Naturalness and GUT Scale Yukawa Ratios

Vinzenz Maurer

UNI
BASEL

Universität Basel

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Done in collaboration with Stefan Antusch, Lorenzo Calibbi, Maurizio Monaco & Martin Spinrath
(arXiv:1111.6547, PRD **85** 035025 (2012))
(in preparation)

Outline

1. Motivation
2. Fine-Tuning and Yukawa Ratios in the CMSSM
3. Fine-Tuning and Yukawa Ratios near the Constrained MSSM
4. Summary and Conclusions

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Fine-Tuning with Supersymmetry

Naturalness

SUSY has not been found so far

$\downarrow$

How fine tuned is the MSSM?

GUT Relations between Yukawa Couplings

Attractive options:

- $y_\tau = y_b$ [Georgi, Glashow '74]
- $y_\tau = \frac{3}{2} y_b$ [Antusch, Spinrath '09]

$\downarrow$

Which one is preferred?

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Putting the Price Tag on Fine-Tuning

Parameter Sensitivity of the Z Mass

Definition:

$$\Delta_a := \left| \frac{\partial \log M_Z(a, \dots)}{\partial \log a} \right| = \left| \frac{a}{2M_Z} \frac{\partial M_Z^2(a, \dots)}{\partial a} \right|$$

Interpretation: Tuning to 1 part in Δ_a

- Fine-tuning of one point in parameter space

$$\Delta := \max_a \Delta_a$$

- Depends on parametrisation
  $\Rightarrow$ Beware when comparing numbers

[Barbieri, Giudice '88]

Semi-Analytical Treatment

- Tree-level relation in the MSSM

$$\frac{M_Z^2}{2} = -|\mu|^2 + \frac{1}{2}\tan 2\beta(m_{H_u}^2 \tan\beta - m_{H_d}^2 \cot\beta) \approx -|\mu|^2 - m_{H_u}^2$$

- Polynomial Approximation in the CMSSM @ $M_{\text{SUSY}} \approx 1\text{TeV}$

$$m_{H_u}^2 = 0.058 m_0^2 - 0.094 A_0^2 + 0.317 A_0 M_{1/2} - 1.304 M_{1/2}^2$$

- Consider (with $a_0 = \frac{A_0}{M_{1/2}}$)

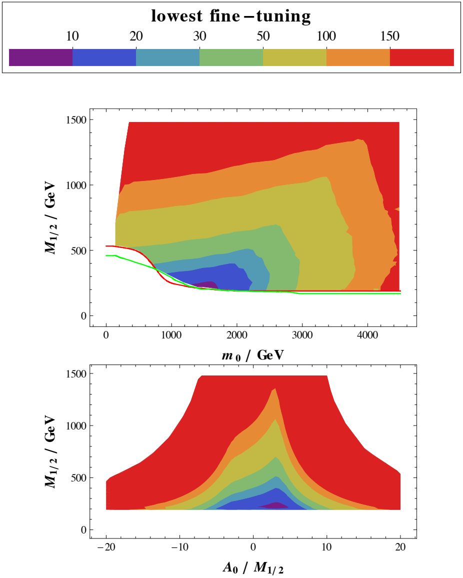
$$\max\{\Delta_{A_0}, \Delta_{M_{1/2}}\} = \left|\frac{M_{1/2}^2}{M_Z^2}\right| \max\left\{\begin{array}{l} |0.32a_0 - 0.19a_0^2| \\ |0.32a_0 - \quad 2.61| \end{array}\right\}$$

$\Rightarrow$ minimal for $a_0 \approx 3$, steep rising for $|a_0| > 4$.

Status in the CMSSM (Nov 2011)

Constraints:

- LEP
- LHC 1 fb^{-1}
- $b \to s\gamma$
- $B_s \to \mu^+\mu^-$
- a_μ @ 5σ



[Antusch, Calibbi, V.M., Monaco,

Spinrath '11]

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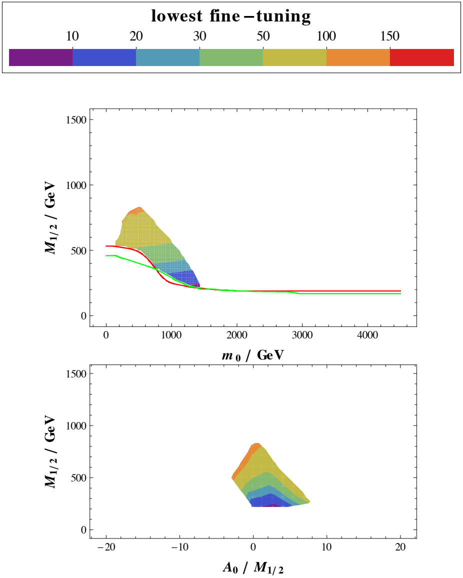

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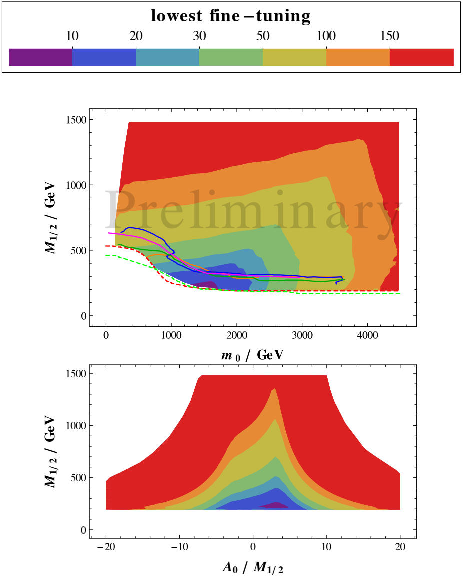

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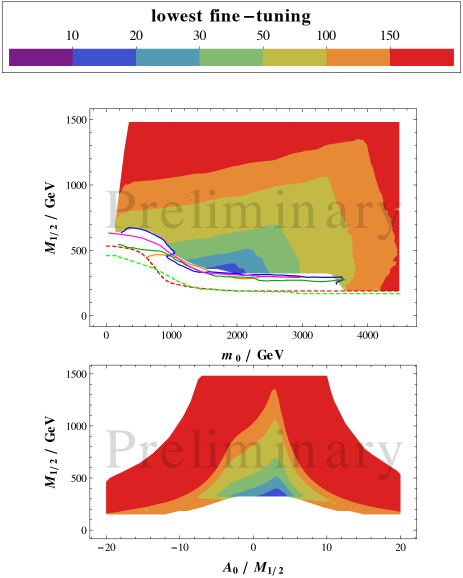

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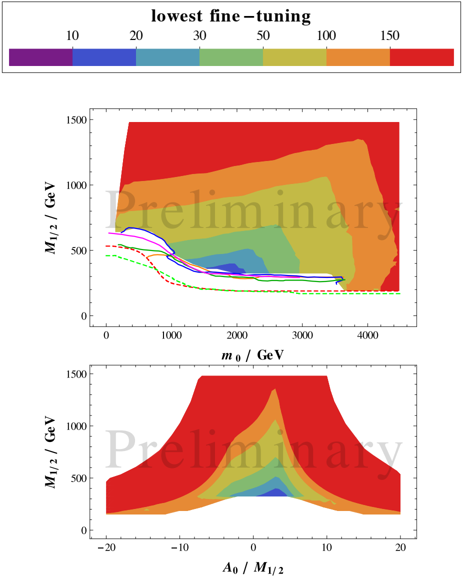

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Constraints:

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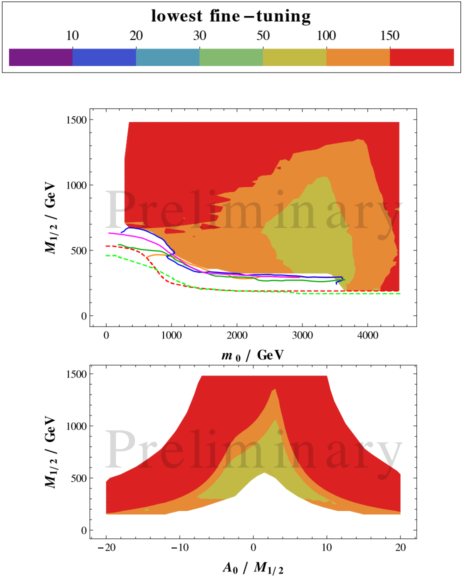
- $m_{h^0} > 120\text{GeV}$



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Yukawa Coupling: What can we expect?

- SUSY threshold correction to τ-b Yukawa coupling ratio:

$$\delta\left(\frac{y_\tau}{y_b}\right) \approx (\epsilon_0 + \epsilon_Y y_t^2)\tan\beta$$

$$\epsilon_0 = -\frac{2}{3\pi}\alpha_s \frac{\mu}{M_3} H_2\left(\frac{m^2_{\tilde{Q}_3}}{M_3^2}, \frac{m^2_{\tilde{d}_3}}{M_3^2}\right)$$

$$\epsilon_Y y_t^2 = -\frac{y_t^2}{16\pi^2}\frac{A_t}{\mu} H_2\left(\frac{m^2_{\tilde{Q}_3}}{\mu^2}, \frac{m^2_{\tilde{u}_3}}{\mu^2}\right)$$

- Large correction for big A_t and $M_3 \Rightarrow$ large fine-tuning
- Example: $m_0 = 1.5$ TeV, $M_{1/2} = 350$ GeV, $a_0 = 3.7$, $\tan\beta = 40$

$$\delta\left(\frac{y_\tau}{y_b}\right) = \begin{cases} 0.088 & \text{for } \mu > 0\ , \\ -0.092 & \text{for } \mu < 0\ . \end{cases}$$

$$\Rightarrow \boxed{\frac{y_\tau}{y_b} = 1.4, 1.1}$$

Yukawa Couplings vs. Fine-Tuning

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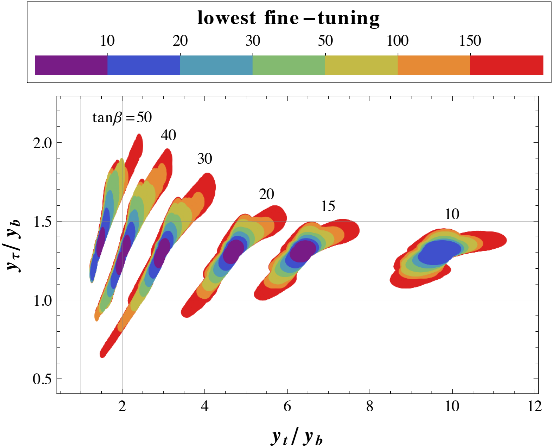

Figure: Lowest fine-tuning with $(g-2)_\mu$ @ 5σ (1σ quark mass errors)

[Antusch, Calibbi, V.M., Monaco, Spinrath '11]

Yukawa Couplings vs. Fine-Tuning

Constraints:

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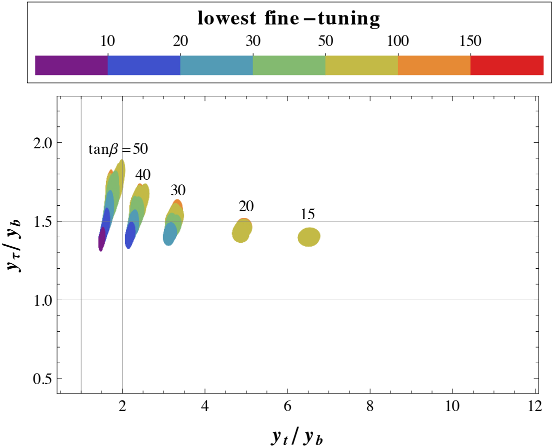
- a_μ @ 2σ



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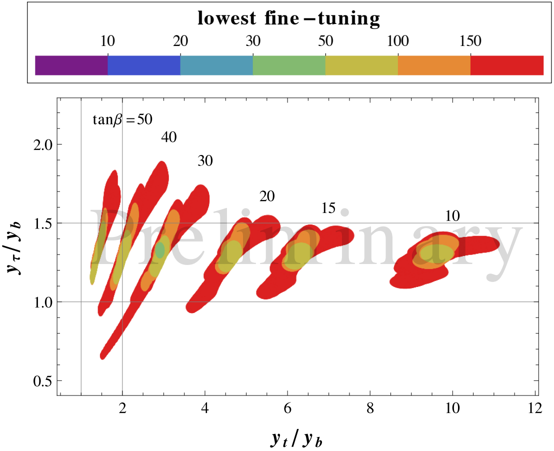

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Non-Universal Gaugino Masses

Motivation

$$G_{\text{GUT}} \xrightarrow{\langle H_{\text{GUT}} \rangle} G_{\text{SM}}$$

$$M_i = M_{1/2} \xrightarrow{\langle H_{\text{GUT}} \rangle} M_i = \eta_i M_{1/2}$$

[Ellis, Enqvist, Nanopoulos, Tamvakis ’84; see also Martin ’09]

Relaxed Setup

Assume η_i fixed, but arbitrary
Set $\eta_3 = 1$, so that $M_{1/2} = M_3(M_{\text{GUT}})$
$\Rightarrow$ Same number of „tunable“ parameters as CMSSM

Fine-Tuning with Non-Universal Gaugino Masses

- Polynomial for $m^2_{H_u}$ takes the form

$$m^2_{H_u} = f_1(\eta_1,\eta_2)M_3^2 + f_2(\eta_1,\eta_2)A_0 M_3 + f_3 A_0^2 + \dots$$

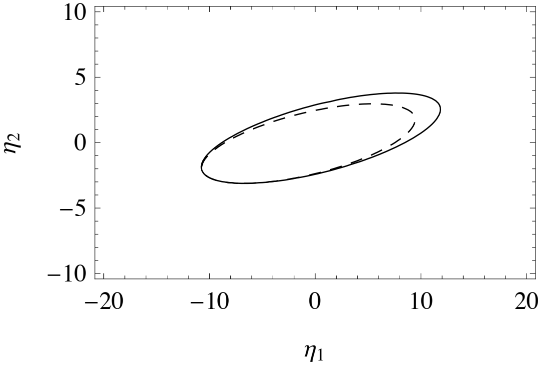
with

$$f_1 = (1.29 - 0.02\eta_1 - 0.02\eta_1^2 + 0.02\eta_2 - 0.22\eta_2^2 + 0.08\eta_1\eta_2)\ ,$$
$$f_2 = (0.29 + 0.01\eta_1 + 0.06\eta_2)\ ,$$
$$f_3 = -0.11\ .$$

- Minimize $\max\{\Delta_{A_0}, \Delta_{M_{1/2}}\} \Rightarrow$ Two solutions:
  - $a_0 = 0, \quad f_1 = 0$ [Horton, Ross '09]

In addition:
  - $a_0 = -\frac{f_2}{2f_3}, \quad f_1 = \frac{f_2^2}{4f_3}$ [Antusch, Calibbi, V.M., Monaco, Spinrath in preparation]

Fine-Tuning with Non-Universal Gaugino Masses



Solutions for $\max\{\Delta_{A_0}, \Delta_{M_{1/2}}\} = 0$

- $a_0 = 0$ (dashed)
- $0.0962 < a_0 < 2.1354$ (solid)

Non-Universal Gaugino Masses: Numerical Results

Constraints:
- LEP
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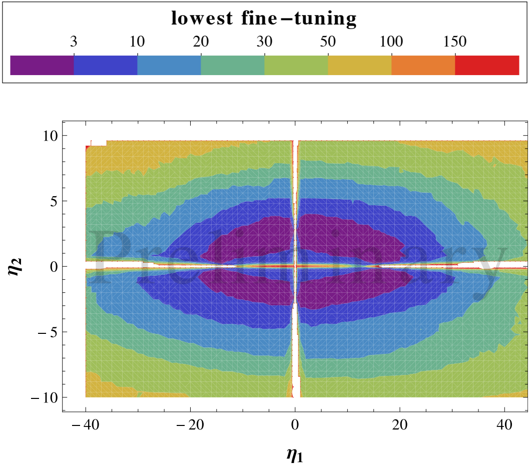
- a_μ @ 5σ

[Antusch, Calibbi, V.M., Monaco, Spinrath in preparation]



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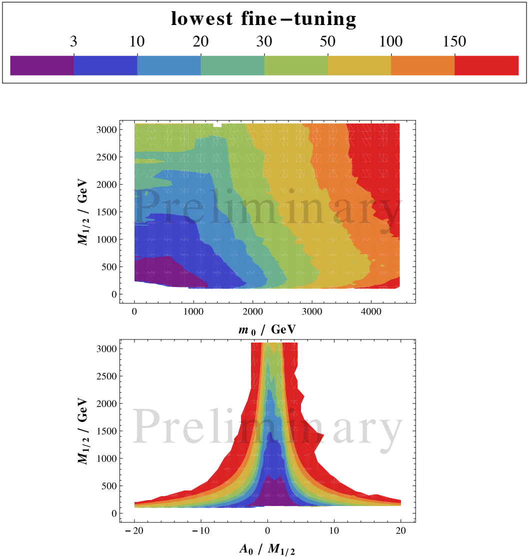

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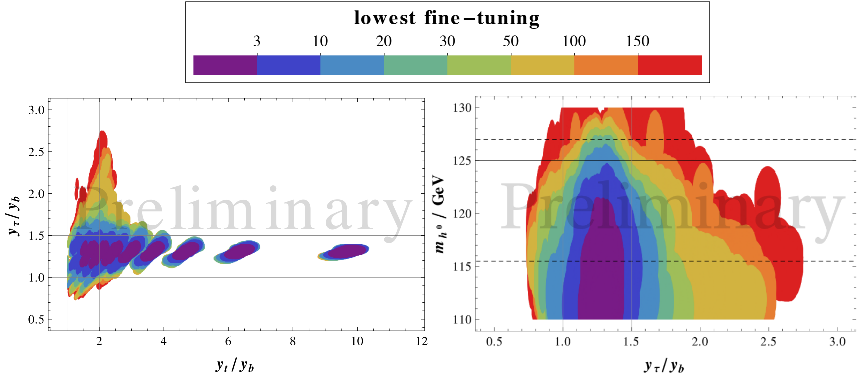

Figure: Lowest fine-tuning with $(g-2)_\mu$ @ 5σ
(1σ quark mass errors + 3 GeV uncertainty for m_{h^0})

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Summary

- Fine-tuning in the CMSSM
  - with universal gaugino masses
  - without universal gaugino masses

- Fine-tuning as a hint on GUT scale Yukawa relations
  - $y_\tau = y_b$
  - $y_\tau = \frac{3}{2} y_b$

Summary

- Fine-tuning in the CMSSM
  - with universal gaugino masses
  - without universal gaugino masses $\rightarrow$ can be significantly improved

- Fine-tuning as a hint on GUT scale Yukawa relations
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Summary

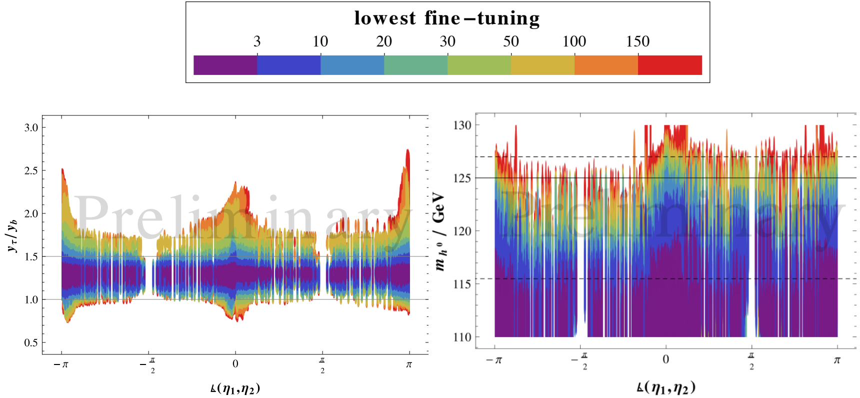
- Fine-tuning in the CMSSM
  - with universal gaugino masses
  - without universal gaugino masses $\rightarrow$ can be significantly improved

- Fine-tuning as a hint on GUT scale Yukawa relations
  - $y_\tau = y_b$
  - $y_\tau = \frac{3}{2} y_b$ $\rightarrow$ often preferred

End of the Talk

Thank you for your attention!

Backup: Where on the ellipsis is this?



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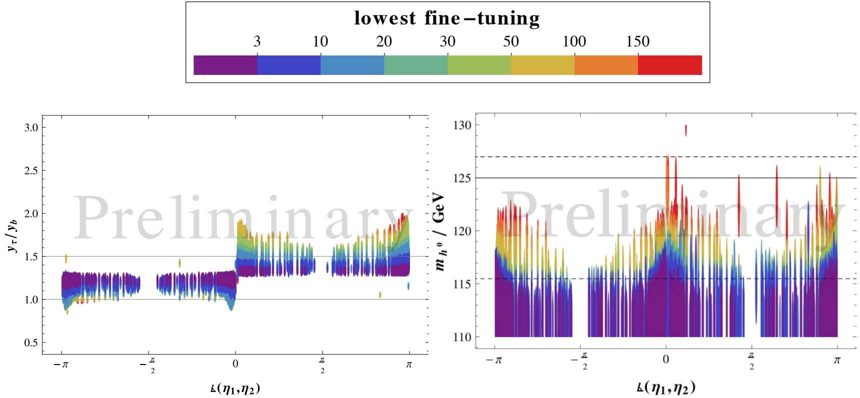

Figure: Lowest fine-tuning with $(g-2)_\mu$ @ 2σ
(1σ quark mass errors + 3 GeV uncertainty for m_{h^0})

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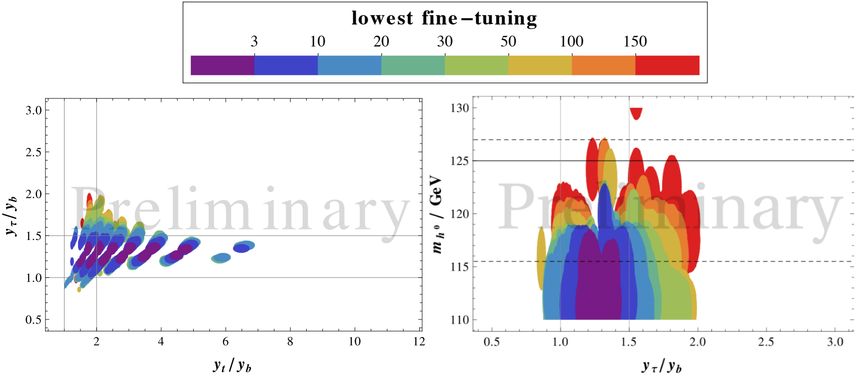
Backup: Yukawa Couplings vs. Fine-Tuning Revisited



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