation of the curvature...

D. H. Lyth, K. A. Malik and M. Sasaki,
JCAP **0505**, 004 (2005) [astro-ph/0411220]

For each component “i”, define the **component perturbation**.

$$\begin{aligned}\zeta_i &= \delta N + \int_{\bar{\rho}_i}^{\rho_i} H \frac{d\tilde{\rho}_i}{3(1+w_i)\tilde{\rho}_i} = \delta N + \frac{1}{3(1+w_i)} \ln\left(\frac{\rho_i}{\bar{\rho}_i}\right) \\ &\equiv \delta N + \zeta_i^{\rm iso} \simeq \delta N + \frac{1}{3(1+w_i)} \frac{\delta\rho_i^{\rm iso}}{\bar{\rho}_i}, \qquad (1)\end{aligned}$$

ρ_i is on the uniform density hypersurface $\delta\rho \equiv \sum \delta\rho_i = 0$.
Calculation gives the δN formalism; $\delta N = [r_1\zeta_1 + (1-r_1)\zeta_2]$,
where $r_1 \equiv \frac{\dot{\rho}_1}{\dot{\rho}_1+\dot{\rho}_2}$.

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where $r_1 \equiv \frac{\dot{\rho}_1}{\dot{\rho}_1+\dot{\rho}_2}$. **Our idea is very simple.**
Use this formulation for the slow-rolling component.

$$w_i + 1 = \epsilon_w \ll 1$$

The inflating curvaton

1) The primordial inflation creates $\delta\sigma$ (isocurvature).
2) Inflating curvaton makes $\delta\sigma \to \zeta$ (Iso $\to$ Adi).
3) ζ_σ will be constant after a few $N_{\rm 2nd}$ (e-foldings).

Simple replacement gives;

$$\begin{aligned} \delta N &= r_1\zeta_\sigma + (1-r_1)\zeta_r, \\ r_1 &\simeq \frac{\epsilon_w\rho_\sigma}{\epsilon_w\rho_\sigma + 4\rho_r} \end{aligned}$$

where $3(1+w_\sigma) = 3$ is replaced by $3(1+w_\sigma) = 3\epsilon_w \ll 1$

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4) The end of the curvaton inflation is identical to the uniform density hypersurfaces if the curvaton inflation is enough. (Almost single component)

This model is very healthy!

We can calculate the perturbation =>

Calculation of the perturbation (up to 2nd)

Consider the simple quadratic potential (actually 3 components..)

Potential

$$\rho_1 = V_0 \pm \frac{1}{2} m^2 \sigma^2$$

Assuming that the end of curvaton inflation is identical to the uniform density hypersurfaces, (true for the inflating curvaton) we find at first order

Curvature perturbations (1st order)

$$\delta N^{(1)} = r_1 \zeta_1^{(1)}.$$

* $r_1 \simeq 1$ when N_{2nd} is enough.

This is rather trivial (identical to the simple replacement), but $=>$ second order!

At second order, we find defining $R \equiv \frac{\pm\frac{1}{2}m^2\sigma^2}{V_0 \pm \frac{1}{2}m^2\sigma^2}$;

2nd order δN

$$\delta N^{(2)} = \left[\frac{1}{r_1}\frac{3\epsilon_w}{2R}\left(1+\frac{gg''}{g'^2}\right) - 6\epsilon_w + 4 + (3\epsilon_w - 4)r_1\right] \times (\delta N^{(1)})^2,$$

$\sigma_{\rm ini} \equiv g(\sigma_*)$ is the quantity at the beginning of the curvaton mechanism. The non-Gaussianity is ($R < 0$ is possible!)

Non-gaussianity

$$f_{NL} = \frac{1}{r_1}\frac{5\epsilon_w}{4R}\left(1+\frac{gg''}{g'^2}\right) - 5\epsilon_w + \frac{10}{3} + \left(\frac{5}{2}\epsilon_w - \frac{10}{3}\right) r_1.$$

The result coincides with the usual curvaton when $R = 1$ and $\epsilon_w = 1$.

This is the **straight generalization** of the curvaton mechanism.

Next topic is "**Modulation in the curvaton mechanism**" =>

Unconventional Modulation without moduli

The Usual curvaton mechanism

(1) The oscillation **begins** at $H_{\rm osc} = m_\sigma$.
(2) The **end** (decay) is determined by $H_d = \Gamma_\sigma$.

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Both ends are identical to the uniform (total) density.

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(**No additional δN creation** at the boundary.)

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However

This is **not always** the case in the **generalized** curvaton mechanism.

The hybrid curvaton **without** inflation

will be the most significant example. =>

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Critical point σ_c!!

In the multi-component Universe, $\sigma = \sigma_c$ does not coincide with the uniform density hypersurfaces because $\delta\rho \neq \delta\rho_\sigma$, which is due to the conventional isocurvature perturbations.

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Modulation

There is **"Modulation"** without Moduli!!

Modulation in the curvaton mechanism

Non-linear formalism with modulation

$$\zeta_i = \delta N + \int_{\bar{\rho}_i}^{\rho_x} H \frac{d\tilde{\rho}_i}{3(1+w_{i+})\tilde{\rho}_i} + \int_{\rho_x}^{\rho_i} H \frac{d\tilde{\rho}_i}{3(1+w_{i-})\tilde{\rho}_i}, \qquad (2)$$

No modulation when $\rho_x \to \rho_i$

We divide the integral introducing **the phase transition at** ρ_x.
Take $\bar{\rho}_i$ just after the transition and think about the transition from slow to Osc (Matter). ($\epsilon_w \ll 1 \to w = 0$). The component perturbation is conserved after the transition.
Then...
ζ_i calculated above gives the initial quantity $\zeta_i^{\rm ini}$ for the usual curvaton mechanism.
I think we need some examples...

"OUR" Modulation

When the conventional isocurvature perturbation causes $\rho_x \neq \rho_i$, **transition does not coincide** with the uniform density hypersurfaces. Note that no "moduli" is assumed.

Example Hybrid curvaton $\rho_x \to \bar{\rho}_i$ (Opposite!)

Transition is determined by σ_c, not by the total density.
The component perturbation at the beginning of the oscillation is

$$\zeta_i = \delta N + \frac{1}{3\epsilon_w} \ln\left(\frac{\rho_i}{\bar{\rho}_i}\right),$$

which is obviously enhanced by the slow-rolling $\epsilon_w \ll 1$.

In the hybrid (no inflating) model the **modulation always dominates** ζ_i, and δN is created **during oscillation** via the conventional curvaton mechanism.

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Source = Modulation, Evolution = curvaton;

Peculiar collaboration!!

Back to Inflating curvaton /with ?/ without? modulation

Long inflation

($N_{\rm curv} \gg 1$) leads to signidficant decay of the isocurvature perturbations

$$\rho_i \to \rho$$

(At the end of inflation we find "uniform $\rho_i \simeq$ uniform ρ")

* Standard "Moduli-caused Modulation" does not decay because the moduli is supposed to be slow-rolling.
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No modulation when $N_{\rm 2nd}$ is enough

In that case, the curvaton inflation ends almost at

$$H = H_e.$$

We thus find no Modulation after enough e-foldings.

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The curvaton mechanism always assumes Slow-rolling phase, but there has been no detailed discussion about the evolution caused by the slow-rolling component. Especially, the non-Gaussianity parameter has not been calculated.

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Unified with the curvaton in the non-linear formalism.