Discrete Flavour Groups, Neutrino Reactor Angle and LFV

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Outline

- News on neutrino mixings
- Impact on neutrino flavour models [much more in Feruglio's talk on Friday]
- Implications for LFV transitions in supersymmetric models:
  - without RH neutrinos
  - with RH neutrinos
- Correlation with the muon g-2 discrepancy

based on: Altarelli, Feruglio, LM & Stamou, arXiv:1205.4670

Altarelli, Feruglio & LM, arXiv:1205.5133

Bazzocchi & LM, arXiv:1205.5135

Recent Results of Global Fits

	$\sin^2 2\theta_{13}$	$\sin^2 \theta_{13}$
T2K [1106.2822]	$0.11^{+0.11}_{-0.05}$ $(0.14^{+0.12}_{-0.06})$	$0.028^{+0.019}_{-0.024}$ $(0.036^{+0.022}_{-0.030})$
MINOS [1108.0015]	$0.041^{+0.047}_{-0.031}$ $(0.079^{+0.071}_{-0.053})$	$0.010^{+0.012}_{-0.008}$ $(0.020^{+0.019}_{-0.014})$
DC [1112.6353]	$0.086 \pm 0.041 \pm 0.030$	$0.022^{+0.019}_{-0.018}$
DYB [1203.1669]	$0.092 \pm 0.016 \pm 0.005$	0.024 ± 0.005
RENO [1204.0626]	$0.113 \pm 0.013 \pm 0.019$	0.029 ± 0.006

Very recent global fit: Fogli *et al.* 1205.5254 (see also [Forero, Tortola and Valle 1205.4018])

$$\Delta m^2_{\rm sol} = (7.54^{+0.26}_{-0.22}) \times 10^{-5}\,{\rm eV}^2$$

$$\Delta m^2_{\rm atm} = (2.43^{+0.07}_{-0.09})[2.42^{+0.07}_{-0.10}] \times 10^{-3}\,{\rm eV}^2$$

$$\sin^2\theta_{12} = 0.307^{+0.018}_{-0.016}$$

$$\sin^2\theta_{23} = 0.398^{+0.030}_{-0.026}[0.408^{+0.035}_{-0.030}]$$

$$\sin^2\theta_{13} = 0.0245^{+0.0034}_{-0.0031}[0.0246^{+0.0034}_{-0.0031}]$$

$$\delta = \pi(0.89^{+0.29}_{-0.44})[0.90^{+0.32}_{-0.43}]$$

Neutrino Mass Patterns

In the past:

- large atmospheric angle
- only upper bound on the reactor angle

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$$\sin^2\theta_{23} = \frac{1}{2}$$

$$\sin^2\theta_{13} = 0$$

mu-tau symmetry

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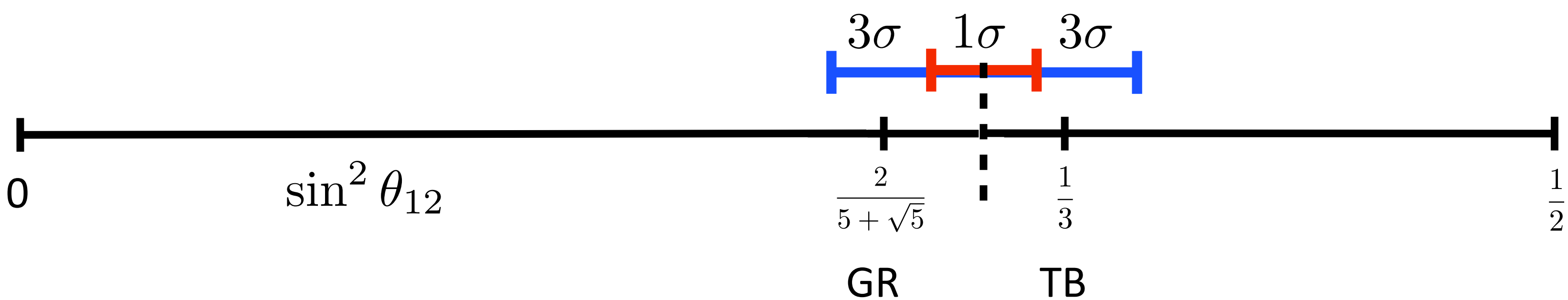

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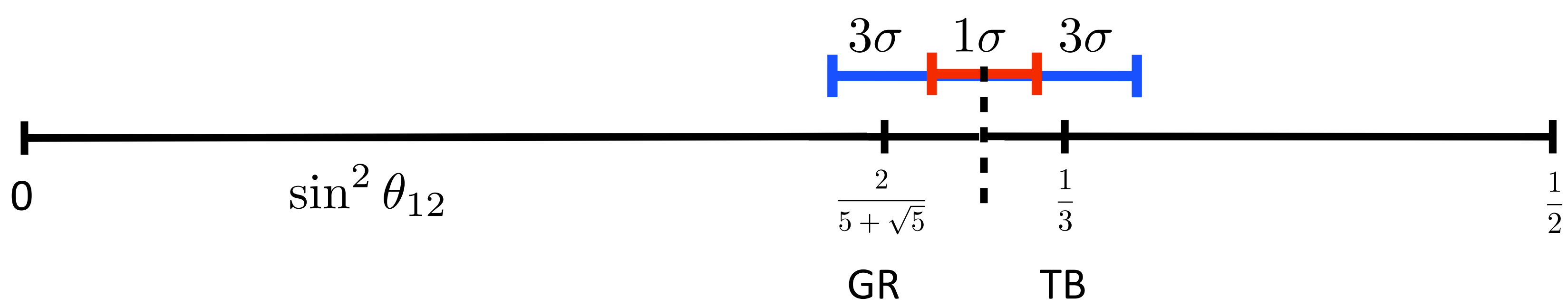

TRI-BIMAXIMAL (TB) [Harrison, Perkins & Scott 2002; Zhi-Zhong Xing 2002]

$$\sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0 \qquad \sin^2\theta_{12} = \frac{1}{3} \longrightarrow \theta_{12} = 35.26^\circ$$

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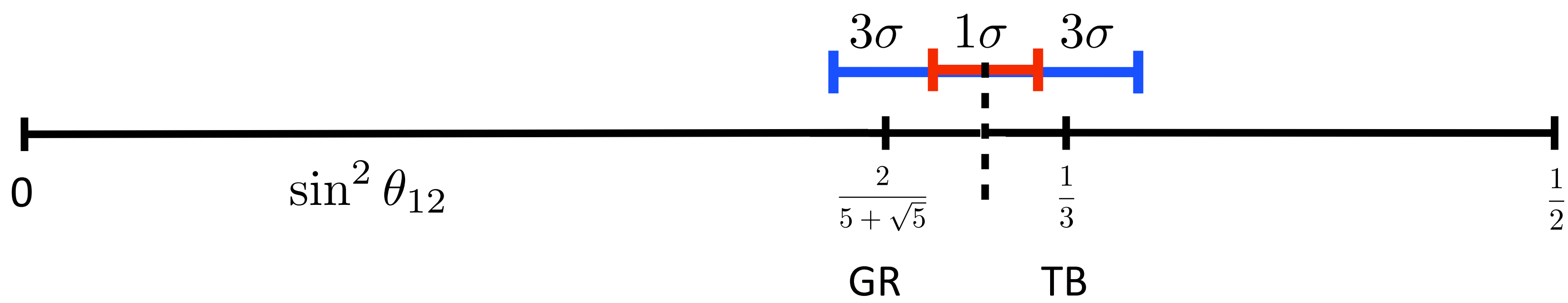

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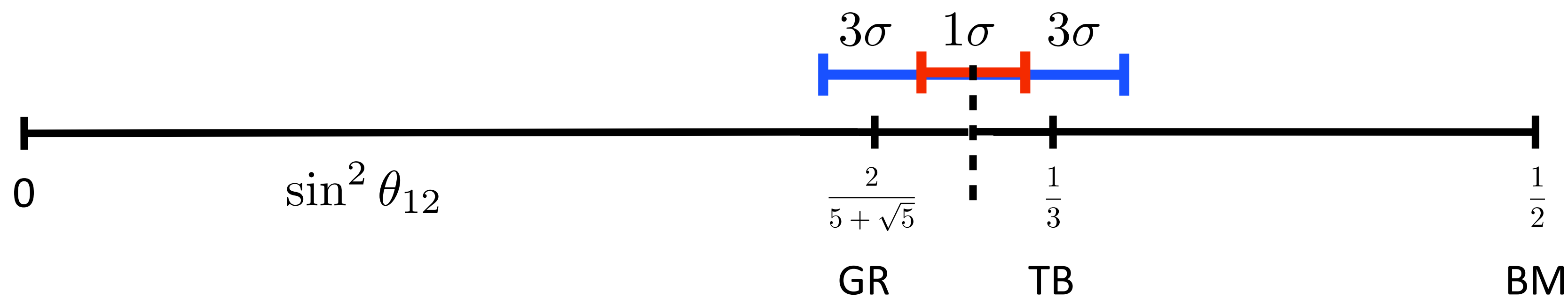
GOLDEN RATIO (GR) [Kajiyama, Raidal & Strumia 2007]

$$\sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0 \quad \tan\theta_{12} = \frac{1}{\phi} \quad \longrightarrow \quad \theta_{12} = 31.72^\circ$$

$$\phi \equiv \frac{1+\sqrt{5}}{2}$$

Neutrino Mass Patterns



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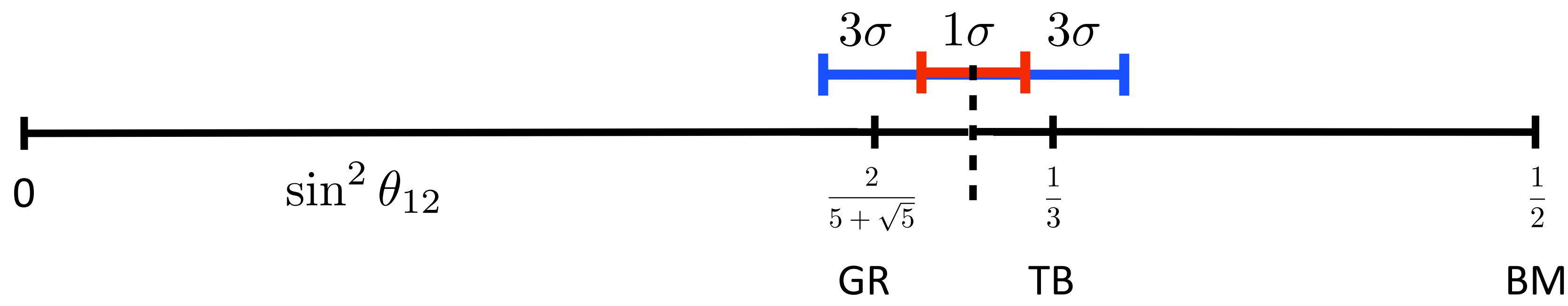

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Maybe related to the
Quark-Lepton Complementarity:
[Smirnov; Raidal; Minakata & Smirnov 2004]

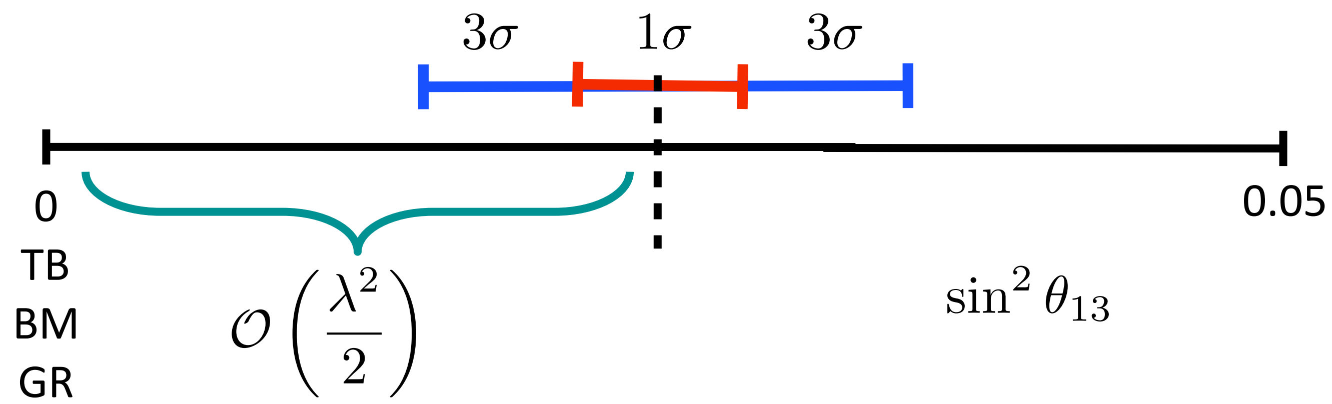
$$\pi/4 \approx \theta_{12} + \lambda$$

$$\longrightarrow \theta_{12}^{Exp} \approx \theta_{12}^{BM} - \lambda$$

[Altarelli, Feruglio and LM 2009,
Adelhart, Bazzocchi and LM 2010,
Meloni 2011]

Need of Corrections!!

$$\sin^2\theta_{13} = 0.0245^{+0.0034}_{-0.0031}[0.0246^{+0.0034}_{-0.0031}]$$



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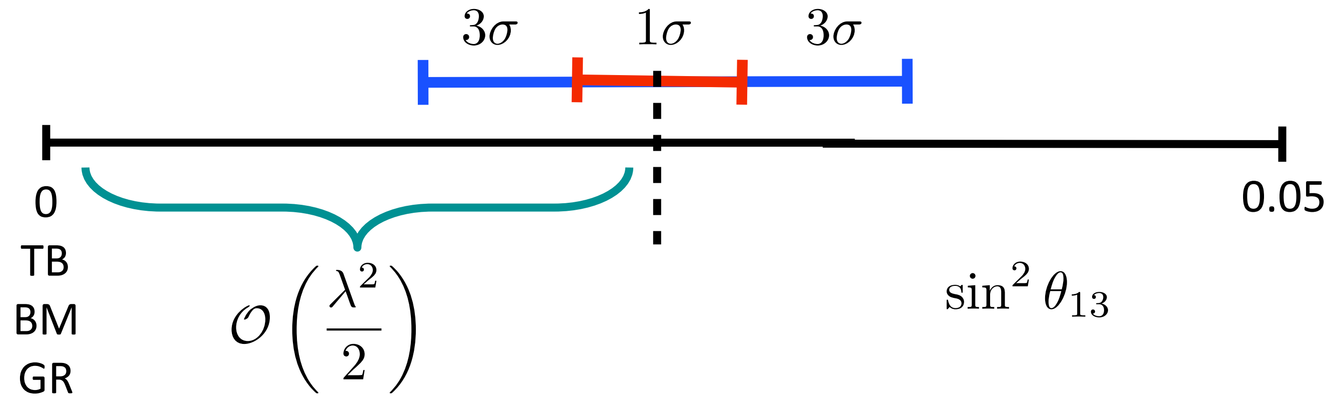

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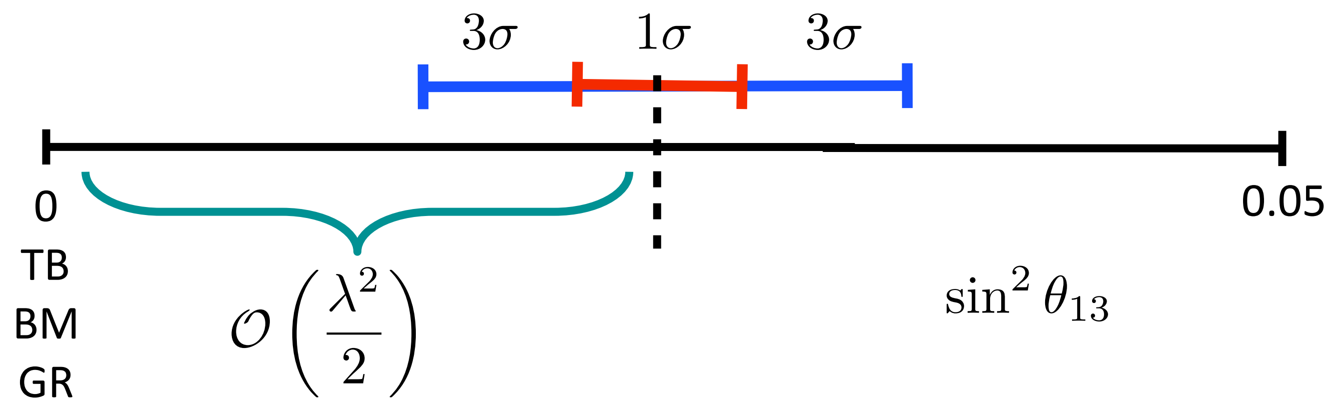

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$$m_e = m_e^{(0)} + \delta m_e \qquad\qquad m_\nu = m_\nu^{(0)} + \delta m_\nu$$

in the basis in which the LO masses satisfy to

$$m_e^{diag} = m_e^{(0)} \qquad m_\nu^{diag} = U_\nu^{0T}\, m_\nu^{(0)}\, U_\nu^0 \qquad U_\nu^0 = \{U_{TB}, U_{GR}, U_{BM}\}$$

then the NLO corrections are encoded in

$$(m_e^{diag})^2 = \delta U_e^\dagger\, m_e^\dagger\, m_e\, \delta U_e$$

$$m_\nu^{diag} = \delta U_\nu^T\, U_\nu^{0T}\, m_\nu\, U_\nu^0\, \delta U_\nu$$

$$\delta U = \begin{pmatrix} 1 & c_{12}\,\xi & c_{13}\,\xi \\ -c_{12}^*\,\xi & 1 & c_{23}\,\xi \\ -c_{13}^*\,\xi & -c_{23}^*\,\xi & 1 \end{pmatrix}$$

Type of Models

[much more in Feruglio's talk on Friday]

- In typical TB (GR) models, the corrections are democratic in all the angles:

$$\xi^e \approx \xi^\nu \equiv \xi \qquad \begin{array}{c} c^e_{12} \approx c^e_{23} \approx c^e_{13} \\ c^\nu_{12} \approx c^\nu_{23} \approx c^\nu_{13} \end{array}$$

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- Which is the meaning of ξ ?
- How can we achieve these flavour structures?

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- Starting from a Yukawa Lagrangian invariant under a Flavour Symmetry, masses and mixings arise only through a **symmetry breaking mechanism**:

$$\mathcal{L}_Y = \frac{(Y_e[\varphi^n])_{ij}}{\Lambda_f^n} e_i^c H^\dagger \ell_j + \frac{(Y_\nu[\varphi^m])_{ij}}{\Lambda_f^m} \frac{(\ell_i \tilde{H}^*)(\tilde{H}^\dagger \ell_j)}{2\Lambda_L}$$

where φ are new heavy scalar fields, singlets under SM, called **flavons**

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- At LO the PMNS can take one of the previous predictive patterns
- At NLO, some corrections arise and they are proportional to the VEV of the flavons: larger is the VEV and larger are the corrections

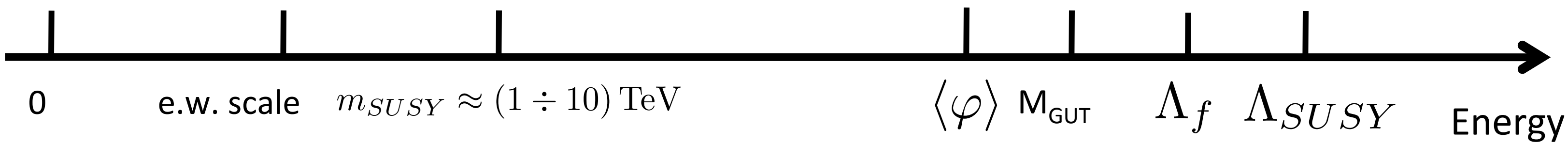
Are there consequences of so large ξ ?

Impact on LFV

Low Energy Observables:
- ν masses
- ν oscillations

- $(g-2)_\mu$ discrepancy
- dark matter
- gauge coupling unification
- hierarchy problem

- GUTs
- flavour symmetries
- ν^c
- superheavy gauge bosons



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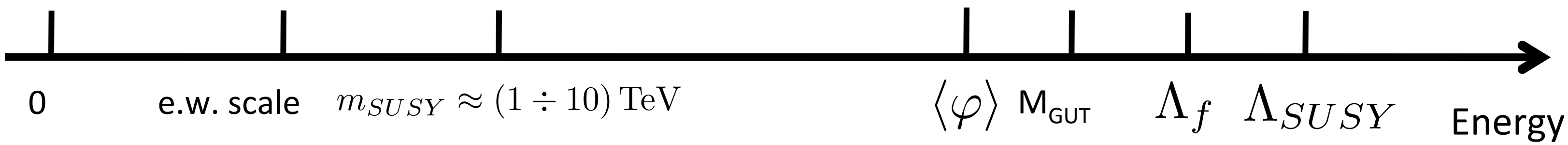

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→ non-universal boundary conditions for the soft terms

→ different results wrt CMSSM scenario

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We focus on the radiative decay $\mu \to e\gamma$:

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The normalized BR is defined by:

$$R_{ij} = \frac{48\pi^3 \alpha_{em}}{G_F^2 m_{SUSY}^4} \left[\left| A_L^{ij} \right|^2 + \left| A_R^{ij} \right|^2 \right]$$

MI

$$A_L^{ij} = a_{LL} \left(\delta_{ij}\right)_{LL} + a_{RL} \frac{m_{SUSY}}{m_i} \left(\delta_{ij}\right)_{RL}$$

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The $a_{CC'}$ are loop factors of the SUSY parameters:

$$\tan\beta = \{2, 25\} \quad \begin{cases} a_{LL} = \{2, 27\} \\ a_{RR} = \{-1.9, -0.6\} \\ a_{RL} = a_{LR} = 0.3 \end{cases}$$

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$$-\mathcal{L}_m \supset \quad (\bar{\tilde{e}} \quad \tilde{e}^c) \begin{pmatrix} m^2_{eLL} & m^2_{eLR} \\ m^2_{eRL} & m^2_{eRR} \end{pmatrix} \begin{pmatrix} \tilde{e} \\ \bar{\tilde{e}}^c \end{pmatrix} + \bar{\tilde{\nu}}\, m^2_{\nu LL}\, \tilde{\nu}$$

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- $m^2_{(e,\nu)LL}$ and m^2_{eRR} are hermitian matrices from the Kähler potential
- $m^2_{eLR} = (m^2_{eRL})^\dagger$ from the superpotential

generated from the SUSY Lagrangian analytically continuing all the couplings constants into superspace:

$$\mathcal{L} \supset \int d^2\theta\, d^2\bar{\theta}\, \bar{\ell}\ell \to \int d^2\theta\, d^2\bar{\theta}\, \left(1 + k\, m_0^2\, \theta^2\bar{\theta}^2\right) \bar{\ell}\ell$$

$$\mathcal{L} \supset \int d^2\theta\, y_e\, e^c\, \ell\, h_d \to \int d^2\theta\, \left(y_e + x_e\, m_0\, \theta^2\right)\, e^c\, \ell\, h_d$$

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→ Non-canonical kinetic terms

→ $$(m_{eLL}^2)_K = \begin{pmatrix} 1 & \mathcal{O}(\xi^n) & \mathcal{O}(\xi^n) \\ \mathcal{O}(\xi^n) & 1 & \mathcal{O}(\xi^n) \\ \mathcal{O}(\xi^n) & \mathcal{O}(\xi^n) & 1 \end{pmatrix} m_0^2$$

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$$\mathcal{L} \supset \int d^2\theta\ \left(Y_e + A_e\, m_0 \theta^2\right)_{ij}\ e^c_i\, \ell_j\, h_d$$

→ $$Y_e = \begin{pmatrix} y_e & y_e\, \mathcal{O}(\xi^n) & y_e\, \mathcal{O}(\xi^n) \\ y_\mu\, \mathcal{O}(\xi^n) & y_\mu & y_\mu\, \mathcal{O}(\xi^n) \\ y_\tau\, \mathcal{O}(\xi^n) & y_\tau\, \mathcal{O}(\xi^n) & y_\tau \end{pmatrix}$$

→ $$m^2_{eRL} = \begin{pmatrix} y_e & y_e\, \mathcal{O}(\xi^n) & y_e\, \mathcal{O}(\xi^n) \\ y_\mu\, \mathcal{O}(\xi^n) & y_\mu & y_\mu\, \mathcal{O}(\xi^n) \\ y_\tau\, \mathcal{O}(\xi^n) & y_\tau\, \mathcal{O}(\xi^n) & y_\tau \end{pmatrix} m_0\, v_d$$

same flavour structure but different coefficients

Typical TB (GR) models

$$\xi \simeq 0.075$$
$$SR \sim 12\%$$

→

$$R_{ij} = \frac{48\pi^3 \alpha_{em}}{G_F^2 m_{SUSY}^4} \left| a_{LL} + a_{RL} \right|^2 \mathcal{O}\left(\xi^4\right)$$

$$R_{\mu e} \approx R_{\tau e} \approx R_{\tau\mu}$$

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- BM models

$$\xi^e \simeq 0.17$$
$$SR \sim 3.4\%$$

→

$$R_{ij} = \frac{48\pi^3\alpha_{em}}{G_F^2 m_{SUSY}^4} |a_{LL} + a_{RL}|^2 \times \begin{cases} \mathcal{O}\left(\xi^{e2}\right) & ij = 21, 31 \\ \mathcal{O}\left(\xi^{e4}\right) & ij = 32 \end{cases}$$
$$R_{\mu e} \approx R_{\tau e} \gg R_{\tau\mu}$$

$m_0 = 200$ GeV & $\tan\beta = 15$

$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$$

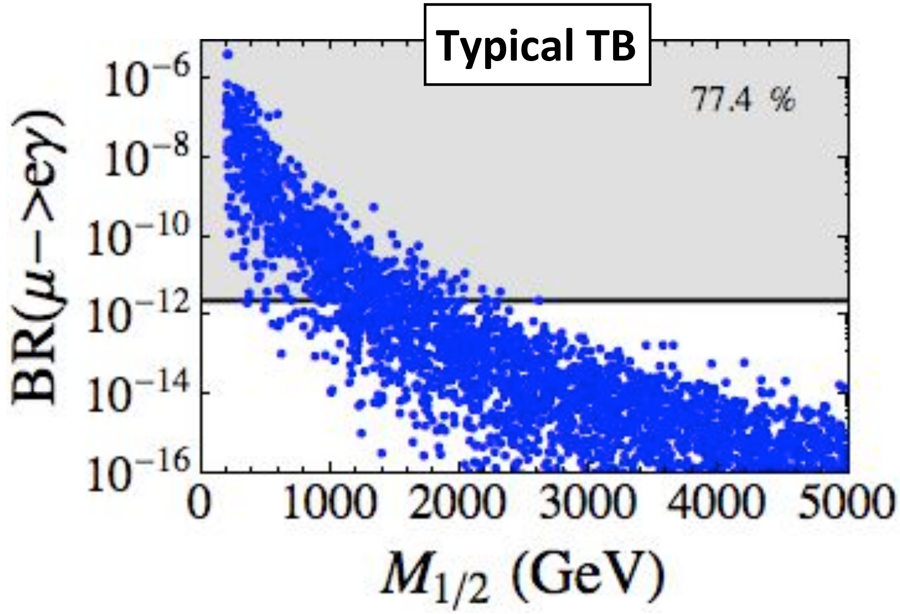
→ $M_{1/2} \lesssim 400$ GeV

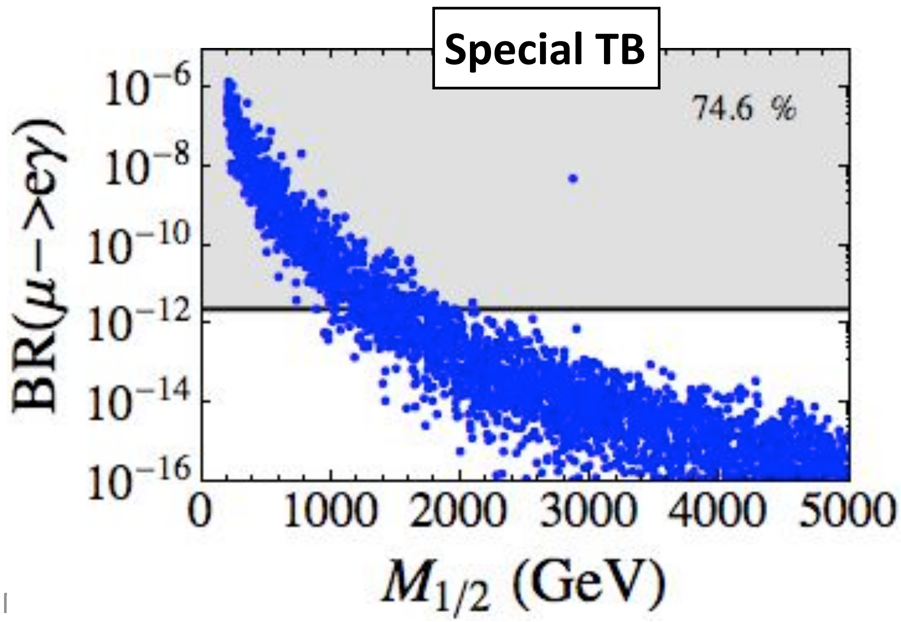
$\chi^0 \approx 156$ GeV

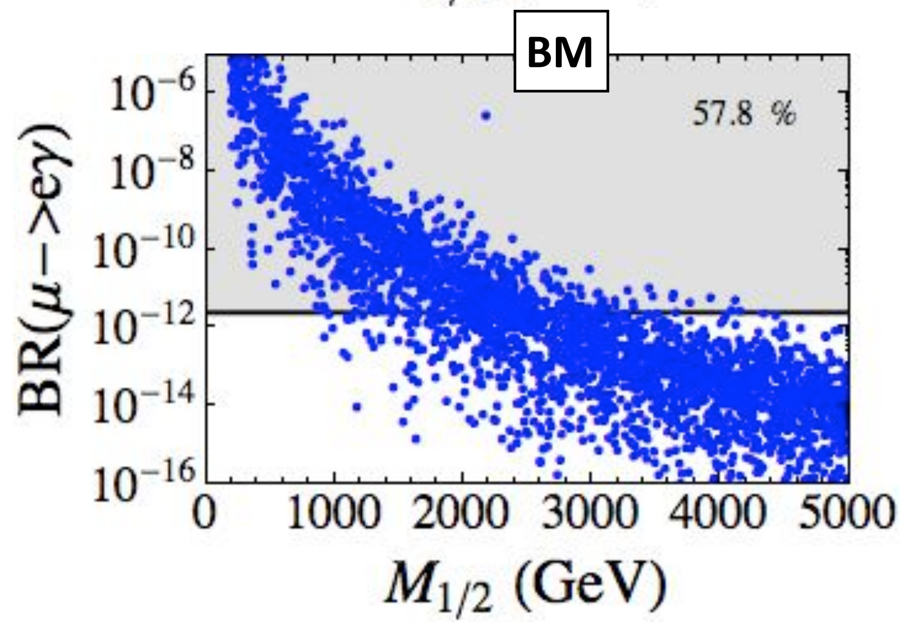
$\chi^\pm \approx 306$ GeV

$\tilde{\ell}_R \approx [160, 350]$ GeV

$\tilde{\ell}_L \approx [230, 500]$ GeV







With RH Neutrino

When RH neutrinos are present in the spectrum, their RGE are important:

$$(m^2_{eLL})_{ij} \simeq -\frac{1}{8\pi^2}\left(3\, m_0^2 + A_0^2\right) \sum_k (\hat{Y}_\nu^\dagger)_{ik} \log\left(\frac{\Lambda}{M_k}\right)(\hat{Y}_\nu)_{kj}$$

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If the RH neutrinos transform as 3dim irreducible representations then

$$\rho(g)\; Y_\nu^\dagger Y_\nu\; \rho(g)^\dagger = Y_\nu^\dagger Y_\nu \to \left[\rho(g),\, Y_\nu^\dagger Y_\nu\right] = 0 \to Y_\nu^\dagger Y_\nu \propto 1 \to Y_\nu \text{ is unitary}$$

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$$\longrightarrow \quad (m^2_{eLL})_{ij} \simeq -\frac{|k|^2}{8\pi^2}\left(3\,m_0^2 + A_0^2\right)\left[U_{i2}\log\frac{m_2}{m_1}U^*_{j2} + U_{i3}\log\frac{m_3}{m_1}U^*_{j3}\right] + \ldots$$

Very predictive relation: it only depends on the LO mixing pattern and neutrino spectrum

- TB pattern

$$\left(m^2_{eLL}\right)_{\mu e} \propto \frac{1}{3}\log\left(\frac{m_2}{m_1}\right)$$

$$\left(m^2_{eLL}\right)_{\tau e} \propto \frac{1}{3}\log\left(\frac{m_2}{m_1}\right)$$

$$\left(m^2_{eLL}\right)_{\tau\mu} \propto \frac{1}{3}\log\left(\frac{m_2}{m_1}\right) - \frac{1}{2}\log\left(\frac{m_3}{m_1}\right)$$

- GR pattern

$$\left(m^2_{eLL}\right)_{\mu e} \propto -\frac{1}{\sqrt{10}}\log\left(\frac{m_2}{m_1}\right)$$

$$\left(m^2_{eLL}\right)_{\tau e} \propto -\frac{1}{\sqrt{10}}\log\left(\frac{m_2}{m_1}\right)$$

$$\left(m^2_{eLL}\right)_{\tau\mu} \propto \frac{5+\sqrt{5}}{20}\log\left(\frac{m_2}{m_1}\right) - \frac{1}{2}\log\left(\frac{m_3}{m_1}\right)$$

- BM pattern

$$\left(m^2_{eLL}\right)_{\mu e} \propto \frac{1}{4}\sqrt{\frac{3}{2}}\log\left(\frac{m_2}{m_1}\right)$$

$$\left(m^2_{eLL}\right)_{\tau e} \propto \frac{1}{4}\sqrt{\frac{3}{2}}\log\left(\frac{m_2}{m_1}\right)$$

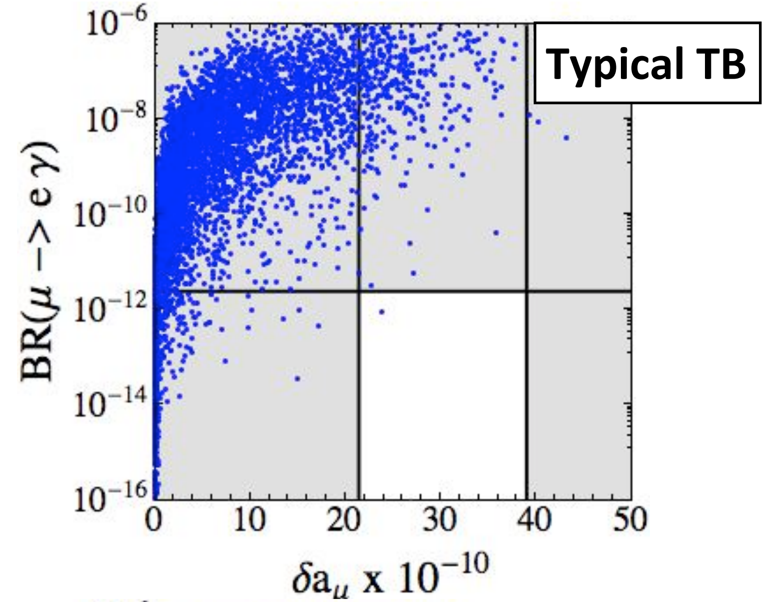
$$\left(m^2_{eLL}\right)_{\tau\mu} \propto \frac{3}{8}\log\left(\frac{m_2}{m_1}\right) - \frac{1}{2}\log\left(\frac{m_3}{m_1}\right)$$

→ Expressing all the neutrino masses in terms of the lightest one, these quantities depend on only 1 parameter

$BR(\mu \to e\gamma)$ & $(g-2)_\mu$

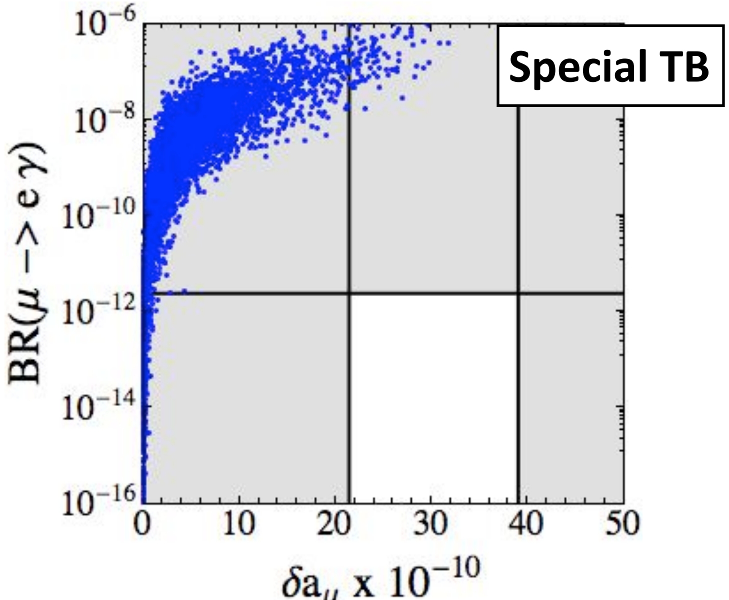
$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$$

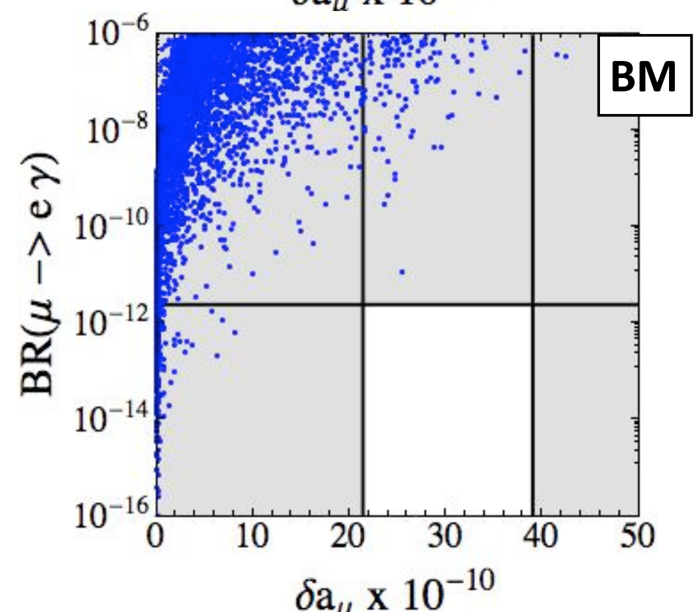
$$\delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = 302(88) \times 10^{-11}$$



$$\tan\beta \in [2, 15]$$

$$m_0,\ M_{1/2} \in [200, 5000]\text{GeV}$$





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Thanks for your attention

Backup Slides

Typical Tri-Bimaximal

[much more in Feruglio's talk on Friday]

In typical TB (GR) models, the corrections are democratic in all the angles:

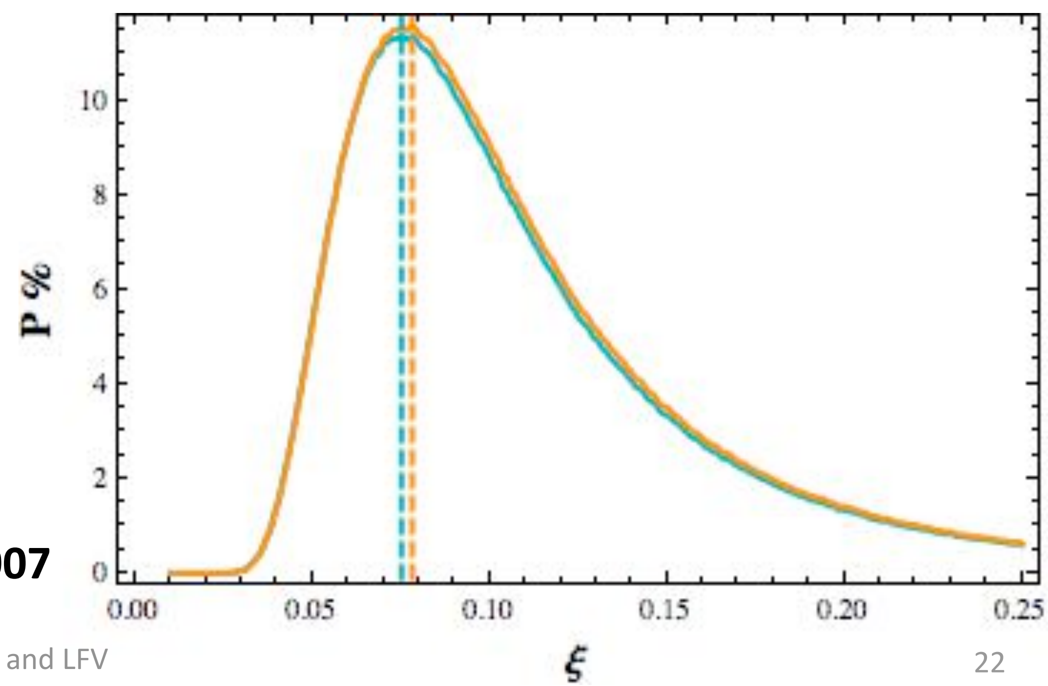
$$c^e_{12} \approx c^e_{23} \approx c^e_{13} \qquad\qquad c^\nu_{12} \approx c^\nu_{23} \approx c^\nu_{13}$$

1. Corrections only from the charged lepton: $\xi^\nu = 0$
2. Corrections only from the neutrino sector: $\xi^e = 0$
3. Correction from both the sector and democratic: $\xi^e \approx \xi^\nu \equiv \xi$

To maximize the success rate for all the three mixing angles inside the 3σ:

1.-2. $\xi^\nu, \xi^e \simeq 0.1$

3. $\xi \simeq 0.075$



A_4: Altarelli & Feruglio 2005
T': Feruglio, Hagedorn, LM & Lin 2007
S_4: Bazzocchi, LM & Morisi 2009

Special Tri-Bimaximal

[much more in Feruglio's talk on Friday]

In special TB models, the corrections are specific in certain flavour directions:

A$_4$: Lin 2009

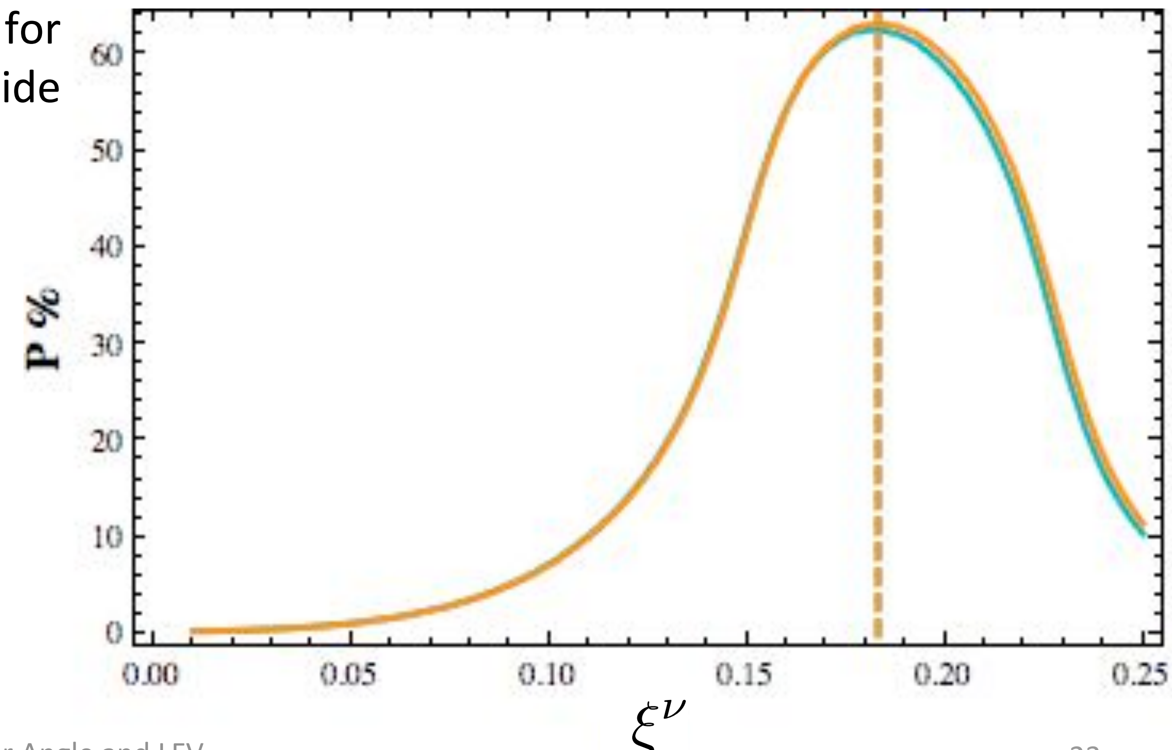
$$\xi^\nu \gg \xi^e$$

$$c^\nu_{12} = c^\nu_{23} = 0 \qquad c^\nu_{13} \neq 0$$

$$c^e_{12} \approx c^e_{23} \approx c^e_{13}$$

To maximize the success rate for all the three mixing angles inside the 3σ:

$$\xi^\nu \simeq 0.18$$



Bimaximal

[much more in Feruglio's talk on Friday]

Also in BM models, the corrections are specific in certain flavour directions:

S4: Altarelli, Feruglio and LM 2009
Adelhart, Bazzocchi and LM 2010

$$\xi^\nu \ll \xi^e$$

$$c^e_{12},\, c^e_{13} \neq 0 \qquad c^e_{13} = 0$$

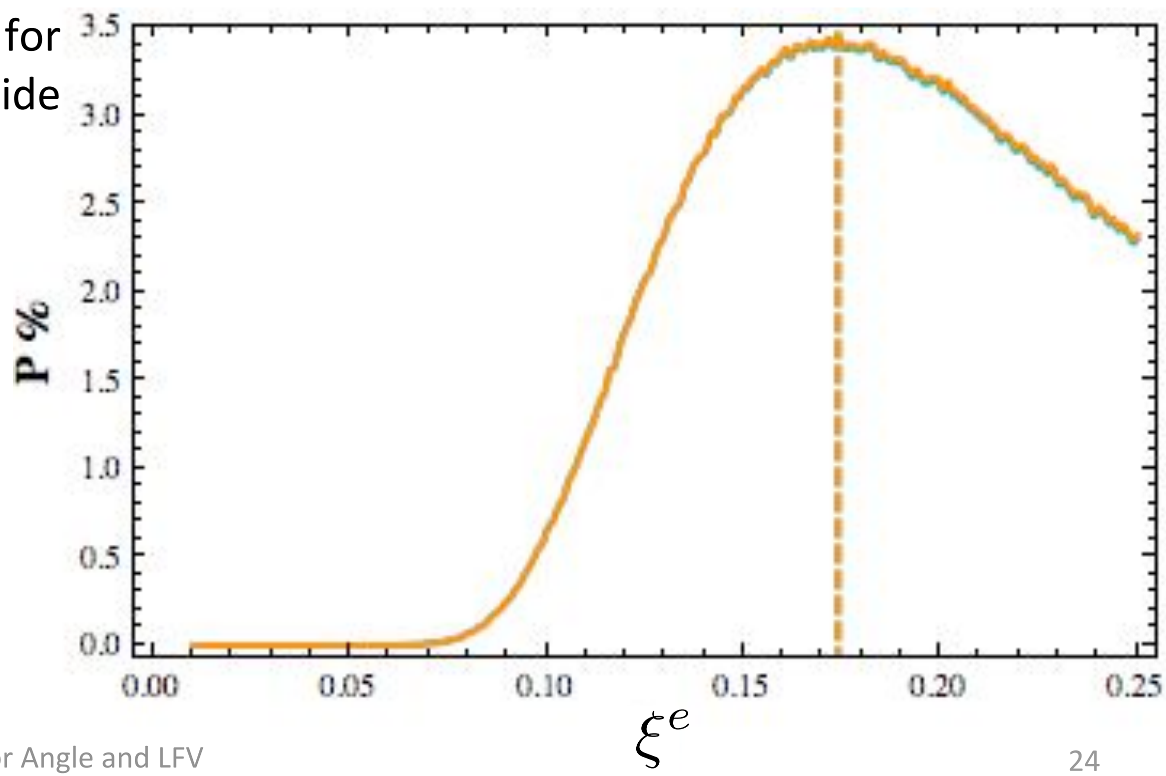
$$c^\nu_{12} \approx c^\nu_{23} \approx c^\nu_{13}$$

To maximize the success rate for all the three mixing angles inside the $3\,\sigma$:

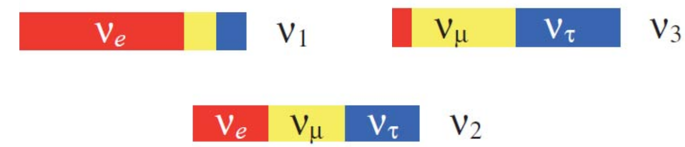
$$\xi^e \simeq 0.17$$

(Similar results when the corrections come from the neutrino sector instead of the charged lepton sector.)

[Bazzocchi & LM, arXiv:1205.5135]



Typical Tri-Bimaximal



$$\sin^2\theta_{12}^{TB} = 1/3$$
$$\sin^2\theta_{23}^{TB} = 1/2$$
$$\sin\theta_{13}^{TB} = 0$$

$$U^{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

In the basis of diagonal charged leptons:

$$M_\nu^{TB} = \begin{pmatrix} x & y & y \\ y & z & x+y-z \\ y & x+y-z & z \end{pmatrix}$$

mu-tau sym
magic sym

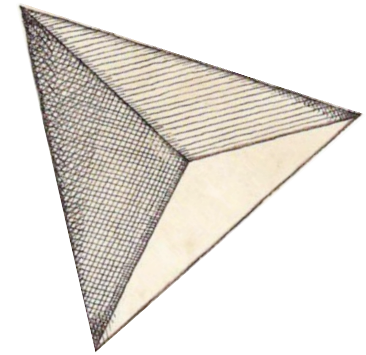
Discrete Symmetries:

[A_4: Adhikary; Altarelli; Aristizabal Sierra; Babu; Bazzocchi; Bertuzzo; Di Bari; Branco; Brahmachari; Chen; Choubey; Ciafaloni; Csaki; Delaunay; Felipe; Feruglio; Frampton; Frigerio; Ghosal; Grimus; Grojean; Grossmann; Hagedorn; He; Hirsch; Honda; Joshipura; Kaneko; Keum; King; Koide; Kuhbock; Lavoura; Lin; Ma; Machado; Malinsky; Matsuzaki; de Medeiros Varzielas; Meloni; LM; Mitra; Molinaro; Morisi; Nardi; Parida; Paris; Petcov; Pleitez; Picariello; Rajasekaran; Riazzudin, Romao; Serodio; Skadhauge; Tanimoto; Torrente-Lujan; Urbano; Valle; Villanova del Moral; Volkas; Yin; Zee; ...;
S_4, T', $\Delta(3n^2)$: de Adelhart Toorop; Altarelli, Bazzocchi; Chen; Ding; Hagedorn; Feruglio; Frampton; Kephart; King; Lam; Lin; Luhn; Ma; Mahanthappa; Matsuzaki; de Medeiros Varzielas; LM; Morisi; Nasri; Ramond; Ross;
...]

The Altarelli-Feruglio Model

[Altarelli & Feruglio 2005]

A_4 is the group of even permutations of 4 objects isomorphic to the group of the rotations which leave a regular tetrahedron invariant (Subgroup of SO(3)).
It has 12 elements and 4 representations: 3, 1, 1', 1''



$$G_f = A_4 \times G_{aux}$$

- keeps separated the two sectors
- explains the hierarchy

$$m_e \ll m_\mu \ll m_\tau$$

$\langle \varphi_\ell \rangle$ → $G_\ell = Z_3$ → M_e^{diag}

$\langle \varphi_\nu \rangle$ → $G_\nu = Z_2$ → M_ν^{TB}

$$\Rightarrow \quad U = U_e^\dagger U_\nu = U^{TB}$$

The Altarelli-Feruglio Model

[Altarelli & Feruglio 2005]

	Matter fields				Higgs	Flavons			
	ℓ	e^c	μ^c	τ^c	$h_{u,d}$	θ	φ_T	φ_S	ξ
A_4	3	1	$1''$	$1'$	1	1	3	3	1

$$w_e = y_e \frac{\theta^2}{\Lambda^3} e^c (\varphi_T \ell) h_d + y_\mu \frac{\theta}{\Lambda^2} \mu^c (\varphi_T \ell)' h_d + y_\tau \frac{1}{\Lambda} \tau^c (\varphi_T \ell)'' h_d$$

$$w_\nu = x_a \frac{\xi}{\Lambda} \frac{h_u \ell h_u \ell}{\Lambda_L} + x_b \left(\frac{\varphi_S}{\Lambda} \frac{h_u \ell h_u \ell}{\Lambda_L} \right)$$

Expansion in ϕ/Λ

vacuum alignment:

$$\frac{\langle \varphi_T \rangle}{\Lambda} = (u, 0, 0)$$

$$\frac{\langle \varphi_S \rangle}{\Lambda} = c_b (u, u, u)$$

$$\frac{\langle \xi \rangle}{\Lambda} = c_a u$$

$$\frac{\langle \theta \rangle}{\Lambda} = t$$

$$M_e = \mathrm{diag}(y_e t^2, \, y_\mu t, \, y_\tau) v_d u$$

$$\frac{m_e}{m_\mu} = \frac{m_\mu}{m_\tau} = t \approx 0.05$$

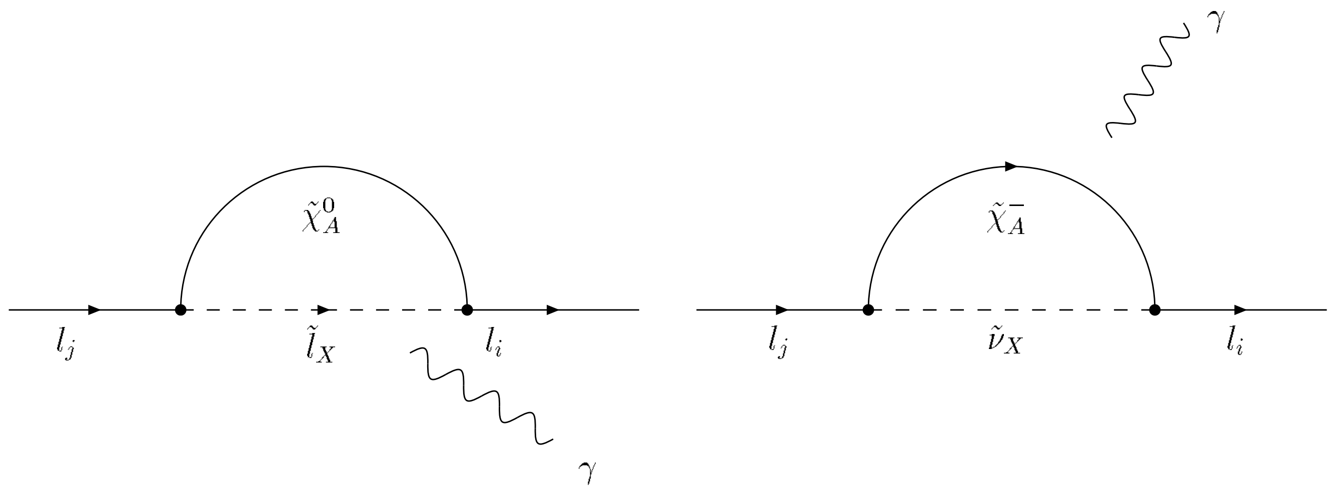
$$M_\nu = \begin{pmatrix} a + \frac{2}{3} b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3} b & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & \frac{2}{3} b \end{pmatrix} v_u^2$$

$$M_\nu^{diag} = v_u^2 \mathrm{diag}(a + b, \, a, \, -a + b)$$

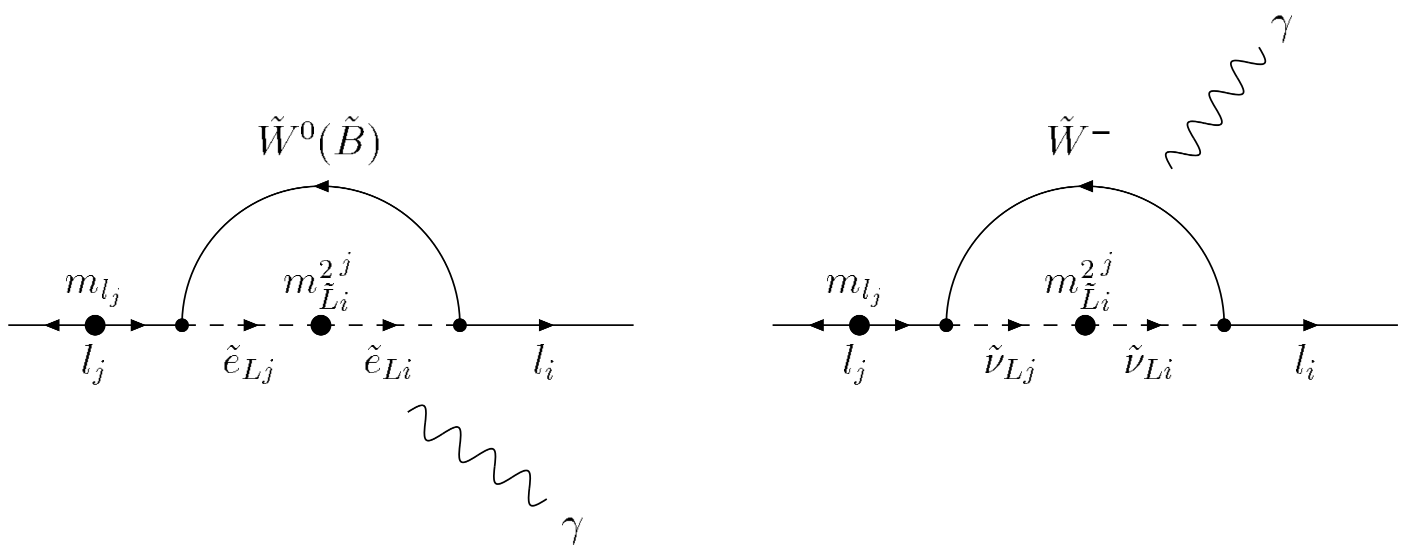
$$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{35}$$

Mass Insertion Approximation

To get EDM, MDM and the LFV transitions we should calculate diagrams as:



A Good analytical approach is the Mass Insertion approximation:



SUSY Parameters

Many parameters: $M_1, M_2, \mu, \tan\beta, m_L^2, m_R^2, A_0$

All of them are not independent:
$$m_L^2(\Lambda_f) = m_R^2(\Lambda_f) = A_0 \equiv m_0$$
$$\tan\beta \approx 100\,\eta\,y_\tau$$

SUGRA context:
$$m_L^2(m_W) \simeq m_L^2(\Lambda_f) + 0.5 M_2^2(\Lambda_f) + 0.04 M_1^2(\Lambda_f)$$

$$m_R^2(m_W) \simeq m_R^2(\Lambda_f) + 1.5 M_1^2(\Lambda_f)$$

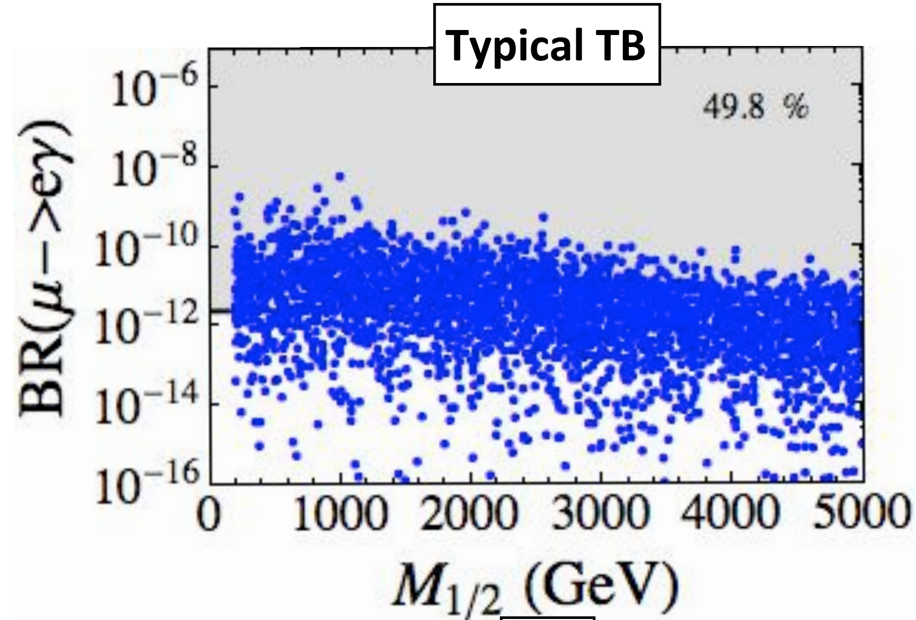
$$M_i(m_W) \simeq \frac{\alpha_i(m_W)}{\alpha_i(\Lambda_f)} M_i(\Lambda_f)$$

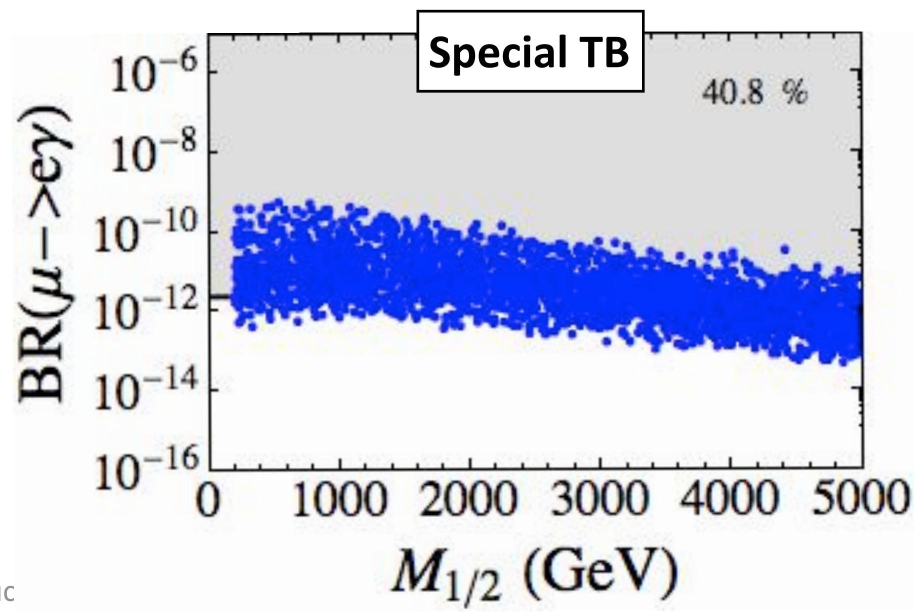
$$\rightarrow \quad M_i(\Lambda_f) \equiv M_{1/2} \qquad \alpha_i(\Lambda_f) = \frac{1}{25}$$

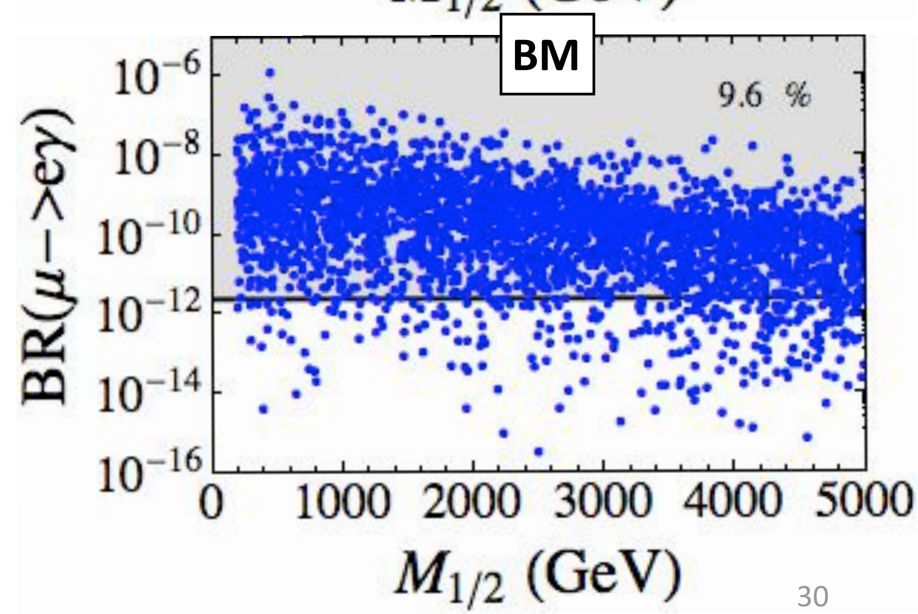
$$|\mu|^2 \simeq \frac{1 + 0.5\tan^2\beta}{\tan^2\beta - 1} m_0^2 + \frac{0.5 + 3.5\tan^2\beta}{\tan^2\beta - 1} M_{1/2}^2 - \frac{1}{2} m_Z^2$$

$m_0 = 5000\ \mathrm{GeV}$ & $\tan\beta = 15$

$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$$







Numerical Accidents?

Are these patterns only numerical accidents? If **Yes** what?

→ **Anarchy (?)**

[Review: Altarelli, Feruglio & Masina 2002]
[Recentrly: Buchmuller, Domcke & Schmitz 2011]

Consist in using a simple U(1) as flavour symmetry:

- many parameters already at the LO
- low predictive power
- no correlations among observables

→ but the mixing angles and the r parameter can be accommodated

$$SU(5) \times U(1)$$

$$\Psi_{10} = (5,3,0)$$

$$\Psi_{\overline{5}} = (2,0,0)$$

$$\Psi_1 = (1,-1,0)$$

$$m_\nu = \begin{pmatrix} \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 0 & 0 \\ \epsilon^2 & 0 & 0 \end{pmatrix}$$

$$\epsilon \approx 0.5$$