Embedding Z' models in GUTs

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Z' model

- No additional fermions (except for right-handed neutrinos) are needed for anomaly cancelation. It's possible only when U(1)^2 group is spanned by Y and B-L .

Abelian Lagrange`an

$$L^{abelian} = -\frac{1}{4}\widetilde{h}_{ab} F_a^{\mu\nu} F_{b\mu\nu} + \sum_f \bar{\Psi}_f \gamma_\mu \left((X^T)_a^f G_{ab} A_b^\mu \right) \Psi_f + L_{int}^{abelian-scalar}$$

$a, b \in \{1, 2\}$ - indices in $U(1)^2$ algebra

We choose $\widetilde{h}_{ab} = \delta_{ab}$ (at tree level)

We have a freedom to transform abelian gauge bosons orthogonaly: $A^\nu_{a'} = O_{a'b} A^\nu_b$

We have a freedom to transform abelian generators linearly: $X_{a'} = L_{a'b} X_b$

Both kind of transformations result in appriopriate change of gauge coupling matrix G_{ab}.

$$G_{a'b'} = (L^T)^{-1}_{a'b} G_{ba} O^T_{ab'}$$

Standard, low energy parametrization

We can choose $X = \begin{bmatrix} Y \\ B-L \end{bmatrix}$

and use the freedom given by O-transformations to demand $G_{21} = 0$.

Standard parametrization

$$(X^T)_a G_{ab} A_b^\mu = \begin{bmatrix} Y \\ B-L \end{bmatrix}^T \begin{bmatrix} g' & g'_{B-L} \\ 0 & g_{B-L} \end{bmatrix} \begin{bmatrix} B_0^\mu \\ Z_0'^\mu \end{bmatrix}$$

g' is equivalent to g' from the SM

g_{B-L}, g'_{B-L} and $M_{Z'}$ are basic parameters additional with respect to the SM

How are they constrained?

Embedding Z’ model in GUTs

- The smallest possible simple unification gauge group is SO(10).

- Grand unification gives constraints on the space of Z’ parameters, to be compared to experimental constraints from the EWPT and the LHC.

- Treshold corrections may play relevant role.

Two specific basis in U(1)^2 algebra

Low energy basis (part of standard parametrization): $X = \begin{bmatrix} Y \\ B-L \end{bmatrix}$

High energy basis (generators of the GUT group): $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

Parametrization, which is natural for SO(10) GUT models is different, than the standard Z' parametrization. After finding constraints from gauge coupling unification in GUT parametrization, one has to make appriopriate transformations to obtain final constraints on g_{B-L} and g'_{B-L}.

Symmetry breaking

Pattern 1 $SO(10)$ Pattern 2

μ_0 $\mu_0 \gtrsim 10^{16}$ GeV

$^*SU(5) \oplus U(1)_X$ $^*SU(3)_c \oplus SU(2)_L \oplus SU(2)_R \oplus U(1)_{B-L}$

$\mu_1 \gtrsim 10^{16}$ GeV μ_1

Z' group: $SU(3)_c \oplus SU(2)_L \oplus U(1)^2$

$\mu_2 = M_{Z'}$

SM Group: $SU(3)_c \oplus SU(2)_L \oplus U(1)_Y$ *Intermediate group

M_Z

Final Group: $SU(3)_c \oplus U(1)_{EM}$

$$U(1)^2 = U(1)_Y \oplus U(1)_X = U(1)_R \oplus U(1)_{B-L}$$

Transormation of U(1) generators

There are two diagonal generators of the intermediate group, that are linear combinations of Y and $B-L$:

In Pattern 1: $$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \widehat{Y} \\ X \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\sqrt{15}}{5} & 0 \\ -\frac{\sqrt{10}}{5} & \frac{\sqrt{10}}{4} \end{bmatrix}}_{L_1^{-1}} \cdot \begin{bmatrix} Y \\ B-L \end{bmatrix}$$

In Pattern 2: $$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} R \\ \widehat{B-L} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{\sqrt{6}}{4} \end{bmatrix}}_{L_2^{-1}} \cdot \begin{bmatrix} Y \\ B-L \end{bmatrix}$$

R - third (diagonal) generator of $SU(2)_R$

The running below the μ_1 scale is considered in the $[Y, B-L]$ basis.

Two specific, supersymmetric models

Model 1 (based on Pattern 1):

Higgs sector: $210_H + 54_H + \overline{126}_H + 126_H + 10_H$ of $SO(10)$

$$SO(10) \xrightarrow{210_H} SU(5) \oplus U(1)_X \xrightarrow{24_{54_H}} \text{Z' Group} \xrightarrow{\chi_{+/-}} \text{SM Group} \xrightarrow{h} \text{Final Group}$$

Model 2 (based on Pattern 2):

Higgs sector: $45_H + 45_H + \overline{126}_H + 126_H + 10_H$ of $SO(10)$

$$SO(10) \xrightarrow{45_H} SU(3)_c \oplus SU(2)_L \oplus SU(2)_R \oplus U(1)_{B-L} \xrightarrow{[1,1,3]_{45_H}} \text{Z' G.} \xrightarrow{\chi_{+/-}} \text{SM G.} \xrightarrow{h} \text{F. G.}$$

In both models:

All SM fermions are embedded into three 16's of $SO(10)$

$h \in [1,2]_{10_H}$ $\qquad$ $\chi_{+/-} = [1,1]_{126_H/\overline{126}_H}$

Treshold corrections

MSSM-fields, RH-(s)neutrinos and $U(1)^2$-breaking higgses: $\chi_{+/-}$ (and their higgsinos) are assumed to have masses below μ_1. Therefore, they contribute to treshold corrections for gauge coupling constants of the Z' Group $(SU(3)_c \oplus SU(2)_L \oplus U(1)_Y \oplus U(1)_{B-L})$ or the SM Group.

All other fields are assumed to have masses between μ_1 and μ_0. Therefore, they contribute to treshold corrections for gauge coupling constants of the Intermediate Group.

To avoid large number of free parameters - masses M_Φ, for each gauge coupling constant g_A one can introduce the so called effective treshold correction. It has a mass-parameter M_A, which is a weighted average of particle masses M_Φ with weights being proportional to corresponding Dynkin indeces. For example, effective treshold corrections for the intermediate group are the following

$$(b_A^{\mu_0} - b_A^{\mu_1}) \ln\left(\frac{M_A}{\mu_1}\right) = \sum_{\Phi:\mu_1<M_\Phi<\mu_0} b_A^\Phi \ln\left(\frac{M_\Phi}{\mu_1}\right) \qquad b_A^\Phi = \frac{d_\Phi}{d_{\Phi A}} S_2^A(\Phi)$$

$$M_A = \prod_{\Phi:\mu_1<M_\Phi<\mu_0} M_\Phi^{\left(\frac{b_A^\Phi}{b_A^{\mu_0} - b_A^{\mu_1}}\right)} \qquad b_A^{\mu_0} - b_A^{\mu_1} = \sum_{\Phi:\mu_1<M_\Phi<\mu_0} b_A^\Phi$$

Some constraints

Planckian constraint: $\mu_0 \leq M_{Pl}$

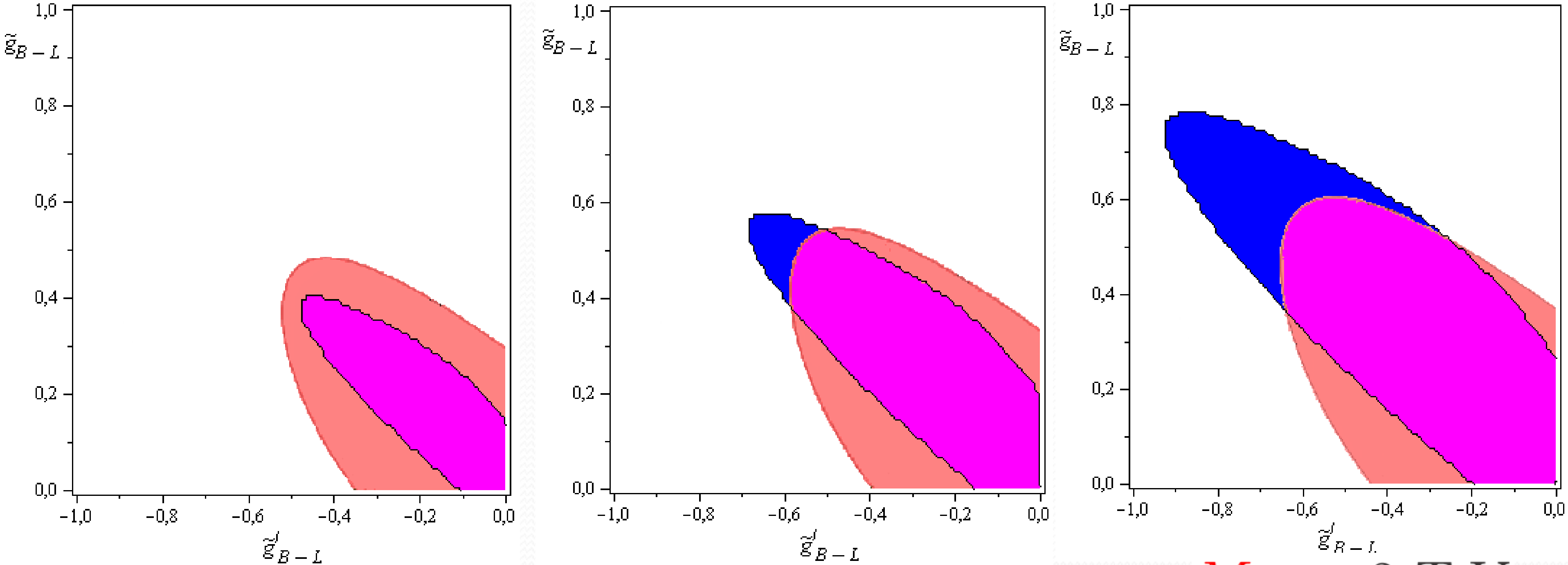
Perturbativity constraint: $g_{10} \leq 4\pi$

All considered values of gauge coupling constants should be perturbative, but the largest one is g_{10} or g_3 (which is perturbative), so the above condition is sufficient.

$\mu_1 \geq 10$ TeV - important only in Pattern 2

The full set of constraints contains both equations and inequalities. For a given value of $M_{Z'}$, it defines the allowed multidimensional polyhedron in the space of parameters spanned by g'_{B-L}, g_{B-L}, treshold mass parameters and jumps. One can project this polyhedron on the 2D plane spanned by g'_{B-L} and g_{B-L} only.

Experimental constraints from ATLAS and EWPT



$M_{Z'} = 1.6$ TeV $M_{Z'} = 1.8$ TeV $M_{Z'} = 2$ TeV

(pink) + (magenta): allowed by EWPT [Salvioni, Villadoro, Zwirner '09]

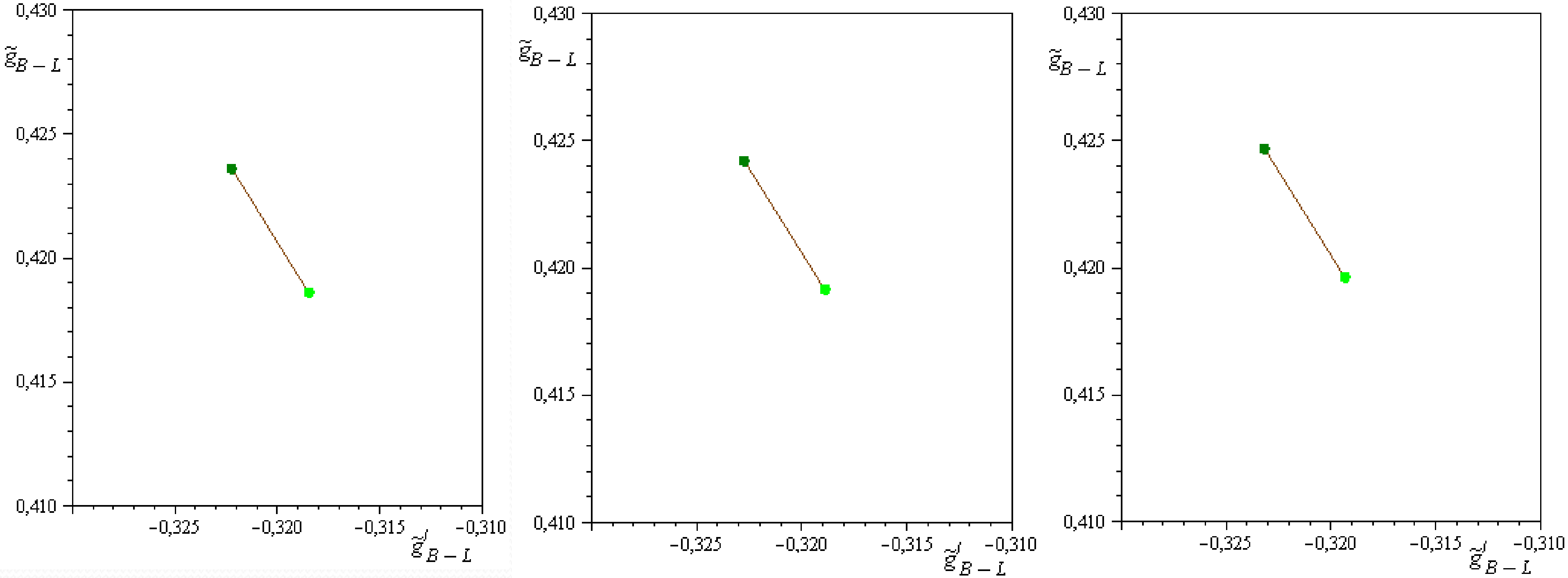
(blue) + (magenta): allowed by ATLAS [ATLAS-CONF-2012-007, 1 March 2012]

95% C.L. exclusion $Z' \longrightarrow l^+ l^-$ $\sigma B <$ Observed limit

σB calculated in narrow width approximation in LO

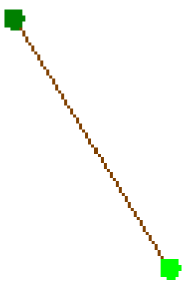
$$\tilde{g}'_{B-L} = \frac{g'_{B-L}}{g_Z} \qquad \tilde{g}_{B-L} = \frac{g_{B-L}}{g_Z}$$

Model 1 – without treshold corrections



$M_{Z'} = 1.6$ TeV $M_{Z'} = 1.8$ TeV $M_{Z'} = 2$ TeV

$\mu_0 = \mu_1$



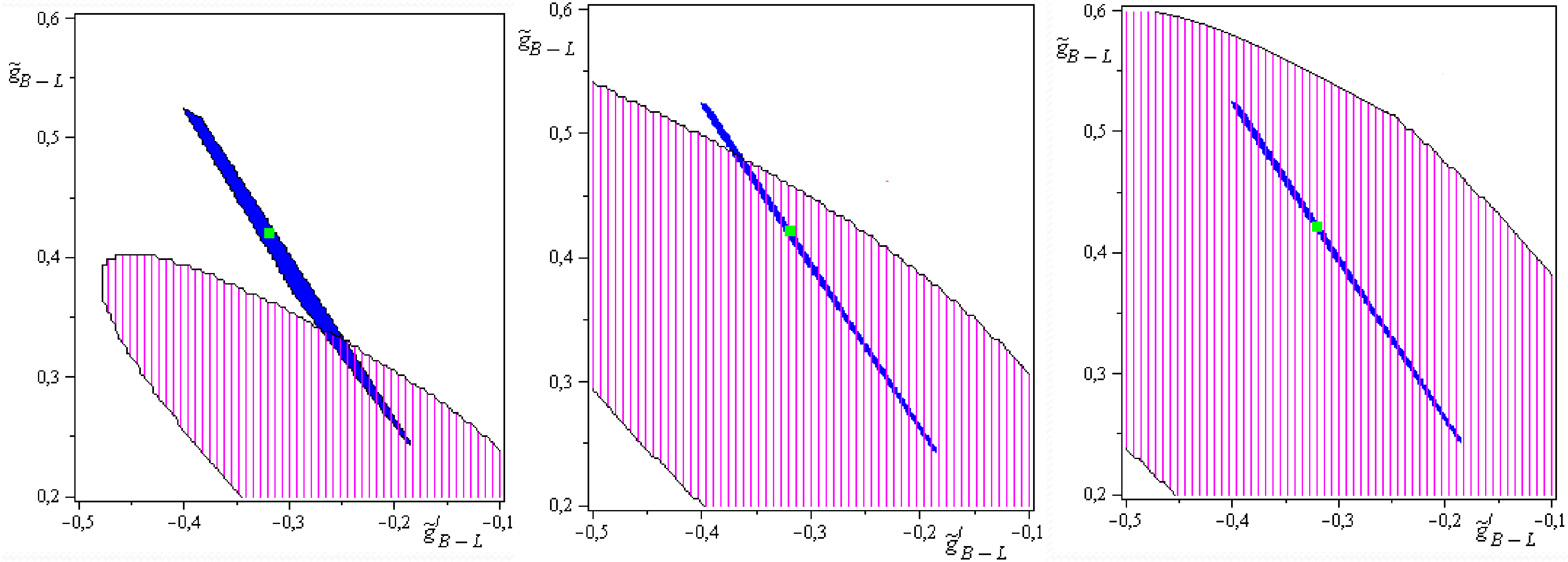
In the $[\mu_1, \mu_0]$ interval: All fields

In the $[M_Z, \mu_1]$ interval: MSSM $+ 3 \cdot \nu_R + \chi_+ + \chi_-$

Perturbativity limit

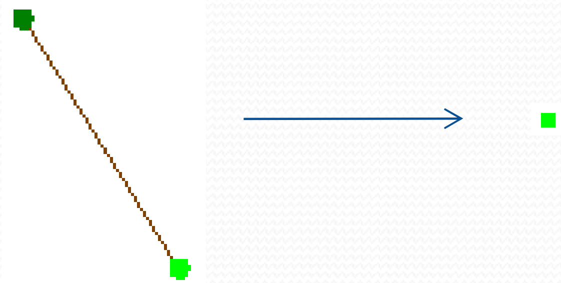
1-loop unification at μ_1 is only approximate.

Constraints on Model 1



$M_{Z'} = 1.6$ TeV $\qquad M_{Z'} = 1.8$ TeV $\qquad M_{Z'} = 2$ TeV

$\mu_0 = \mu_1$

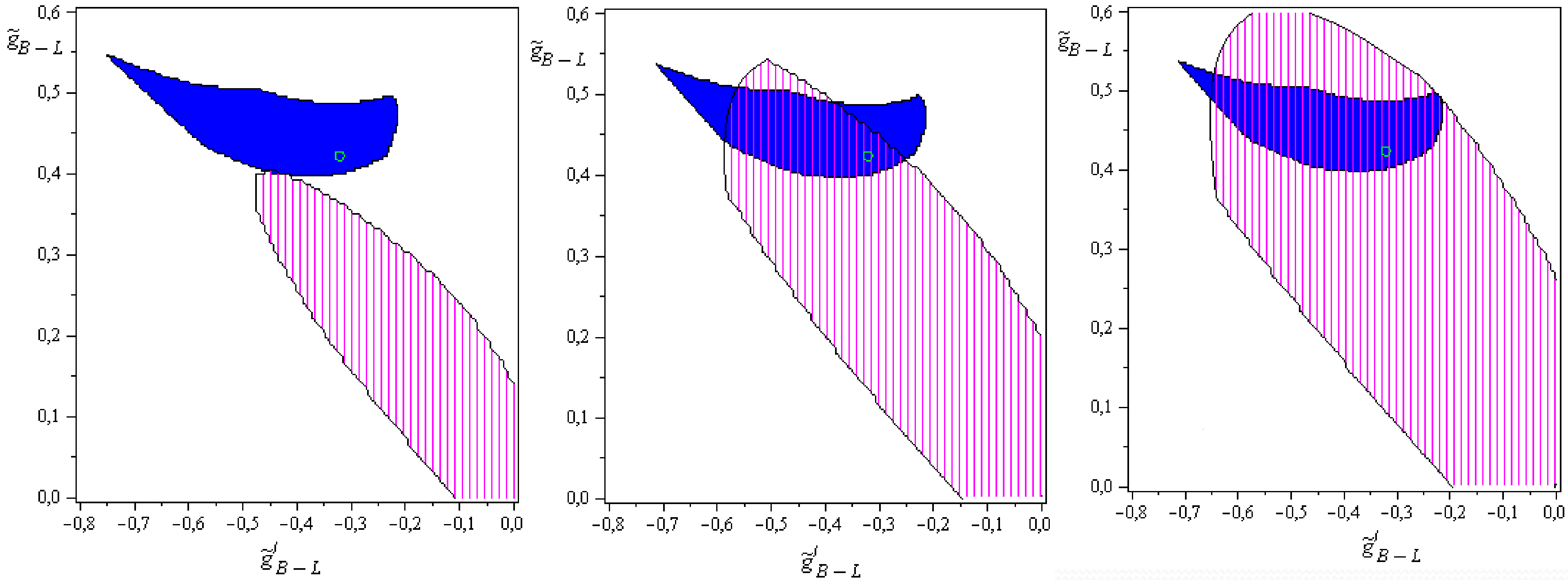


allowed by unification

allowed by ATLAS and EWPT

Perturbativity limit

Constraints on Model 2



$M_{Z'} = 1.6$ TeV $M_{Z'} = 1.8$ TeV $M_{Z'} = 2$ TeV

allowed by unification

allowed by ATLAS and EWPT

unification without treshold corrections
1-loop unification at μ_1 and μ_0 is exact.

Summary and conclusions

- Constraints from Grand Unification decrease experimentally-allowed region of the parameter space in the minimal Z' model. They can provide a lower bound on the Z' mass (like 1.6 TeV in Model 2).
- Treshold corrections are important. They give additional freedom, enlarging allowed region of the parameter space .
- Initial 2-loop results are in agreement with 1-loop ones. Allowed regions are a little bit smaller, because 2-loop corrections make gauge coupling constants larger, so perturbativity constraints are stronger.
- Presented methods can be used beyond minimal Z' models unless there are 3 or more U(1)'s at the same range of scales.

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Backup 1: Gauge boson redefinitions

$$\{W_3^\mu, A_{X_1}^\mu, A_{X_2}^\mu\}$$

rotation $O_{a'b}$ - to obtain $G_{21} = 0$
(standard parametrization)

$$\left\{W_3^\mu, B_0^\mu, Z_0'^\mu\right\}$$

Weinberg rotation (θ_W) - as in SM

$$\left\{A_Q^\mu, Z_0^\mu, Z_0'^\mu\right\}$$

diagonalisation of squared mass matrix $M^2_{Z_0\,Z_0'}$ (θ')

$$\left\{A_Q^\mu, Z^\mu, Z'^\mu\right\} \quad M^2_{Z_0\,Z_0'} = \begin{bmatrix} \frac{1}{2}v_H^2 g_Z^2 & -\frac{1}{2}v_H^2 g_Z g'_{B-L} \\ -\frac{1}{2}v_H^2 g_Z g'_{B-L} & \frac{1}{2}v_H^2 g'^2_{B-L} + 2\left((B-L)_\chi\right)^2 v_\chi^2 g_{B-L}^2 \end{bmatrix}$$

$$M_{A_Q} < 1 \cdot 10^{-18}\ \text{eV}$$

$$M_Z = 91.1876 \pm 0.0021\ \text{GeV} \qquad M_{Z'} = ?$$

[Amsler et al. '08]

Backup 2: Symmetry breaking – higgs fields

Names	Groups and possible higgses				Scales
Initial group	$SO(10)$				$\mu_0 - M_{Pl}$
Possible higgses	45, 210	45	45	54, 210	μ_0
Possible intermediate (maximal little) groups	$SU(5) \oplus U(1)_X$	$SU(3)_c \oplus SU(2)_L \oplus SU(2)_R \oplus U(1)_{B-L}$	$SU(4) \oplus SU(2)_L \oplus U(1)_R$	$SU(4) \oplus SU(2)_L \oplus SU(2)_R$	$\mu_1 - \mu_0$
Possible higgses	$24_{45}, 24_{54}, 24_{210}, 75_{210}$	$[1,1,3]_{45}, [1,1,3]_{210}$	$[15,1]_{45}, [15,1]_{210}$	$[15,1,3]_{210}$ $([15,1,1]+[1,1,3])_{45}$	μ_1
$X_{\hat{a}}$ generators	$\widehat{Y}, X$	$R, \widehat{B-L}$			
Z' group	$SU(3)_c \oplus SU(2)_L \oplus U(1)^2$				$\mu_2 - \mu_1$
Possible higgses	$[1,1]_{16}, [1,1]_{\overline{16}}, [1,1]_{126}, [1,1]_{\overline{126}}, [1,1]_{144}, [1,1]_{\overline{144}}, [1,1]_{560}, [1,1]_{\overline{560}}$				μ_2
SM group	$SU(3)_c \oplus SU(2)_L \oplus U(1)_Y$				$M_Z - \mu_2$
Possible higgses	$[1,2]_{10}, [1,2]_{120}, [1,2]_{\overline{126}}, [1,3]_{\overline{126}}$				M_Z
Final group	$SU(3)_c \oplus U(1)_{EM}$				$\mu < M_Z$

▭ - chosen Higgs fields

In many $SO(10)$ models $\overline{126} = \overline{126}$

Backup 3: RGE's for gauge couplings constants

RGE's for non-abelian gauge couplings:

$$\mu \frac{d}{d\mu} g_A = \frac{1}{(4\pi)^2} g_A^3 b_A \qquad (b_A)_{SUSY}^{1-loop} := \sum_{\Phi} \left(\frac{d_\Phi}{d_{\Phi A}} S_2^A(\Phi) \right) - 3C_2(A)$$

$$b_A^{1-loop} := \frac{2}{3} \sum_f \left(\frac{d_f}{d_{fA}} S_2^A(f) \right) + \frac{1}{3} \sum_s \left(\frac{d_s}{d_{sA}} S_2^A(s) \right) - \frac{11}{3} C_2(A)$$

RGE's for abelian gauge couplings:

$$\mu \frac{d}{d\mu} G_{ab} = \frac{1}{(4\pi)^2} G_{ac} G_{cd}^T b_{de} G_{eb} \qquad (b_{de})_{SUSY}^{1-loop} := \sum_{\Phi} (d_\Phi X_d^\Phi X_e^\Phi)$$

$$b_{de}^{1-loop} := \frac{2}{3} \sum_f (d_f X_d^f X_e^f) + \frac{1}{3} \sum_s (d_s X_d^s X_e^s)$$

Can we find analytic 1-loop solutions not only for g_A, but also for G_{ab} ?

Backup 4: Analytic 1-loop solutions

For g_A we can define $\alpha_A := \frac{g_A^2}{4\pi}$ and obtain a solution:

$$\alpha_A^{-1}(\mu_1) = \alpha_A^{-1}(\mu_2) - \frac{b_A}{2\pi} \ln\left(\frac{\mu_1}{\mu_2}\right)$$

Let's define $h := (GG^T)^{-1}$
h is O-invariant (but not L-invariant)

$$h_{a'b'} = L_{a'b} h_{ba} L^T_{ab'}$$

Then one can obtain:

$$h_{ab}(\mu_1) = h_{ab}(\mu_2) - \frac{b_{ab}}{8\pi^2} \ln\left(\frac{\mu_1}{\mu_2}\right)$$

[Salvioni, Villadoro, Zwirner '09]

Backup 5: h matrices

$$h^{Y\,B-L}(\mu_2) = \begin{bmatrix} \frac{1}{g'^2} & -\frac{g'_{B-L}}{g_{B-L}}\frac{1}{g'^2} \\ -\frac{g'_{B-L}}{g_{B-L}}\frac{1}{g'^2} & \frac{1}{g^2_{B-L}} + \left(\frac{g'_{B-L}}{g_{B-L}}\right)^2 \frac{1}{g'^2} \end{bmatrix}$$

(standard parametrization)

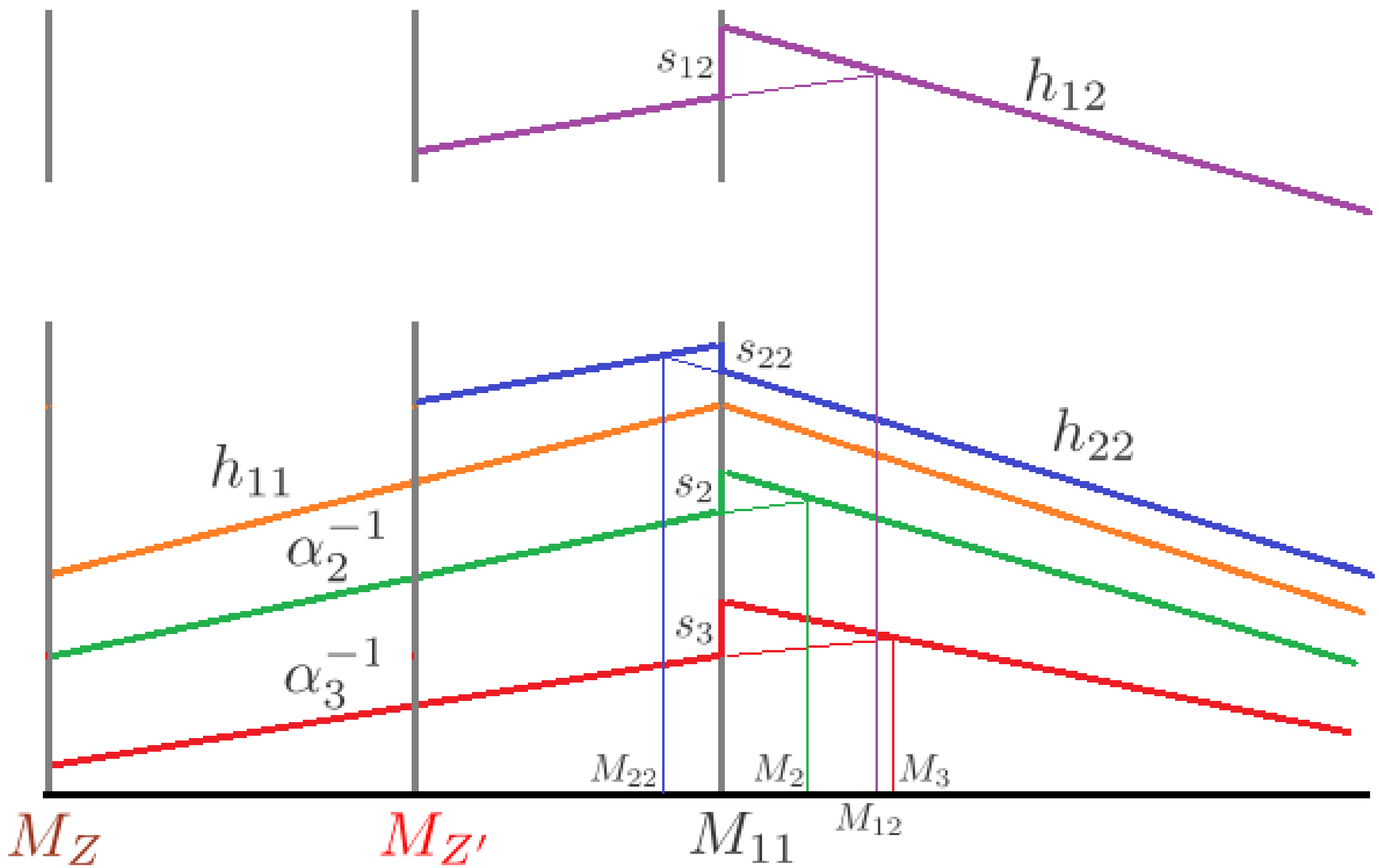
[Salvioni, Villadoro, Zwirner '09]

$$h^{Y\,B-L}(\mu_1) = L h^{X_1 X_2}(\mu_1) L^T = \begin{bmatrix} \frac{a^2}{g^2_{X_1}} + \frac{b^2}{g^2_{X_2}} & \frac{ac}{g^2_{X_1}} + \frac{bd}{g^2_{X_2}} \\ \frac{ac}{g^2_{X_1}} + \frac{bd}{g^2_{X_2}} & \frac{c^2}{g^2_{X_1}} + \frac{d^2}{g^2_{X_2}} \end{bmatrix}$$

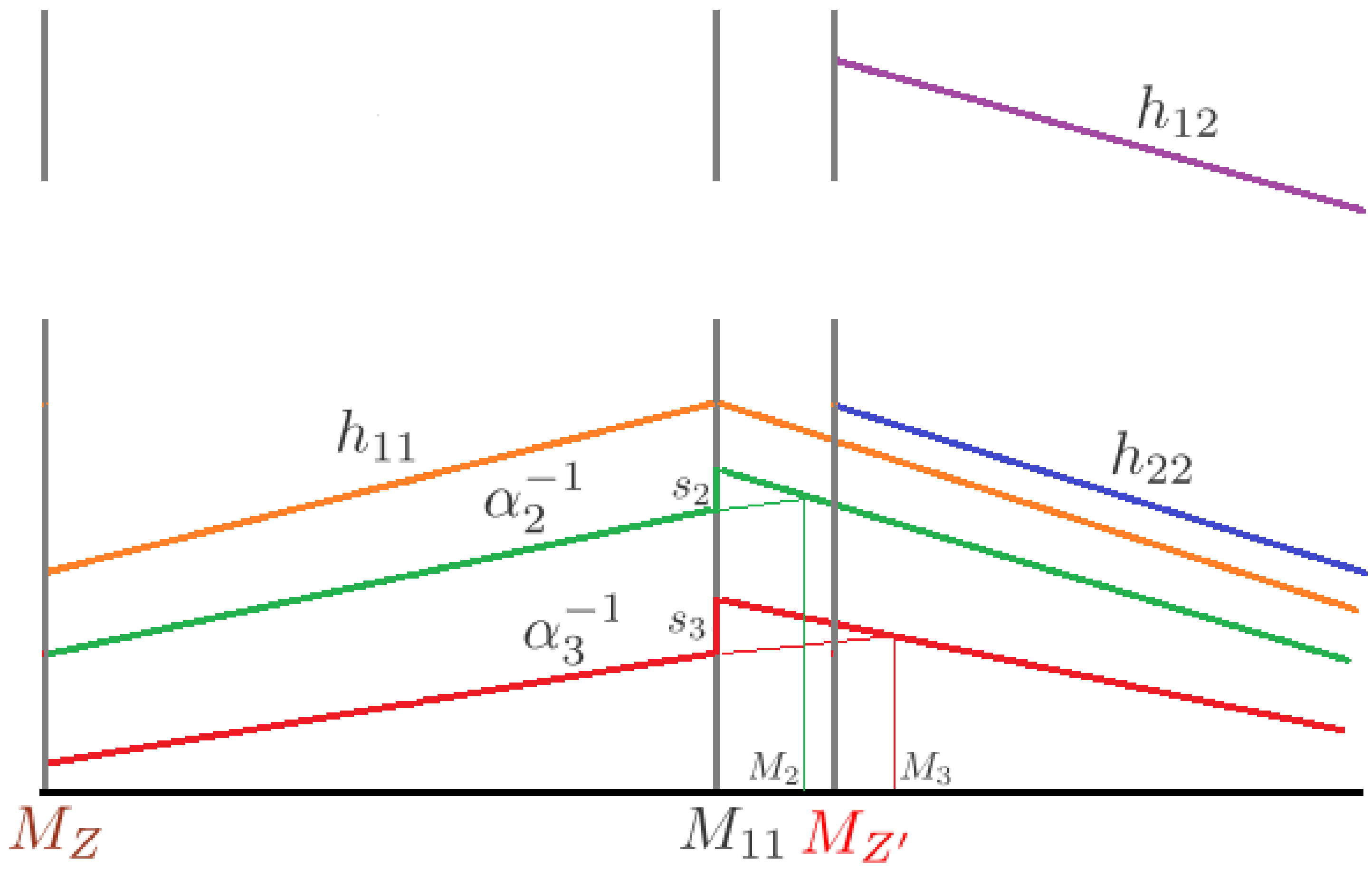
where $L = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Moreover $G^{X_1 X_2}(\mu_1) = \begin{bmatrix} g_{X_1} & 0 \\ 0 & g_{X_2} \end{bmatrix}$ so $h^{X_1 X_2}(\mu_1) = \begin{bmatrix} \frac{1}{g_{X_1}} & 0 \\ 0 & \frac{1}{g_{X_2}} \end{bmatrix}$

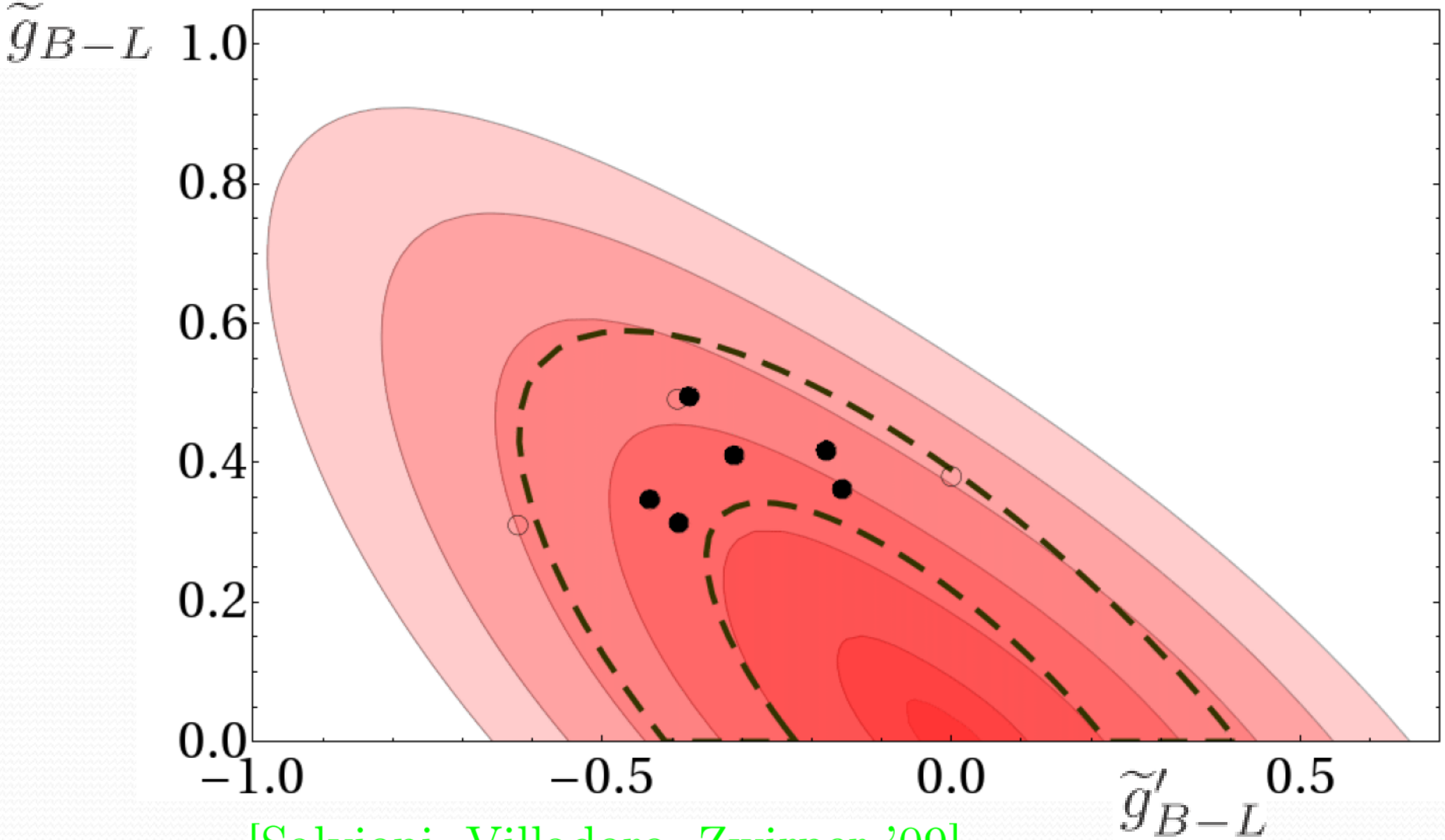
Backup 6: Treshold corrections below μ_1 $M_{11} > M_{Z'}$



Backup 7: Treshold corrections below μ_1 $M_{11} < M_{Z'}$



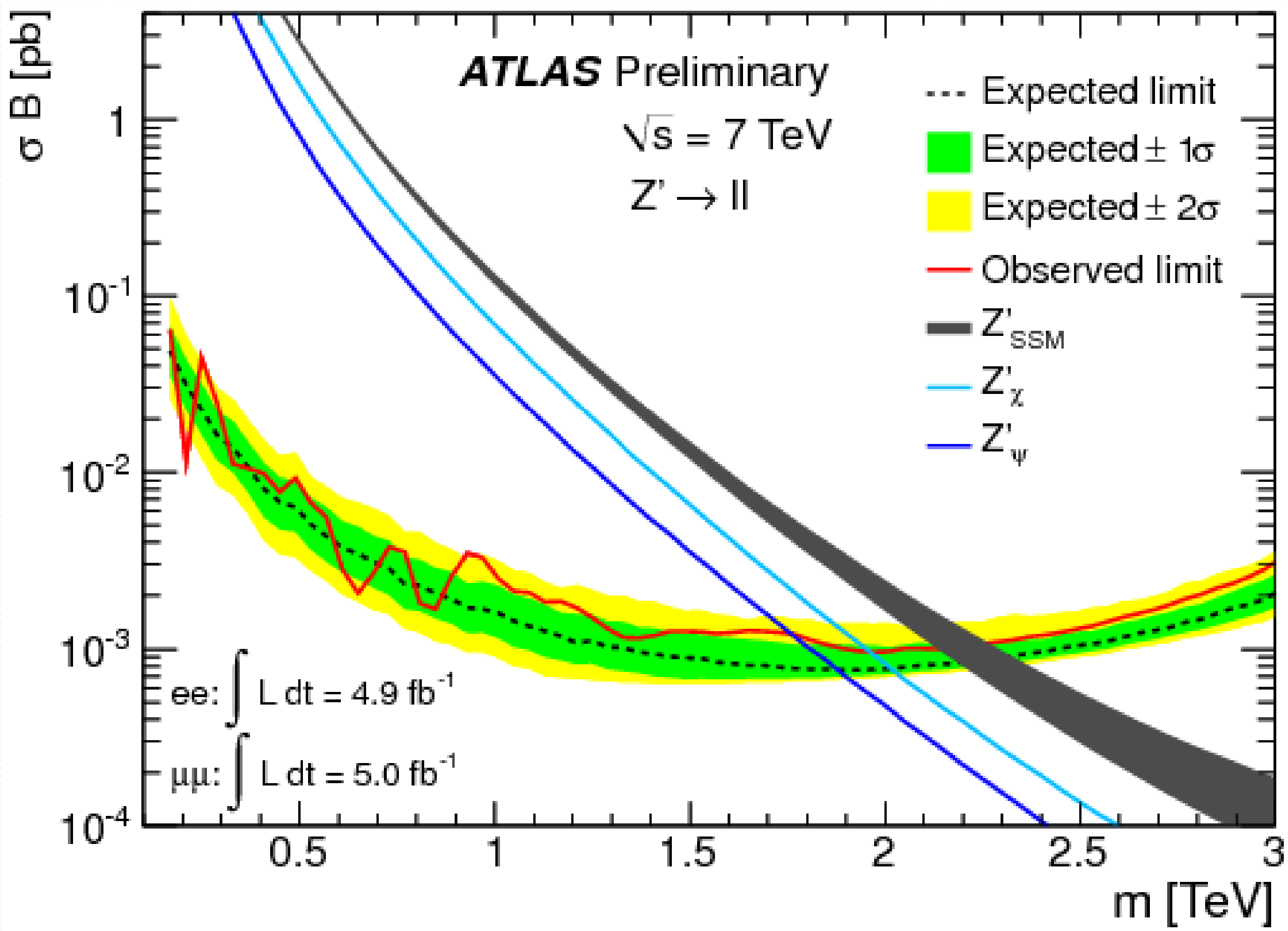
Backup 8: Constraints from LEP II EWPT



[Salvioni, Villadoro, Zwirner '09]

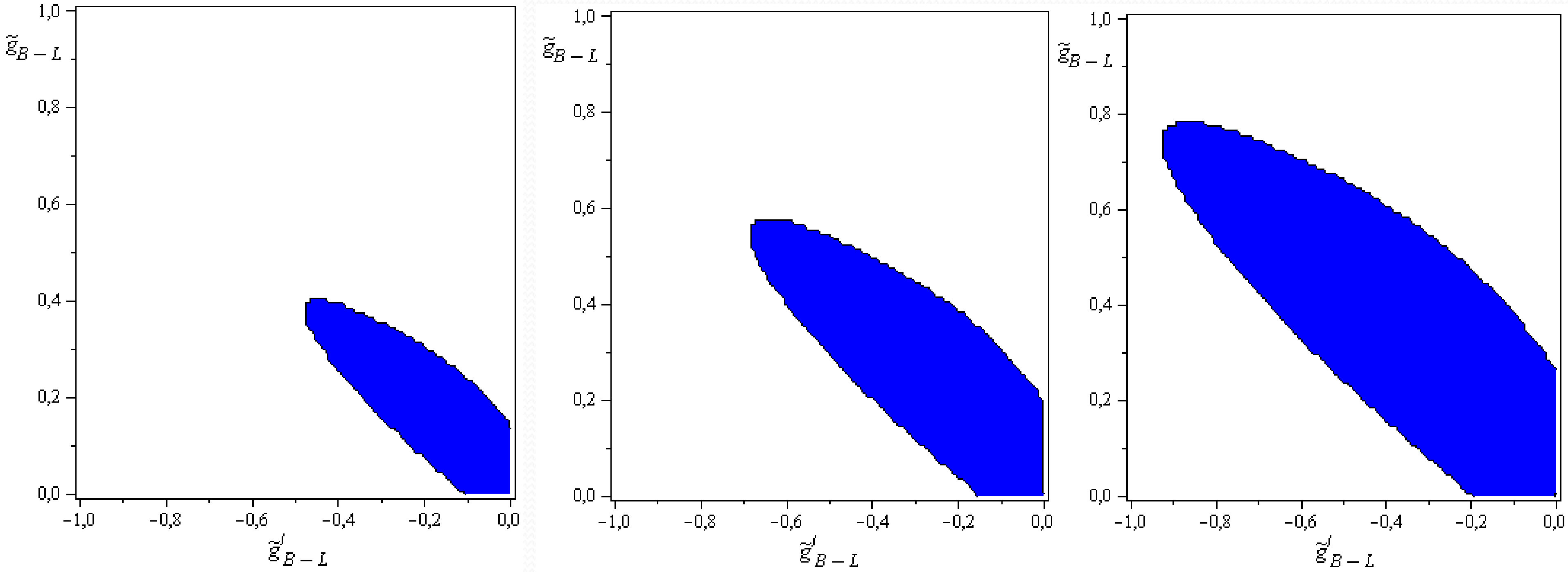
from inner to outer bound: $M_{Z'} \in \{0.2, 0.5, 1, 1.5, 2, 2.5, 3\}$ TeV

Backup 9: Latest constraints from ATLAS



95% C.L. exclusion [ATLAS-CONF-2012-007, 1 March 2012]

Backup 10: Latest constraints from ATLAS



$M_{Z'} = 1.6$ TeV $M_{Z'} = 1.8$ TeV $M_{Z'} = 2$ TeV

■ $\sigma\, B\, <$ Observed limit

$\sigma\, B$ calculated in narrow width approximation in LO

Backup 11: Model 1

Higgs sector: $210_H + 54_H + \overline{126}_H + 126_H + 10_H$ of $SO(10)$

$$SO(10) \xrightarrow{210_H} SU(5) \oplus U(1)_X \xrightarrow{24_{54_H}} \text{Z' Group} \xrightarrow{\chi_{+/-}} \text{SM Group} \xrightarrow{h} \text{Final Group}$$

$$\mu_1 \leq M_{210_H}, M_{54_H}, M_{126_H}, M_{\overline{126}_H} \leq \mu_0$$

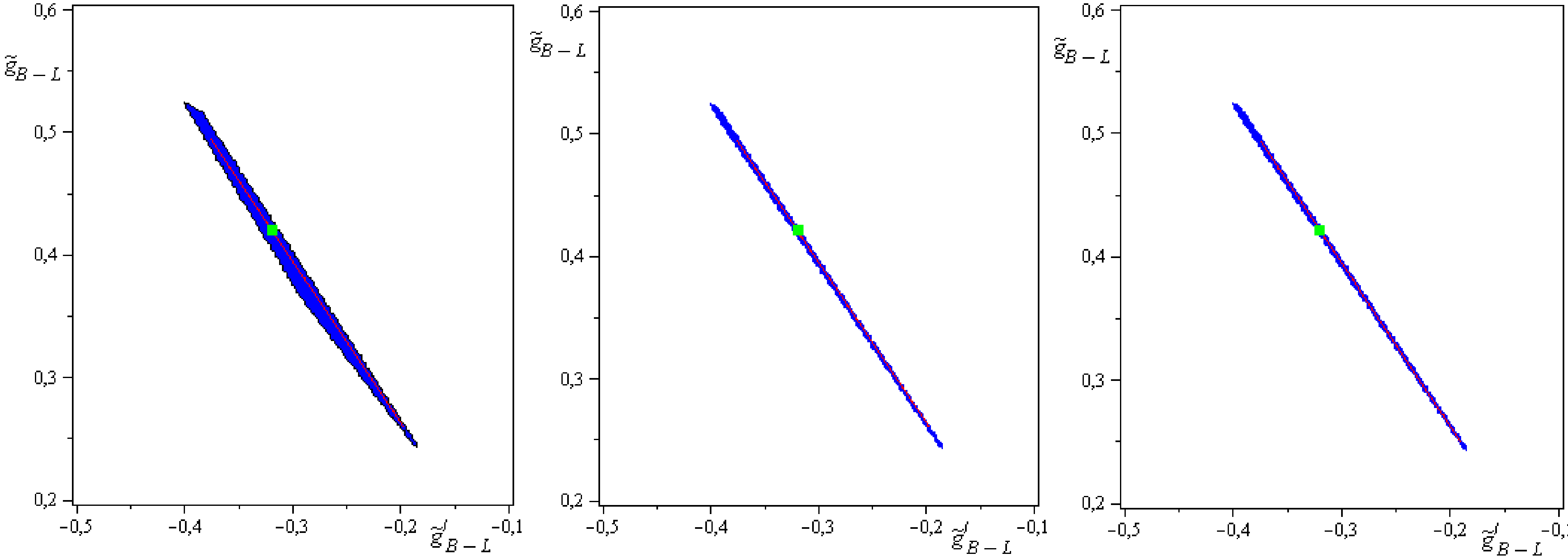
$$M_{[3,1]_{10_H}}, M_{[\overline{3},1]_{10_H}}, M_{24_{54_H}} \approx \mu_1$$

Exception: $M_\chi < \mu_1$

$$\chi_{+/-} = [1,1]_{126_H/\overline{126}_H} = 1_{126_H/\overline{126}_H}$$

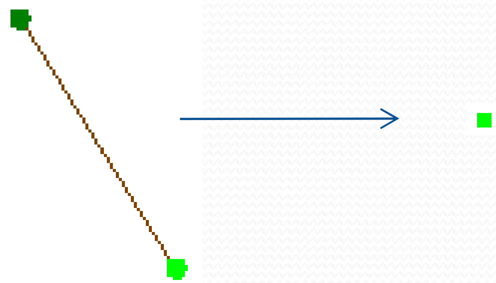
$$10 \text{ TeV} \gtrsim M_{MSSM-SM}, M_{\nu_R}, M_\chi \gtrsim 1 \text{ TeV}$$

Backup 12: Model 1 – with treshold corrections



$M_{Z'} = 1.6$ TeV $M_{Z'} = 1.8$ TeV $M_{Z'} = 2$ TeV

$\mu_0 = \mu_1$



Perturbativity limit

■ $1 \text{ TeV} \leq M_{11} \leq M_{Z'}$

■ + ■ $M_{Z'} \leq M_{11} \leq 10 \text{ TeV}$

Backup 13: Model 2

Higgs sector: $45_H + 45_H + \overline{126}_H + 126_H + 10_H$ of $SO(10)$

$$SO(10) \xrightarrow{45_H} SU(3)_c \oplus SU(2)_L \oplus SU(2)_R \oplus U(1)_{B-L} \xrightarrow{[1,1,3]_{45_H}} \text{Z' G.} \xrightarrow{\chi_{+/-}} \text{SM G.} \xrightarrow{h} \text{F. G.}$$

$$\mu_1 \leq M_{45_H}, M_{45_H}, M_{126_H}, M_{\overline{126}_H}, M_{[3,1,1]_{10_H}}, M_{[\overline{3},1,1]_{10_H}} \leq \mu_0$$

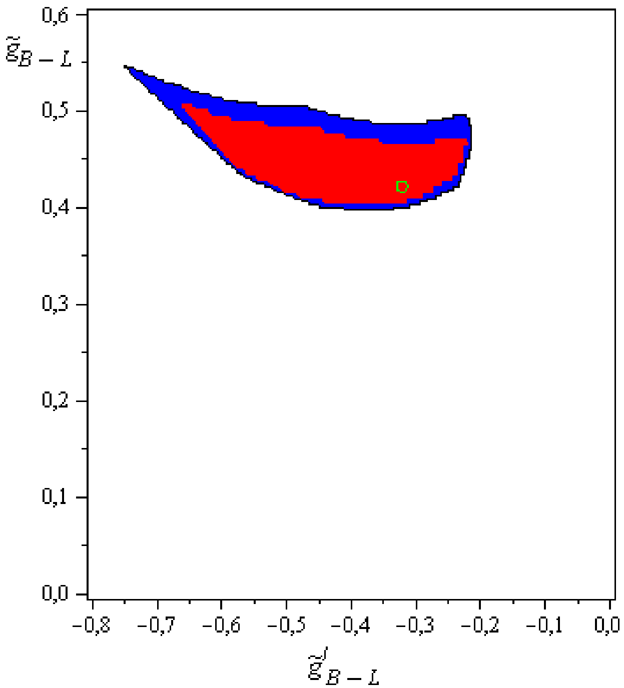
$$M_{[1,1,3]_{45_H}}, M_{[1,1,3]_{126_H/\overline{126}_H}} \approx \mu_1$$

Exception: $M_\chi < \mu_1$

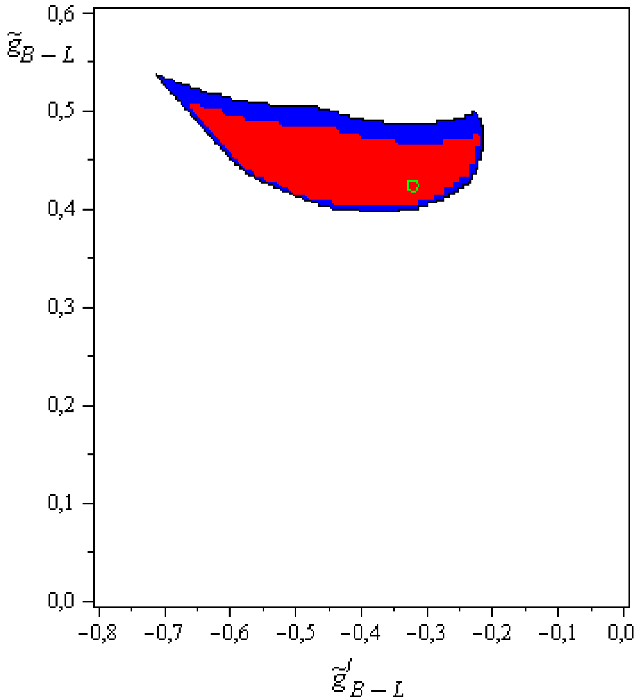
$$\chi_{+/-} = [1,1]_{126_H/\overline{126}_H} \subset [1,1,3]_{126_H/\overline{126}_H}$$

$$10 \text{ TeV} \gtrsim M_{MSSM-SM}, M_{\nu_R}, M_\chi \gtrsim 1 \text{ TeV}$$

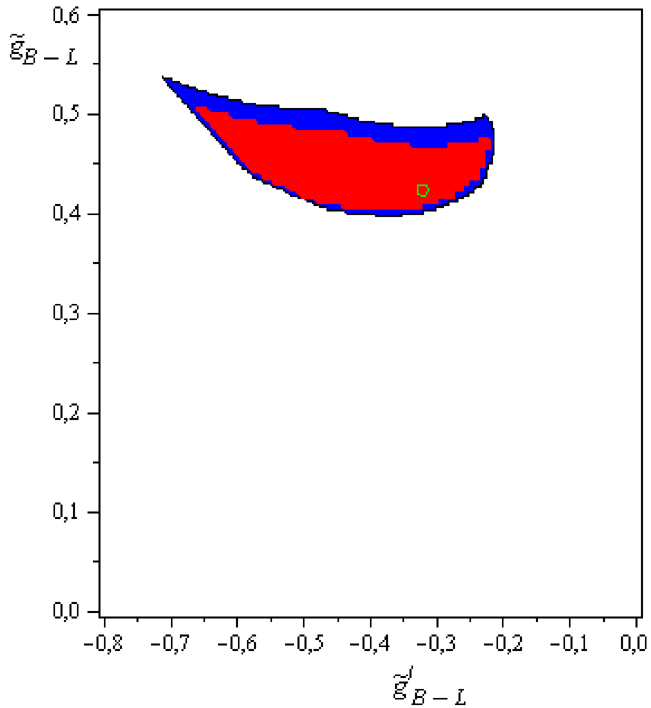
Backup 14: Model 2



$M_{Z'} = 1.6$ TeV



$M_{Z'} = 1.8$ TeV



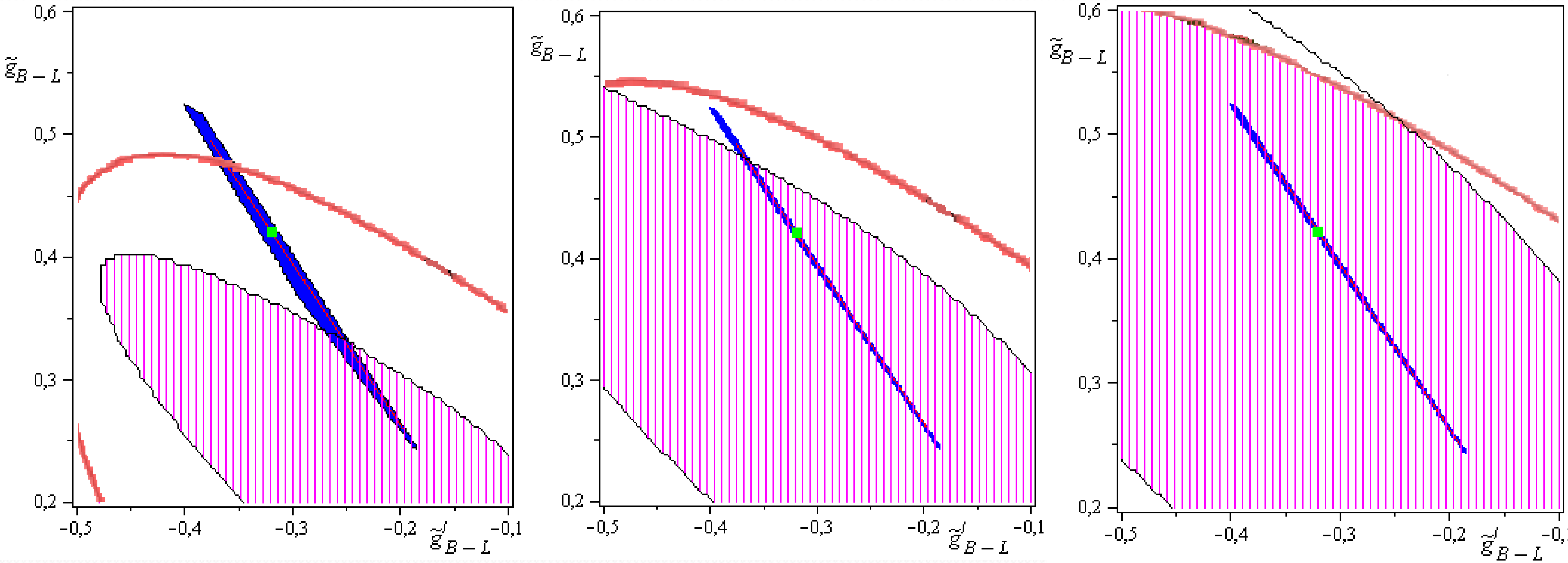
$M_{Z'} = 2$ TeV

(red) $1\ \text{TeV} \leq M_{11} \leq M_{Z'}$

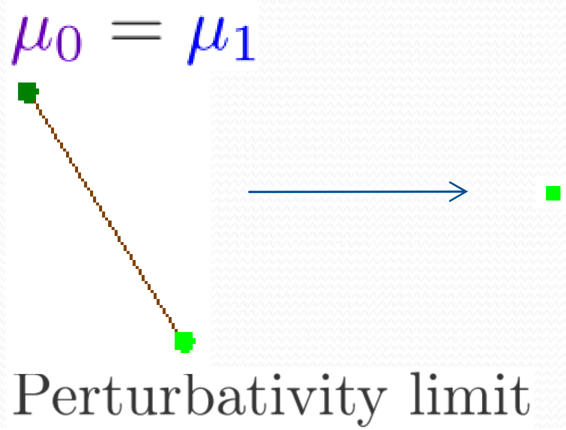
(blue) + (red) $M_{Z'} \leq M_{11} \leq 10\ \text{TeV}$

(green) unification without treshold corrections 1-loop unification at μ_1 and μ_0 is exact.

Backup 15: Experimental constraints on Model 1

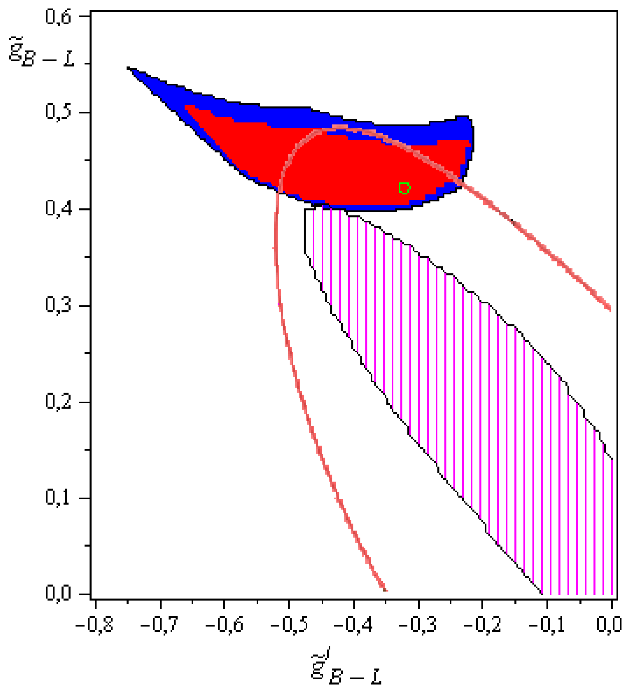


$M_{Z'} = 1.6$ TeV $\quad$ $M_{Z'} = 1.8$ TeV $\quad$ $M_{Z'} = 2$ TeV

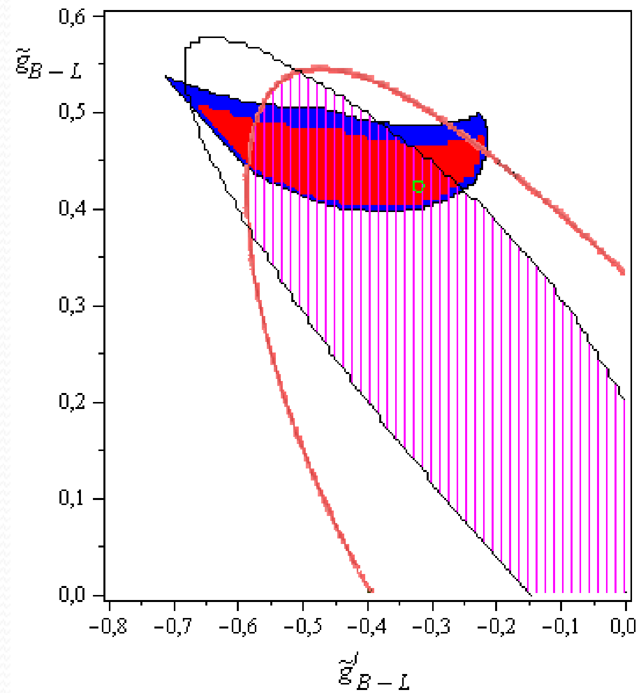


- (red) $1 \text{ TeV} \leq M_{11} \leq M_{Z'}$
- (blue) + (red) $M_{Z'} \leq M_{11} \leq 10 \text{ TeV}$
- (hatched) allowed by ATLAS and EWPT

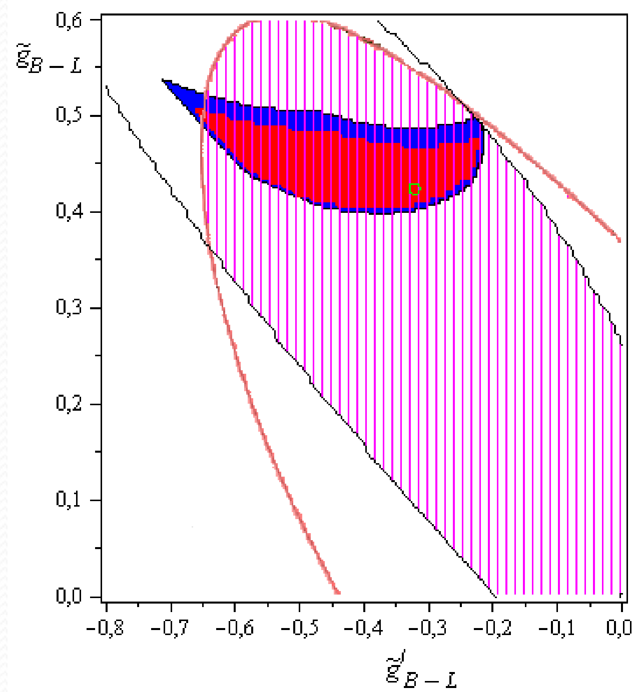
Backup 16: Experimental constraints on Model 2



$M_{Z'} = 1.6$ TeV



$M_{Z'} = 1.8$ TeV



$M_{Z'} = 2$ TeV

[red] $1 \text{ TeV} \leq M_{11} \leq M_{Z'}$

[blue] + [red] $M_{Z'} \leq M_{11} \leq 10 \text{ TeV}$

[hatched] allowed by ATLAS and EWPT

[green square] unification without treshold corrections
1-loop unification at μ_1 and μ_0 is exact.