



# Predictions for Neutrino Masses and $\beta\beta 0\nu$ -Decay in a $SUSY SU(5) \times T'$ Unified Model of Flavour with large $\theta_{13}$

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arXiv:1205.5241

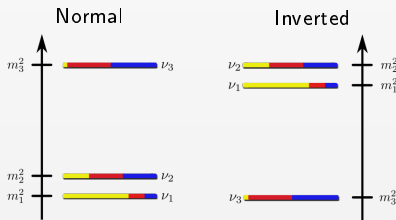
- Motivations for a  $SU(5) \otimes T'$  Unified Model of Flavour
- General Setup of the Model
- Neutrino Sector
- Conclusions

Global fit analysis including the Daya Bay and RENO results  
 D. V. Forero, M. Tortola, J. W. F. Valle [ArXIV: 1205.4018](https://arxiv.org/abs/1205.4018).

Parameter	best-fit ( $\pm 1\sigma$ )	$3\sigma$
$\Delta m_{\odot}^2 [10^{-5} \text{ eV}^2]$	$7.62 \pm 0.19$	7.12 - 8.20
$ \Delta m_A^2  [10^{-3} \text{ eV}^2]$	$2.53^{+0.08}_{-0.10}$	2.26 - 2.77
$\sin^2 \theta_{12}$	$0.320^{+0.015}_{-0.017}$	0.27 - 0.37
$\sin^2 \theta_{23}$	$0.49^{+0.08}_{-0.02}$ $0.53^{+0.05}_{-0.07}$	0.39-0.64
$\sin^2 \theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	0.015-0.036 0.016-0.037

## Open Questions in Neutrino Physics

- Majorana or Dirac?
- Absolute values of neutrino masses
- Hierarchy (normal  $m_1 < m_2 < m_3$  or inverted  $m_3 < m_1 < m_2$ )
- CP-phases:  $\delta$ ,  $\alpha_{31}$  and  $\alpha_{21}$



- GUT:  $SU(5)$
- \*  $SU(5)$ , Unified picture of quarks and leptons
  - \* possibly leads to sizeable  $\theta_{13}$

- discrete symmetry:  $T'$
- \* treats quarks and lepton mixing **simultaneously**
  - \* can allow spinorial unitary irreducible reps
  - \* “geometrical” CP violation
  - \* corrections to  $U_{\text{TBM}}$  via lepton sector

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix},$$

## Unified model of flavour

- Model based on  $SU(5) \otimes T'$
- In this model  $\sin^2 \theta_{13} \cong 0.02$

## Discrete symmetry: the group $T'$

- $T'$  is the Double-valued group of  $T \sim A_4$  (even permutations of 4 objects)
- inequivalent UIRs :  $\underbrace{1, 1', 1'', 3}_{\text{TBM for Neutrinos}} + \underbrace{2, 2', 2''}_{2+1 \text{ assignments for } q \text{ and } \ell}$
- **Complex CG coefficients** when spinorial UIRs are involved !

Clebsch-Gordan coefficients can be **complex**  $\Rightarrow$  geometrical origin of **CP** violation!

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{2(2')} \otimes \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix}_{2(2'')} = \begin{pmatrix} \frac{x_1 x'_2 - x_2 x'_1}{\sqrt{2}} \end{pmatrix}_1 \oplus \begin{pmatrix} -\frac{(1+i)}{2}(x_1 x'_2 + x_2 x'_1) \\ x_1 x'_1 \\ -i x_2 x'_2 \end{pmatrix}_3$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_3 \sim \left[ \begin{pmatrix} (1+i)x_2 u_2 + x_1 u_1 \\ (1-i)x_1 u_3 - x_2 u_1 \end{pmatrix}_2 \oplus \begin{pmatrix} (1+i)x_2 u_3 + x_1 u_2 \\ (1-i)x_1 u_1 - x_2 u_2 \end{pmatrix}_{2'} \oplus \begin{pmatrix} (1+i)x_2 u_1 + x_1 u_3 \\ (1-i)x_1 u_2 - x_2 u_3 \end{pmatrix}_{2''} \right]$$

M.-C. Chen, K.T. Mahanthappa Phys.Lett. B652 (2007)

F. Feruglio, C. Hagedorn, Y. Lin, L. Merlo, Nucl.Phys. B775 (2007)

Fundamental Symmetries  $SU(5) \otimes T'$  (SUSY!)

Shaping symmetry  $Z_{12} \otimes Z_8^3 \otimes Z_6^2 \otimes Z_4$

$U(1)_R$  Continuous version of the usual  $R$ -parity

Matter Content  $\mathbf{10}(Q, u^c, e^c)_L$  : ten-plets into  $(T1, T2) \sim \mathbf{2}$ ,  $T3 \sim \mathbf{1}$

$\bar{\mathbf{5}}(d^c, \ell)_L$  : five-plets into  $(F1, F2, F3) \sim \mathbf{3}$

Three Heavy RH Majorana  $N_k$ ,  $(N_1, N_2, N_3) \sim \mathbf{3}$

light active neutrinos via Type I See-Saw

Higgs sector: copies of  $\mathbf{5}$ ,  $\bar{\mathbf{5}}$  and  $\mathbf{24}$  assigned to  $\mathbf{1}$ ,  $\mathbf{1}''$

up and down quark sector  $\langle \tilde{\psi}' \rangle \sim \langle \psi' \rangle \propto \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  ,  $\langle \tilde{\psi}'' \rangle \sim \langle \psi'' \rangle \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\langle \tilde{\phi} \rangle \sim \langle \phi \rangle \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$  , singlets e.g.  $\langle \tilde{\zeta}' \rangle$

in the  $\nu$  sector  $\langle \xi \rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \xi_0$  ,  $\langle \rho \rangle = \rho_0$  ,  $\langle \tilde{\rho} \rangle = \tilde{\rho}_0$

**Flavons fields take all real vevs when  $T'$  is broken!**

**Flavon Alignment problem**  $\Leftrightarrow$  “driving fields” ( $S_\rho, S_\xi, D_\xi, \dots$ )  
for example in the neutrino sector

$$\mathcal{W}_{\xi, \rho, \tilde{\rho}} = \frac{D_\xi}{\Lambda} (\xi^2 \epsilon_9 + \xi \rho \epsilon_9 + \xi \tilde{\rho} \epsilon_9) + S_\xi (\xi^2 - M_\xi^2) + S_\rho (\rho^2 + \tilde{\rho}^2 - M_\rho^2) .$$

$$\xi_i = \xi_0 \neq 0 \quad \text{if} \quad \rho_0 = -\tilde{\rho}_0$$

S. Antusch, S. King, C. Luhn, M. Spinrath ArXiv: 1103.5930

RL convention, i.e.  $-\mathcal{L} = Y_{ij} \bar{f}_R^i f_L^j H + \text{H.c.}$

$$Y_u = \begin{pmatrix} (1-i)a_u & i b_u & 0 \\ i b_u & c_u & (1+i)d_u \\ 0 & (1+i)d_u & e_u \end{pmatrix},$$

$$Y_d = \begin{pmatrix} (1+i)a_d & i b'_d & 0 \\ (1-i)b_d & c_d & 0 \\ 0 & 0 & d_d \end{pmatrix} \quad \text{and} \quad Y_\ell = \begin{pmatrix} -\frac{3}{2}(1+i)a_d & (1-i)b_d & 0 \\ 6 i b'_d & 6 c_d & 0 \\ 0 & 0 & -\frac{3}{2}d_d \end{pmatrix}$$

S. Antusch, M. Spinrath arXiv:0902.4644

$$|V_{us}| \sim \left| \frac{b_d}{c_d} \right|$$

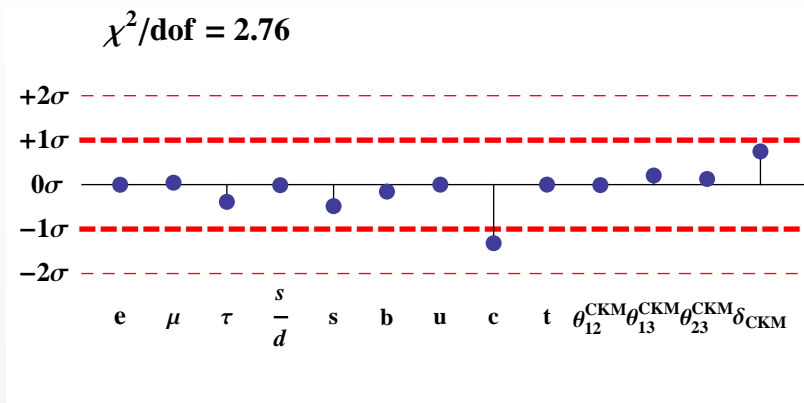
$$\theta_{12}^e \simeq \left| \frac{6 i b'_d}{6 c_d} \right| \sim \left| \frac{b'_d}{b_d} \right| \theta^c, \quad (b'_d = 0.9 b_d)$$

$$\theta_{13} = \frac{1}{\sqrt{2}} \theta_{12}^e = \frac{0.9}{\sqrt{2}} \theta^c$$

Talk by M. Spinrath

D. Marzocca, S. T. Petcov, A. Romanino, M. Spinrath arXiv:1108.0614  
Antusch, Maurer arXiv:1107.3728





A. M., S. T. Petcov, M. Spinrath arXiv:1205.5241

$$\mathcal{W}_\nu = \underbrace{\lambda_1 NN\xi + NN(\lambda_2\rho + \lambda_3\tilde{\rho})}_{\text{RH Majorana Mass } M_R} + \underbrace{\frac{y_\nu}{\Lambda}(N\bar{F})_1(H_5^{(2)}\rho)_1 + \frac{\tilde{y}_\nu}{\Lambda}(N\bar{F})_1(H_5^{(2)}\tilde{\rho})_1}_{\text{Dirac Yukawa coupling } M_D}$$

$$\langle \xi \rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \xi_0, \quad \langle \rho \rangle = \rho_0, \quad \langle \tilde{\rho} \rangle = \tilde{\rho}_0$$

$$M_R = \begin{pmatrix} 2Z + X & -Z & -Z \\ -Z & 2Z & -Z + X \\ -Z & -Z + X & 2Z \end{pmatrix}, \quad M_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{\rho'}{\Lambda}$$

$X, Z$  and  $\rho'$  are real parameters

$M_R$  is form diagonalizable

$$U_{\text{TBM}}^T M_R U_{\text{TBM}} = \text{Diag}(3Z + X, X, 3Z - X)$$

## Diagonal Majorana Mass Matrix

$$\begin{pmatrix} 3Z + X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & 3Z - X \end{pmatrix} \rightarrow |X| \begin{pmatrix} |1 + \alpha e^{i\phi}| e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & |1 - \alpha e^{i\phi}| e^{i\phi_3} \end{pmatrix}$$

$$\alpha \equiv |3Z/X| > 0, \quad \phi \equiv \arg(Z) - \arg(X)$$

$$\phi_i = 0, \pi$$

Light neutrino Majorana mass term via type I see-saw mechanism:

$$M_\nu = -M_D^T M_R^{-1} M_D = U_\nu^* \text{Diag}(m_1, m_2, m_3) U_\nu^\dagger$$

$$U_\nu = i U_{\text{TBM}} \text{Diag}(e^{i\phi_1/2}, e^{i\phi_2/2}, e^{i\phi_3/2})$$

$$m_i = \left(\frac{\rho'}{\Lambda}\right)^2 \frac{1}{M_i}, \quad i = 1, 2, 3 \quad m_i > 0$$

(remember :  $\alpha > 0$   $\cos \phi = \pm 1!$ )

$$\Delta m_{31}^2 \equiv \Delta m_A^2 = \frac{1}{|X|^2} \left( \frac{\rho'}{\Lambda} \right)^4 \frac{4\alpha \cos \phi}{|1 + \alpha e^{i\phi}|^2 |1 - \alpha e^{i\phi}|^2}.$$

$\left\{ \begin{array}{l} \text{for } \cos \phi = +1 \quad \text{mass spectrum with NO} \\ \text{for } \cos \phi = -1 \quad \text{mass spectrum with IO} \end{array} \right.$

$$\Delta m_{21}^2 \equiv \Delta m_{\odot}^2 = \frac{1}{|X|^2} \left( \frac{\rho'}{\Lambda} \right)^4 \frac{\alpha (\alpha + 2 \cos \phi)}{|1 + \alpha e^{i\phi}|^2}.$$

$$r = \frac{\Delta m_{\odot}^2}{\Delta m_A^2} = \frac{1}{4} (\alpha + 2 \cos \phi) (1 - 2\alpha \cos \phi + \alpha^2) = \mathbf{0.032 \pm 0.006}.$$

## NO Solution A

$$m_1 \cong 4.44 \times 10^{-3} \text{ eV}$$

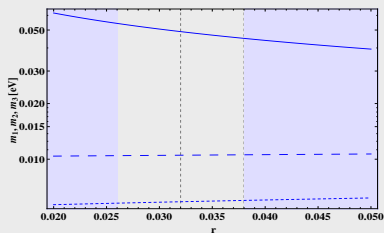
$$m_2 \cong 9.77 \times 10^{-3} \text{ eV}$$

$$m_3 \cong 4.89 \times 10^{-2} \text{ eV}$$

RH Majorana  $N_k$ :  $M_3 < M_2 < M_1$

$$M_1/M_3 \cong 11.0$$

$$M_2/M_3 \cong 5.0.$$



## NO Solution B

$$m_1 \cong 5.89 \times 10^{-3} \text{ eV}$$

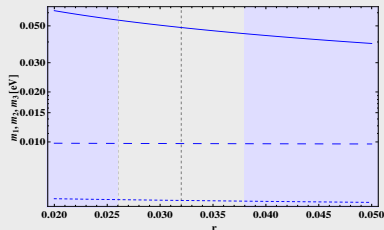
$$m_2 \cong 1.05 \times 10^{-2} \text{ eV}$$

$$m_3 \cong 4.90 \times 10^{-2} \text{ eV}$$

RH Majorana  $N_k$ :  $M_3 < M_2 < M_1$

$$M_1/M_3 \cong 8.33$$

$$M_2/M_3 \cong 4.67.$$



IO

$$m_1 \cong 5.17 \times 10^{-2} \text{ eV}$$

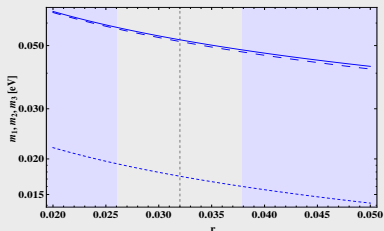
$$m_2 \cong 5.24 \times 10^{-2} \text{ eV}$$

$$m_3 \cong 1.74 \times 10^{-2} \text{ eV}$$

 RH Majorana  $N_k$ :  $M_1 \cong M_2 < M_3$ 

$$M_1/M_2 \cong 1.014$$

$$M_3/M_2 \cong 3.01.$$



$$U^{\text{PMNS}} = U_{eL}^\dagger U_\nu, \quad U_{eL} = \text{diag}(1, e^{i\varphi}, 1) R_{12}(\theta_{12}^e), \quad U_\nu = U_{\text{TBM}} \tilde{Q}$$

$$U_{\text{appx}}^{\text{PMNS}} = \begin{pmatrix} \frac{e^{-i\frac{\beta_1}{2}}}{\sqrt{6}} & \frac{e^{-i\frac{\beta_2}{2}}}{\sqrt{3}} & \frac{\sin\theta_{12}^e e^{-i(\pi-\theta_{12}^e)/2}}{\sqrt{2}} \\ * & * & * \\ * & * & * \end{pmatrix}$$

$$\theta_{12} = \arcsin(1/\sqrt{3}) + \sqrt{2}/8(\theta_{12}^e)^2$$

$$\delta = \frac{\pi}{2} - \frac{1}{2}\theta_{12}^e$$

$$\theta_{13} = \theta_{12}^e/\sqrt{2}$$

$$\beta_1 = 2\pi - 2\theta_{12}^e + \phi_3$$

$$\theta_{23} = \pi/4 - 1/4(\theta_{12}^e)^2$$

$$\beta_2 = 2\pi + \theta_{12}^e + \phi_3 - \phi_2$$

Quantity	Experiment ( $2\sigma$ ranges)	Model
$\sin^2 \theta_{12}$	0.275 – 0.342	0.340
$\sin^2 \theta_{23}$	0.36 – 0.60	0.490
$\sin^2 \theta_{13}$	0.015 – 0.032	0.020
$\delta$	-	$84.3^\circ$
$\beta_1$	-	$337.1^\circ + \phi_3$
$\beta_2$	-	$11.5^\circ + \phi_3 - \phi_2$
$J_{\text{CP}}$	-	0.0324

NO Solution A

$$\sum_{k=1}^3 m_k = 6.31 \times 10^{-2} \text{ eV}$$

$$|\langle m \rangle| = 4.90 \times 10^{-3} \text{ eV}$$

NO Solution B

$$\sum_{k=1}^3 m_k = 6.54 \times 10^{-2} \text{ eV}$$

$$|\langle m \rangle| = 7.95 \times 10^{-3} \text{ eV}$$

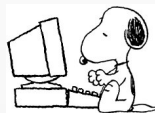
IO

$$\sum_{k=1}^3 m_k = 12.1 \times 10^{-2} \text{ eV}$$

$$|\langle m \rangle| = 2.17 \times 10^{-2} \text{ eV}$$



- We construct a unified model of flavor with large  $\theta_{13}$  based on  $S(5) \otimes T'$  with type I See-Saw
- $T'$  leads to TBM in the neutrino sector + corrections coming from the charged lepton sector
- Geometrical CP violation via CGs of the  $T'$  group
  - Essential ingredient: real flavon vev alignment
- Predictions:
  - $\sin^2 \theta_{13} \sim 0.02$
  - $\delta \cong \pi/2 - 0.45\theta^c \sim 84.3^\circ$
  - Neutrino mass spectra: NO and IO possible
  - unambiguous predictions for  $\beta\beta 0\nu$ -Decay  $| \langle m \rangle |$



Thank you

Back up slides

# Discrete symmetry: UIRs of the group $T'$

- $T'$  is the Double-valued group of  $T \sim A_4$  (even permutations of 4 objects)
- inequivalent UIRs:  $\underbrace{1, 1', 1'', 3}_{\text{TBM for Neutrinos}} + \underbrace{2, 2', 2''}_{2+1 \text{ assignments for } q \text{ and } \ell}$
- **Complex CG coefficients** when spinorial UIRs are involved !
- 3 generators:  $s^2 = r, \quad r^2 = t^3 = (st)^3 = e, \quad rt = tr$

$$3: \quad r = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad s = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix}$$

where  $1 + \omega + \omega^2 = 0$  conventionally.

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$$\begin{aligned}
 1 \otimes \Gamma^p &= \Gamma^p, & 1 \otimes 1'(1'') &= 1'(1''), & 1' \otimes 1'(1'') &= 1''(1), & 1'(1'') \otimes 1'' &= 1(1') \\
 2 \otimes 1'(1'') &= 2'(2''), & 2' \otimes 1'(1'') &= 2''(2), & 2'' \otimes 1'(1'') &= 2(2') \\
 2(2') \otimes 2(2'') &= 1 \oplus 3, & 2'(2) \otimes 2'(2'') &= 1 \oplus 3, & 2''(2) \otimes 2''(2') &= 1 \oplus 3 \\
 3 \otimes 1'(1'') &= 3, & 3 \otimes 2 &= 2 \oplus 2' \oplus 2'', & 3 \otimes 2'(2'') &= 2'(2'') \oplus 2''(2) \oplus 2(2') \\
 & & 3 \otimes 3 &= 1 \oplus 1' \oplus 1'' \oplus 3 \oplus 3
 \end{aligned}$$


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$T'$  group: Clebsch-Gordan series  $\Gamma^p \otimes \Gamma^q = \Gamma^q \otimes \Gamma^p$ . Here  $\Gamma^p = 1, 2, 3$

# Table of Charge Assignments: Matter and Higgs fields

	$T_3$	$T_a$	$\bar{F}$	$N$	$H_5^{(1)}$	$H_5^{(2)}$	$H_5^{(3)}$	$\bar{H}_5^{(1)}$	$\bar{H}_5^{(2)}$	$\bar{H}_5^{(3)}$	$\bar{H}_5''$	$H_{24}''$	$\tilde{H}_{24}''$
$SU(5)$	10	10	5	1	5	5	5	5	5	5	5	24	24
$T'$	1	2	3	3	1	1	1	1	1	1	1''	1''	1''
$U(1)_R$	1	1	1	1	0	0	0	0	0	0	0	0	0
$Z_{12}^u$	2	11	1	9	8	8	2	9	3	6	3	0	3
$Z_8^d$	4	0	2	6	0	4	0	1	4	7	7	4	2
$Z_8^v$	7	6	2	0	2	6	4	1	1	5	7	4	0
$Z_8$	0	5	2	2	0	0	6	0	0	6	6	4	2
$Z_6$	5	0	1	0	2	5	2	2	0	2	2	0	0
$Z_6'$	2	3	1	0	2	5	2	5	0	2	2	0	0
$Z_4$	3	3	0	0	2	0	2	0	1	1	0	0	1

# Table of Charge Assignments: Flavons

	$\tilde{\phi}$	$\tilde{\psi}''$	$\tilde{\psi}'$	$\tilde{\zeta}''$	$\tilde{\zeta}'$	$\phi$	$\psi''$	$\psi'$	$\zeta''$	$\zeta'$	$\xi$	$\rho$	$\tilde{\rho}$
$SU(5)$	1	1	1	1	1	1	1	1	1	1	1	1	1
$T'$	3	2''	2'	1''	1'	3	2''	2'	1''	1'	3	1	1
$U(1)_R$	0	0	0	0	0	0	0	0	0	0	0	0	0
$Z_{12}^u$	0	3	9	0	0	6	3	9	6	0	6	6	6
$Z_8^d$	0	0	0	0	0	2	1	7	6	4	4	4	4
$Z_8^v$	4	1	7	0	0	2	7	1	6	4	0	0	0
$Z_8$	4	7	5	4	0	2	5	3	6	4	4	4	4
$Z_6$	4	4	2	4	2	0	3	3	0	0	0	0	0
$Z_6'$	4	4	2	4	2	3	0	0	0	0	0	0	0
$Z_4$	0	2	2	0	0	0	3	1	2	0	0	0	0

$$\begin{aligned} \mathcal{W}_{Y_u} = & y_{33} H_5^{(1)} T_3 T_3 + \frac{y_{23}}{\Lambda^2} (T_a \tilde{\phi})_{2'} H_5^{(2)} (T_3 \tilde{\psi}'')_{2''} + \frac{y_{22}}{\Lambda^3} (T_a \tilde{\psi}'')_3 (H_5^{(1)} \tilde{\zeta}')_{1'} (T_a \tilde{\psi}'')_3 \\ & + \frac{y_{21}}{\Lambda^4} (T_a \tilde{\phi})_{2'} (H_5^{(1)} \tilde{\zeta}')_{1'} (\tilde{\psi}' (T_a \tilde{\psi}')_3)_{2'} + \frac{y_{11}}{\Lambda^4} ((T_a \tilde{\phi})_2 \tilde{\zeta}'')_{2''} H_5^{(3)} (\tilde{\zeta}'' (T_a \tilde{\phi})_{2''})_{2'} , \end{aligned}$$

$$\begin{aligned} \mathcal{W}_{Y_{d,e}} = & \frac{y_{33}}{\Lambda^2} ((\bar{H}_5^{(2)} \bar{F})_3 \phi)_{1'} (H_{24}'' T_3)_{1''} + \frac{y_{22}}{\Lambda^3} ((\phi T_a)_2 H_{24}'')_2 (\psi' (\bar{H}_5^{(1)} \bar{F})_3)_2 \\ & + \frac{y_{12}}{\Lambda^4} (((T_a \tilde{H}_{24}'')_{2''} (\bar{F} \psi')_{2''})_3 \psi')_{2''} (\bar{H}_5^{(3)} \psi')_{2'} + \frac{y_{21}}{\Lambda^4} ((\bar{F} \psi')_{2''} (\zeta'' \bar{H}_5^{(1)})_{1''} \zeta'')_2 (T_a \phi)_2 \\ & + \frac{y_{11}}{\Lambda^4} ((\bar{F} \psi'')_{2'} (H_{24}'' \psi'')_{2'} \bar{H}_5'')_{1'} (T_a \psi'')_{1''} , \end{aligned}$$