Stitching the Yukawa quilt in the light of θ_{13}

G. Ross, Planck 2012

$$\sin^2\theta_{13} = 0.022 \pm 0.004\ (0.023 \pm 0.004)$$

8.5^0

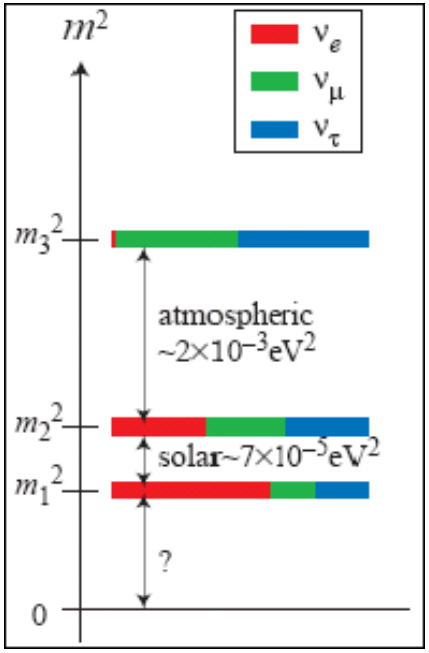
Chooz, Daya Bay, RENO, T2K, MINOS

Neutrino mixing

…near tri-bi-maximal

$$\sin^2\theta_{12} = 0.320^{+0.015}_{-0.017}$$
$$\sin^2\theta_{23} = 0.49^{+0.08}_{-0.05} \qquad \pm 1\sigma$$
$$\sin^2\theta_{13} = 0.026^{+0.003}_{-0.004}$$

Forero, Tortola, Valle see also:
Fogli, Lisi, Marrone, Montanino, Palazzo, Rotuno



Cabibbo haze:

$$\theta_{13}^{\nu} = 0, \quad \theta_{13} \approx \theta_{12}^{l}\sin\theta_{12}^{l} \approx \frac{\theta_{12}^{l}}{\sqrt{2}}$$

$$\begin{cases} M^{q,l} & \text{: small mixing …dominated by } \theta_C \\ M^{\nu} & \text{: tri-bi-maximal} \end{cases}$$

$$L_m^{\nu} = m_{@}\left[\tfrac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)\right]^2 + m_{\odot}\left[\tfrac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)\right]^2$$

Datta, Everett, Ramond
Marzocca, Petkov, Romanino, Spinrath
Antusch et al

If $\theta_{12}^{l} = \theta_C$ (GUT?), $\theta_{13} = 9^0$!

…but $\theta_{12}^{l} = \theta_C$ inconsistent with other plausible mass & GUT relations

Mass relations:

$$\theta_C = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

$$m_\tau(M_{GUT}) = m_b(M_{GUT})$$

$$m_\mu(M_{GUT}) = 3m_s(M_{GUT})$$

$$m_e(M_{GUT}) = \frac{1}{3} m_s(M_{GUT})$$

$$M^d = m_b \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & . \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

Gatto et al, Weinberg, Fritzsch

$$M^l = m_b \begin{pmatrix} <\varepsilon^4 & 1\varepsilon^3 & . \\ 1\varepsilon^3 & 3\varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

Georgi-Jarlskog

Parameters	Input SUSY Parameters					
$\tan\beta$	1.3	10	38	50	38	38
γ_b	0	0	0	0	-0.22	$+0.22$
γ_d	0	0	0	0	-0.21	$+0.21$
γ_t	0	0	0	0	0	-0.44
Parameters	Comparison with GUT Mass Ratios					
$(m_b/m_\tau)(M_X)$	$1.00^{+0.04}_{-0.4}$	0.73(3)	0.73(3)	0.73(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	$0.70^{+0.8}_{-0.05}$	0.69(8)	0.69(8)	0.69(8)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.82(7)	0.83(7)	0.83(7)	0.83(7)	1.05(8)	0.68(6)
$(\frac{\det Y^d}{\det Y^e})(M_X)$	$0.57^{+0.08}_{-0.26}$	0.42(7)	0.42(7)	0.42(7)	0.92(14)	0.39(7)

GGR, Serna

Mass relations: $(\sim 10\%)$

$$\theta_C = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

$$M^d = m_b \begin{pmatrix} < \varepsilon^4 & \varepsilon^3 & \varepsilon^4 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

$$m_\tau(M_{GUT}) = m_b(M_{GUT})$$

$$m_\mu(M_{GUT}) = 3m_s(M_{GUT})$$

$$m_e(M_{GUT}) = \frac{1}{3} m_s(M_{GUT})$$

$$\left| \frac{V_{td}}{V_{ts}} \right| = \sqrt{\frac{m_d}{m_s}}$$

Hall, Rasin

…but $\theta_{12}^{l} = \theta_C$ inconsistent with other plausible GUT relations

$$\theta_C = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

$$m_\tau(M_{GUT}) = m_b(M_{GUT})$$

$$m_\mu(M_{GUT}) = 3m_s(M_{GUT})$$

$$m_e(M_{GUT}) = \frac{1}{3} m_s(M_{GUT})$$

$$M^d = m_b \begin{pmatrix} < \varepsilon^4 & \varepsilon^3 & . \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

$$M^l = m_b \begin{pmatrix} < \varepsilon^4 & 1\varepsilon^3 & . \\ 1\varepsilon^3 & 3\varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

$$\theta_{12}^{l} = \sqrt{\frac{m_e}{m_\mu}} = \frac{\theta_C}{3}$$

$$\theta_{13} = \frac{\theta_{12}^{l}}{\sqrt{2}} = 3^0$$

How do we distinguish between these possibilities?

…correlations between mixing angles

…correlations between mixing angles

$$s_{23} \approx s_{23}^{\nu} - \theta_{23}^{l} c_{23}^{\nu} e^{i\delta_{23}}$$

$$s_{12} \approx s_{12}^{\nu} - \theta_{12}^{l} c_{23}^{\nu} c_{12}^{\nu} e^{i\delta_{12}}$$

$$\theta_{13} e^{-i\delta_{13}} = \theta_{13}^{\nu} e^{-i\delta_{13}^{\nu}} - \theta_{12}^{l} s_{23}^{\nu} e^{-i(\delta_{23}^{\nu} + \delta_{12}^{e})}$$

$\theta_{13}^{l} = 0$

$\delta_{23} = \beta_2$

$\delta_{12} = \beta_1 - \beta_2$

$s_{ij} = \sin(\theta_{ij})..$

Antusch King

Charged lepton contribution:

$\theta_{12}^{l} = \theta_C$ GUT + dimension 5 operator

$\theta_{12}^{l} = \theta_C / 3$ GUT + dimension 4 operator

$\theta_{12}^{l} = 0$

Antusch, Maurer, Gross, Sluka

Neutrino contribution : perturb around TBM mixing

Hall, GGR

$$\nu_{@} \approx \nu_a = \frac{1}{\sqrt{2}}\left(\nu_\mu - \nu_\tau\right)$$
$$\nu_{\odot} \approx \nu_b = \frac{1}{\sqrt{3}}\left(\nu_e + \nu_\mu + \nu_\tau\right)$$
$$\nu_{\perp} \approx \nu_c = \frac{1}{\sqrt{6}}\left(2\nu_e - \nu_\mu - \nu_\tau\right)$$

$$L_m^{\nu} = m_{@}(\nu_a + \varepsilon_{ab}\nu_b + ..)^2 + m_{\odot}(\nu_b - \varepsilon_{ab}\nu_a + ..)^2 + m_{?}(\nu_c + ..)^2$$

- ν_a, ν_b mixing

$$\sin(\theta_{23}^{\nu}) = \left|\frac{1}{\sqrt{3}} + e^{-i\delta_{13}^{\nu}}\sin(\theta_{13}^{\nu})\right|$$

- ν_a, ν_c mixing

$$\sin(\theta_{23}^{\nu}) = \left|\frac{1}{\sqrt{3}} - \frac{1}{2}e^{-i\delta_{13}^{\nu}}\sin(\theta_{13}^{\nu})\right|$$

- ν_b, ν_c mixing

$$\theta_{13}^{\nu} = 0$$

Implications of $\theta_{13} = 0.15$ ($\nu_a \nu_b$ mixing)

- $\theta_{12}^l = 0, \quad \theta_{13}^\nu = \theta_{13}$

  $\sin(\theta_{23}^\nu) = \left| \frac{1}{\sqrt{3}} + e^{-i\delta^\nu} \sin(\theta_{13}^\nu) \right|$

  $\delta = \delta^\nu = 90^{+19}_{-16} \quad (1\sigma) \quad (> 78^0, 3\sigma)$

- $\theta_{12}^l = \theta_C, \quad \theta_{13}^\nu = 0$

  $\sin(\theta_{12}) \simeq \sin(\theta_{12}^\nu) + \theta_{12}^l c_{23}^\nu c_{12}^\nu \cos(\delta)$

  $\delta = 95^{+7}_{-6} \quad \begin{pmatrix} +25 \\ -20 \end{pmatrix}$

- $\theta_{12}^l = \sqrt{\frac{m_e}{m_\mu}}, \quad e^{-i\delta^\nu} s_{13}^\nu = e^{-i\delta} s_{13} + \sqrt{\frac{m_e}{2m_\mu}}$

  $0.36 > \sin^2(\theta_{12}) > 0.306$

Origin of approximateTBM : an example

$$L_m^{\nu} = m_{@}\left[\tfrac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)\right]^2 + m_{\odot}\left[\tfrac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)\right]^2$$

$$\equiv m_{@}\left(\tfrac{1}{\sqrt{2}}\theta_{23}^{i}\nu_i\right)^2 + m_{\odot}\left(\tfrac{1}{\sqrt{3}}\theta_{123}^{i}\nu_i\right)^2$$

$$\theta_{23} = (0,1,-1), \quad \theta_{123} = (1,1,1)$$

Relate θ_i to dynamical variables, familons ϕ_i $\qquad \theta_i \propto < \phi_i >$

ϕ_i triplets of non Abelian discrete family symmetry, $G \subset SU(3)_f$

Vacuum alignment

$$< \phi_{23} >= \varepsilon(0,1,-1), \quad < \phi_{123} >= \varepsilon^2(1,1,1)$$

Vacuum alignment

$e.g. \qquad Z_3 \ltimes Z_n$

ϕ_i	$Z_3\phi_i$	$Z_n'\phi_i$
ϕ_1	$\to \phi_2$	$\to \alpha\,\phi_1$
ϕ_2	$\to \phi_3$	$\to \alpha^2\phi_2$
ϕ_3	$\to \phi_1$	$\to \alpha^{-3}\phi_3$

$$\alpha^n = 1$$

Choice of discrete symmetry

Vacuum structure : $Z_3 \ltimes Z_n \to$

$$\begin{cases} Z_3, & \langle\phi\rangle = (1,1,1) \quad \lambda>0 \\ Z_n, & \langle\phi\rangle = (0,0,1) \quad \lambda<0 \end{cases}$$

$$V(\phi) = -m^2\ \phi^{\dagger i}\phi_i + \ldots + \lambda\ m^2\phi^{\dagger i}\phi_i\phi^{\dagger i}\phi_i$$

Vacuum alignment

$e.g.$ $\quad Z_3 \ltimes Z_n$

ϕ_i	$Z_3\phi_i$	$Z_n'\phi_i$
ϕ_1	$\to \phi_2$	$\to \alpha\,\phi_1$
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ϕ_3	$\to \phi_1$	$\to \alpha^{-3}\phi_3$

$\alpha^n = 1$

Choice of discrete symmetry

Vacuum structure : $Z_3 \ltimes Z_n \to \begin{cases} Z_3, & \langle\phi\rangle = (1,1,1) \quad \lambda>0 \\ Z_n, & \langle\phi\rangle = (0,0,1) \quad \lambda<0 \end{cases}$

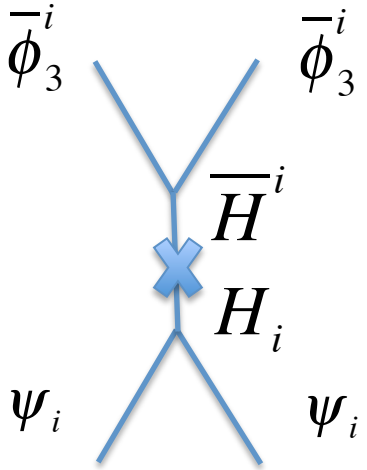
$$V(\phi) = -m^2\,\phi^{\dagger i}\phi_i + \ldots + \lambda\, m^2 \phi^{\dagger i}\phi_i \phi^{\dagger i}\phi_i$$

$$+ \left|\phi_{123}\phi_{23}\right|^2 \quad \Rightarrow \quad \phi_{123} \propto (0,1,-1)$$

$e.g.\quad G = \Delta(6n^2) \sim \left(Z_n \ltimes Z_n\right) \times S_3$ non Abelian discrete group

Escobar, Luhn

Field	$SU(4)$	$SU(2)_L$	$SU(2)_R$	$\Delta(6n^2)$	Z_5	Z_4^R
Ψ	4	2	1	3_{1_l}	0	1
Ψ^c	$\bar{4}$	1	2	3_{1_l}	0	1
θ	8	1	2	1	1	0
ϕ_1	1	1	1	3_{1_l}	2	0
$\bar{\phi}_3$	1	1	1	$3_{1_{-l}}$	2	0
$\bar{\phi}_{23}$	1	1	1	$3_{1_{-l}}$	3	0
$\bar{\phi}_{123}$	1	1	1	$3_{2_{-l}}$	1	0



GGR, Varzielas

Dirac mass structure

$$L_{Dirac}^{q,l,\nu} = \alpha\, \psi_i \bar{\phi}_3^i \psi_i^c \bar{\phi}_3^i + \beta \left(\psi_i \bar{\phi}_{123}^i \psi_j^c \bar{\phi}_{23}^j + \psi_i \bar{\phi}_{23}^i \psi_j^c \bar{\phi}_{123}^j \right) + \gamma\, \psi_i \bar{\phi}_{23}^i \psi_j^c \bar{\phi}_{23}^j \Sigma_{45}$$

$$M^d = m_b \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon^2 & 1 \end{pmatrix} \qquad M^l = m_b \begin{pmatrix} <\varepsilon^4 & 1\varepsilon^3 & 1\varepsilon^3 \\ 1\varepsilon^3 & 3\varepsilon^2 & 3\varepsilon^2 \\ 1\varepsilon^3 & 3\varepsilon^2 & 1 \end{pmatrix}$$

$e.g.\quad G = \Delta(6n^2) \sim \left(Z_n \ltimes Z_n\right) \times S_3$ non Abelian discrete group

Field	$SU(4)$	$SU(2)_L$	$SU(2)_R$	$\Delta(6n^2)$	Z_5	Z_4^R
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θ	8	1	2	1	1	0
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$\bar{\phi}_{123}$	1	1	1	$3_{2_{-l}}$	1	0

Majorana mass structure

$$L_{Dirac}^{q,l,\nu} = \alpha\, \psi_i \bar{\phi}_3^i \psi_i^c \bar{\phi}_3^i + \beta \left(\psi_i \bar{\phi}_{123}^i \psi_j^c \bar{\phi}_{23}^j + \psi_i \bar{\phi}_{23}^i \psi_j^c \bar{\phi}_{123}^j \right) + \gamma\, \psi_i \bar{\phi}_{23}^i \psi_j^c \bar{\phi}_{23}^j \Sigma_{45}$$

$$L_M = \theta \psi_i^c \psi_j^c \left(a \bar{\phi}_3^i \bar{\phi}_3^i + b\varepsilon^2 \bar{\phi}_{23}^i \bar{\phi}_{23}^i + c\varepsilon \bar{\phi}_{123}^i \bar{\phi}_{123}^i + d\varepsilon^2 \bar{\phi}_{23}^i \bar{\phi}_{123}^i \varepsilon \right) \qquad b \sim c \sim d \ll a$$

$e.g.\quad G = \Delta(6n^2) \sim \left(Z_n \ltimes Z_n\right) \times S_3$ non Abelian discrete group

Field	$SU(4)$	$SU(2)_L$	$SU(2)_R$	$\Delta(6n^2)$	Z_5	Z_4^R
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$\bar{\phi}_{123}$	1	1	1	$3_{2_{-l}}$	1	0

$M_1 < M_2 << M_3$

small

Majorana mass structure

$$L^{q,l,\nu}_{Dirac} = \alpha\, \psi_i \bar{\phi}^i_3 \psi^c_i \bar{\phi}^i_3 + \beta \left(\psi_i \bar{\phi}^i_{123} \psi^c_j \bar{\phi}^j_{23} + \psi_i \bar{\phi}^i_{23} \psi^c_j \bar{\phi}^j_{123} \right) + \gamma\, \psi_i \bar{\phi}^i_{23} \psi^c_j \bar{\phi}^j_{23} \Sigma_{45}$$

$$L_M = \theta \psi^c_i \psi^c_j \left(a \bar{\phi}^i_3 \bar{\phi}^i_3 + b\varepsilon^2 \bar{\phi}^i_{23} \bar{\phi}^i_{23} + c\varepsilon \bar{\phi}^i_{123} \bar{\phi}^i_{123} + d\varepsilon^2 \bar{\phi}^i_{23} \bar{\phi}^i_{123} \varepsilon \right) \qquad b \sim c \sim d \ll a$$

$$L^{\nu}_{eff} = \frac{\beta^2}{M_1} \psi_i \phi^i_{123} \psi_j \phi^j_{123} + \frac{\beta^2}{M_2} \psi_i \phi^i_{23} \psi_j \phi^j_{23} + \frac{(\alpha+\beta)^2}{M_3} \psi_i \phi^i_3 \psi_j \phi^j_3$$

$e.g.\quad G = \Delta(6n^2) \sim \left(Z_n \ltimes Z_n\right) \times S_3$ non Abelian discrete group

Field	$SU(4)$	$SU(2)_L$	$SU(2)_R$	$\Delta(6n^2)$	Z_5	Z_4^R
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ϕ_1	1	1	1	3_{1_l}	2	0
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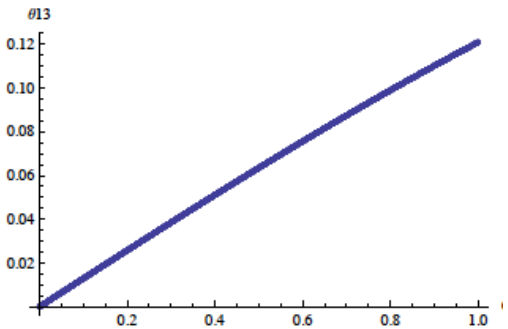
$\bar{\phi}_{123}$	1	1	1	$3_{2_{-l}}$	1	0

θ_{13}



$\frac{d}{c}$

Majorana mass structure

breaks $\phi_{123} \to -\phi_{123}$ symmetry

$$L_{Dirac}^{q,l,\nu} = \alpha\, \psi_i \bar{\phi}_3^i \psi^c{}_i \bar{\phi}_3^i + \beta \left(\psi_i \bar{\phi}_{123}^i \psi^c{}_j \bar{\phi}_{23}^j + \psi_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{123}^j \right) + \gamma\, \psi_i \bar{\phi}_{23}^i \psi^c{}_j \bar{\phi}_{23}^j \Sigma_{45}$$

$$L_M = \theta \psi_i^c \psi_j^c \left(a \bar{\phi}_3^i \bar{\phi}_3^i + b\varepsilon^2 \bar{\phi}_{23}^i \bar{\phi}_{23}^i + c\varepsilon \bar{\phi}_{123}^i \bar{\phi}_{123}^i + d\varepsilon^2 \bar{\phi}_{23}^i \bar{\phi}_{123}^i \varepsilon \right)$$

$\nu_a \nu_b$ mixing

$$L_{eff}^{\nu} = \frac{\beta^2}{M_1} \psi_i \phi_{123}^i \psi_j \phi_{123}^j + \frac{\beta^2}{M_2} \psi_i \phi_{23}^i \psi_j \phi_{23}^j + \frac{\beta^2 (d/c)\varepsilon}{M_2} \psi_i \phi_{23}^i \psi_j \phi_{123}^j$$

Summary

- $\theta_{13} \approx 9^0$ unexpected and requires modification of Yukawa quilt

- Can be accommodated by perturbation of TBM mixing and/or leptonic contribution

  Correlated changes in θ_{12}, θ_{23}

  +Large CP violation

- Discrete non Abelian family symmetry can generate approximate TBM mixing +

$$\theta_C = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

$$\left| \frac{V_{ub}}{V_{cb}} \right| \approx \sqrt{\frac{m_u}{m_c}}$$

$$\left| \frac{V_{td}}{V_{ts}} \right| \approx \sqrt{\frac{m_d}{m_s}}$$

$$m_\tau(M_{GUT}) = m_b(M_{GUT})$$

$$m_\mu(M_{GUT}) = 3m_s(M_{GUT})$$

$$m_e(M_{GUT}) = \frac{1}{3} m_s(M_{GUT})$$

...6+ parameter free postdictions

Vacuum alignment

$$P \supset \langle P \rangle \phi_{23} \phi_{123}^2 \to m_{3/2} \phi_{23} \phi_{123}^2$$

$$V_{tree} = m_{3/2}^2 |\phi_{123}|^4 + m_{3/2}^2 |\phi_{123}\phi_{23}|^2$$

$$V_{rad} = \alpha m_{3/2}^2 |\phi_2|^4 + \beta m_{3/2}^2 |\phi_3|^4 + \gamma m_{3/2}^2 |\phi_2\phi_3|^4 + \delta m_{3/2}^2 |\phi_2\phi_{23}|^2 + \ldots$$

$$\phi_{123} \propto (1,1,1), \quad \phi_2 \propto (1,0,0), \quad \phi_3 \propto (0,0,1), \quad \phi_{23} \propto (0,1,-1)$$

$$\alpha, \beta < 0, \quad \gamma, \delta > 0$$

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	7.62 ± 0.19	7.27–8.01	7.12–8.20
Δm^2_{31} $[10^{-3}\text{eV}^2]$	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	$2.34 - 2.69$ $-(2.25 - 2.59)$	$2.26 - 2.77$ $-(2.15 - 2.68)$
$\sin^2\theta_{12}$	$0.320^{+0.015}_{-0.017}$	0.29–0.35	0.27–0.37
$\sin^2\theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.05}_{-0.07}$	0.41–0.62 0.42–0.62	0.39–0.64
$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	0.019–0.033 0.020–0.034	0.015–0.036 0.016–0.037
δ	$(0.83^{+0.54}_{-0.64})\ \pi$ 0.07π [a]	$0 - 2\pi$	$0 - 2\pi$

Forero, Tortola, Valle

TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. We remind that Δm^2 is defined herein as $m^2_3 - (m^2_1 + m^2_2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH.

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.34 – 2.50	2.26 – 2.58	2.15 – 2.66
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.32 – 2.49	2.25 – 2.56	2.14 – 2.65
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.45	2.14 – 2.79	1.81 – 3.11	1.49 – 3.44
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.46	2.15 – 2.80	1.83 – 3.13	1.50 – 3.47
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.98	3.72 – 4.28	3.50 – 4.75	3.30 – 6.38
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.08	3.78 – 4.43	3.55 – 6.27	3.35 – 6.58
δ/π (NH)	0.89	0.45 – 1.18	—	—
δ/π (IH)	0.90	0.47 – 1.22	—	—

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotuno