

# Right unitarity triangles from discrete symmetries and unification

Martin Spinrath



PLANCK 2012 - 30th May

Based on collaborations with S.Antusch, S.F.King, C. Luhn and M. Malinsky:  
PRD 81 (2010) 033008, PRD 83 (2011) 013005, NPB 850 (2011) 477

# The Sum Rules

[Antusch, King, Malinsky, MS 2010]

$$\theta_{12}^u = \frac{\theta_{13}}{\theta_{23}} = (4.96 \pm 0.30)^\circ$$

$$\theta_{12}^d = \left| \theta_{12} - \frac{\theta_{13}}{\theta_{23}} e^{-i\delta_{\text{CKM}}} \right| = (12.0^{+0.39}_{-0.22})^\circ$$

$$\longrightarrow \alpha = \delta_{12}^d - \delta_{12}^u = (89.0^{+4.4}_{-4.2})^\circ \quad \longleftarrow$$

[Numbers based on PDG]

Idea: Mass matrices with purely  
real/imaginary elements!

[Antusch, King, Malinsky, MS 2010]  
see also [Fritzsch and Xing; Masina and Savoy 2006;  
Harrison, Dallison, Roythorne, Scott 2009]

# The Method

[Antusch, King, Luhn, MS 2011]

- Use family symmetry to align flavon, e.g.:

$$\langle \phi \rangle \propto (0, 0, x)^T \quad \text{or} \quad \langle \phi \rangle \propto (x, x, x)^T$$

- Add term to  $W$  compatible with shaping symmetry:

$$P \left( \frac{\phi^n}{\Lambda^{n-2}} \mp M^2 \right)$$

- Solve F-term conditions ( $|F_P| = 0$ )

$$\arg(\langle \phi \rangle) = \arg(x) = \begin{cases} \frac{2\pi}{n}q, & q = 1, \dots, n \quad \text{for “-”} \\ \frac{2\pi}{n}q + \frac{\pi}{n}, & q = 1, \dots, n \quad \text{for “+”} \end{cases}$$

[Real alignment used in Meroni, Petcov, MS 2012, see also talk by A. Meroni]

And now for something  
(completely) different...

# Two Approaches for Flavour Models with Large $\Theta_{13}$

Martin Spinrath



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Based on D. Marzocca, S.T. Petcov, A. Romanino, MS JHEP 1111 (2011) 009 [arXiv:1108.0614]  
S. Antusch, S.F. King, C. Luhn, MS Nucl. Phys. B856 (2012) 328-341 [arXiv:1108.4278]

# Outline

- Status
- Possibility I: Charged Lepton Corrections
- Possibility II: Trimaximal Mixing
- Summary and Conclusions

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# May 2012

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	March 2011, NH	May 2012, NH
$\sin^2 \theta_{12}$	$0.312^{+0.017}_{-0.015}$	$0.320^{+0.015}_{-0.017}$
$\sin^2 \theta_{23}$	$0.51 \pm 0.06$	$0.49^{+0.08}_{-0.05}$
$\sin^2 \theta_{13}$	$0.010^{+0.009}_{-0.006}$	$0.026^{+0.003}_{-0.004}$
$\delta$	?	$(0.83^{+0.54}_{-0.64})\pi$

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[Schwetz, Tortola, Valle 2011]

[Tortola, Valle, Vanegas 2012]

# So what?

- Very popular before: (Tri-)Bimaximal Mixing

$$\sin^2 \theta_{12} = \left(\frac{1}{3}\right) \frac{1}{2}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = 0$$

- Possible modifications? Plenty...

- Solution I: Charged Lepton Sector Corrections

[Antusch, Maurer `11; Marzocca, Petcov, Romanino, MS `11]

- Solution II: Trimaximal Mixing

[Antusch, King, Luhn, MS `11]

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# The Setup

[Marzocca et al. `11, see also Antusch, Maurer `11]

- No 1-3 mixing in the neutrino and only 1-2 mixing in the charged lepton sector:

$$\sin \theta_{13} \approx \sin \theta_{12}^e \sin \theta_{23}^\nu \approx \frac{1}{\sqrt{2}} \frac{\beta'}{\gamma} \frac{b'}{c}$$

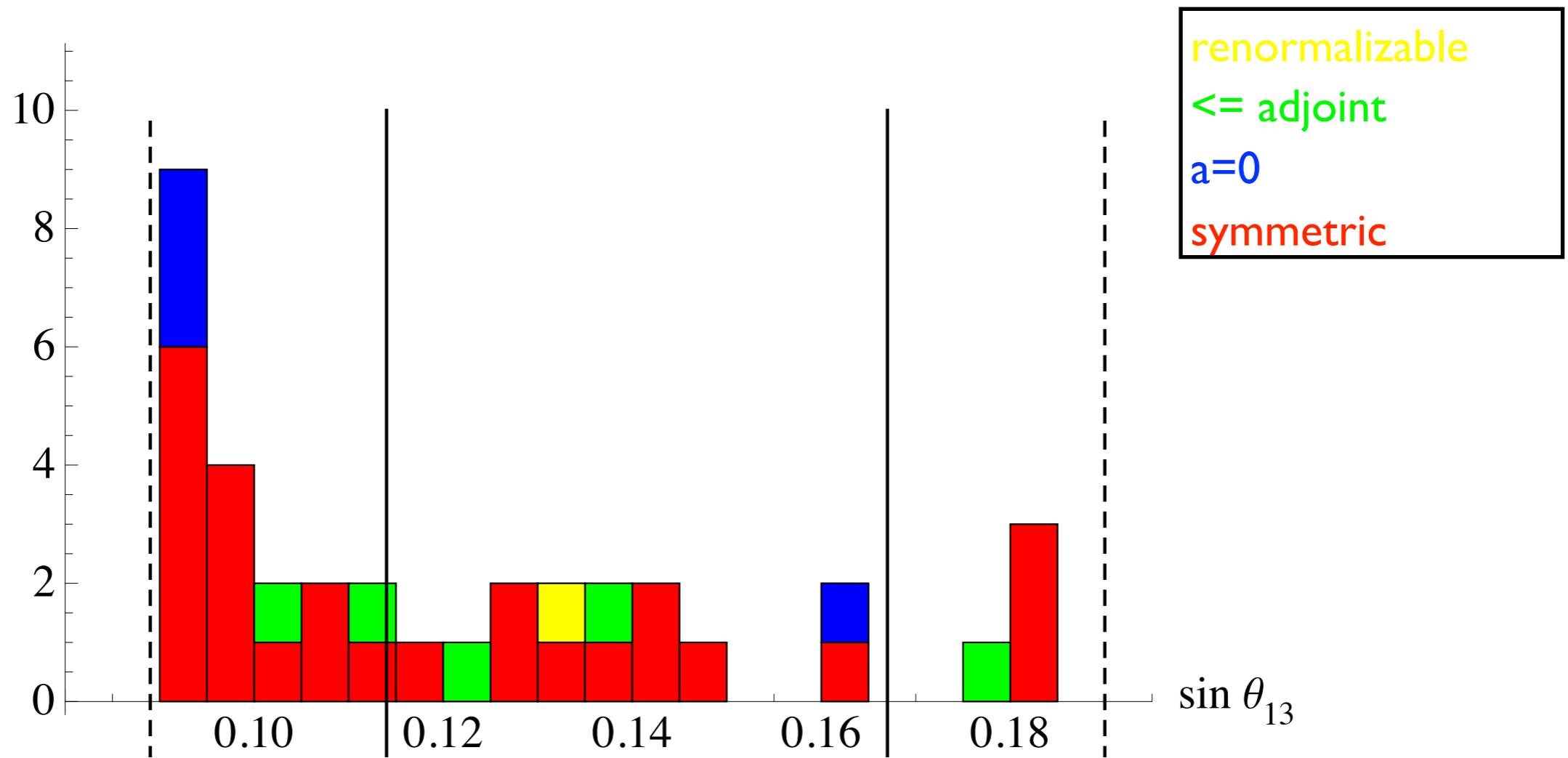
- Assume SU(5):

$$\hat{\lambda}_{[12]}^D = \begin{pmatrix} a & b' \\ b & c \end{pmatrix} \quad \hat{\lambda}_{[12]}^E = \begin{pmatrix} \alpha a & \beta b \\ \beta' b' & \gamma c \end{pmatrix}$$

- $\alpha, \beta, \beta', \gamma$  are SU(5) Clebsch-Gordan coefficients (1, -3/2, -3, 9/2, 6, ...)

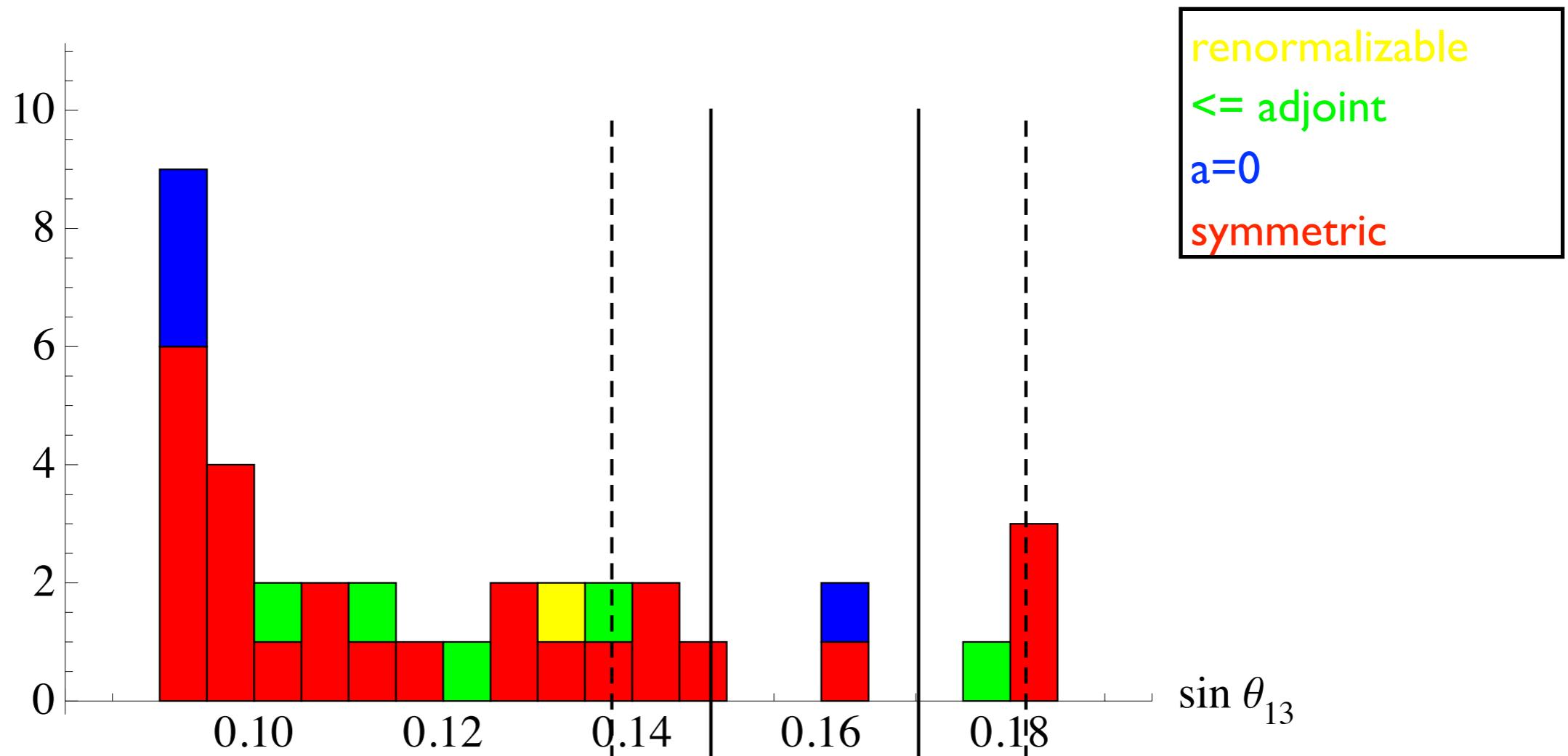
[For non-standard relations, see Antusch, MS `09]

# After Fitting to Exp. Data (June '11)



[Taken from Marzocca *et al.* '11, based on the global fit by Fogli *et al.* '11]

# After Fitting to Exp. Data (May `12)



[Based on the global fit by Tortola, Valle, Vanegas `11, Thanks to D. Marzocca for providing this plot]

# The Good Cases

$\{\alpha, \beta, \beta', \gamma\}$	$\sin \theta_{13}$
$\{-, -1/2, 6, 6\}$	$0.164 \pm 0.013$
$\{-3/2, -3, -3, -3\}$	$0.164 \pm 0.007$
$\{-18, 9/2, 9/2, 9/2\}$	$0.149 \pm 0.003$

[Taken from Marzocca *et al.* `11]

For a model implementation see talk by A. Meroni

[A. Meroni, S. T. Petcov, MS `11]

# Corrections to other Mixing Angles

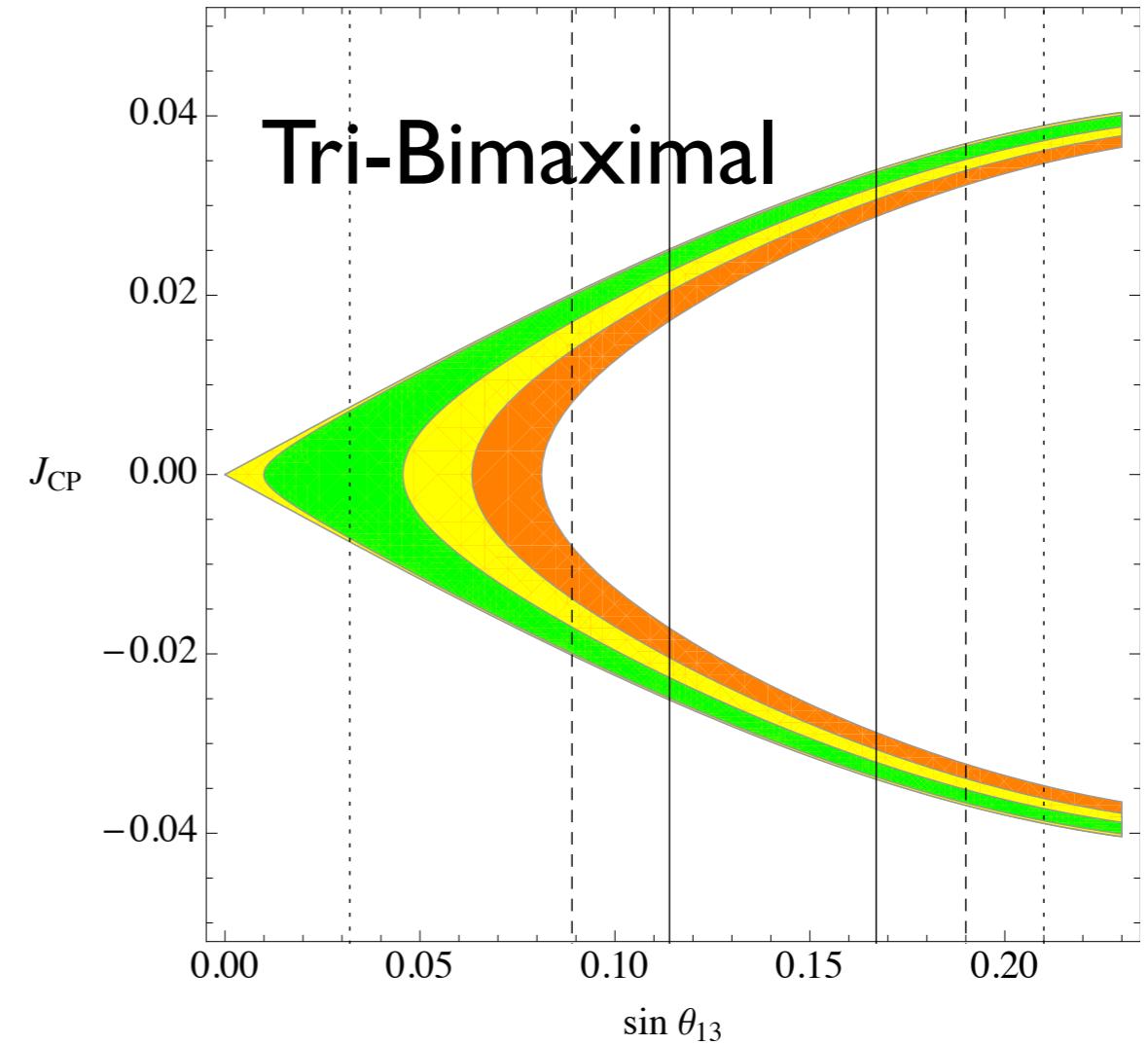
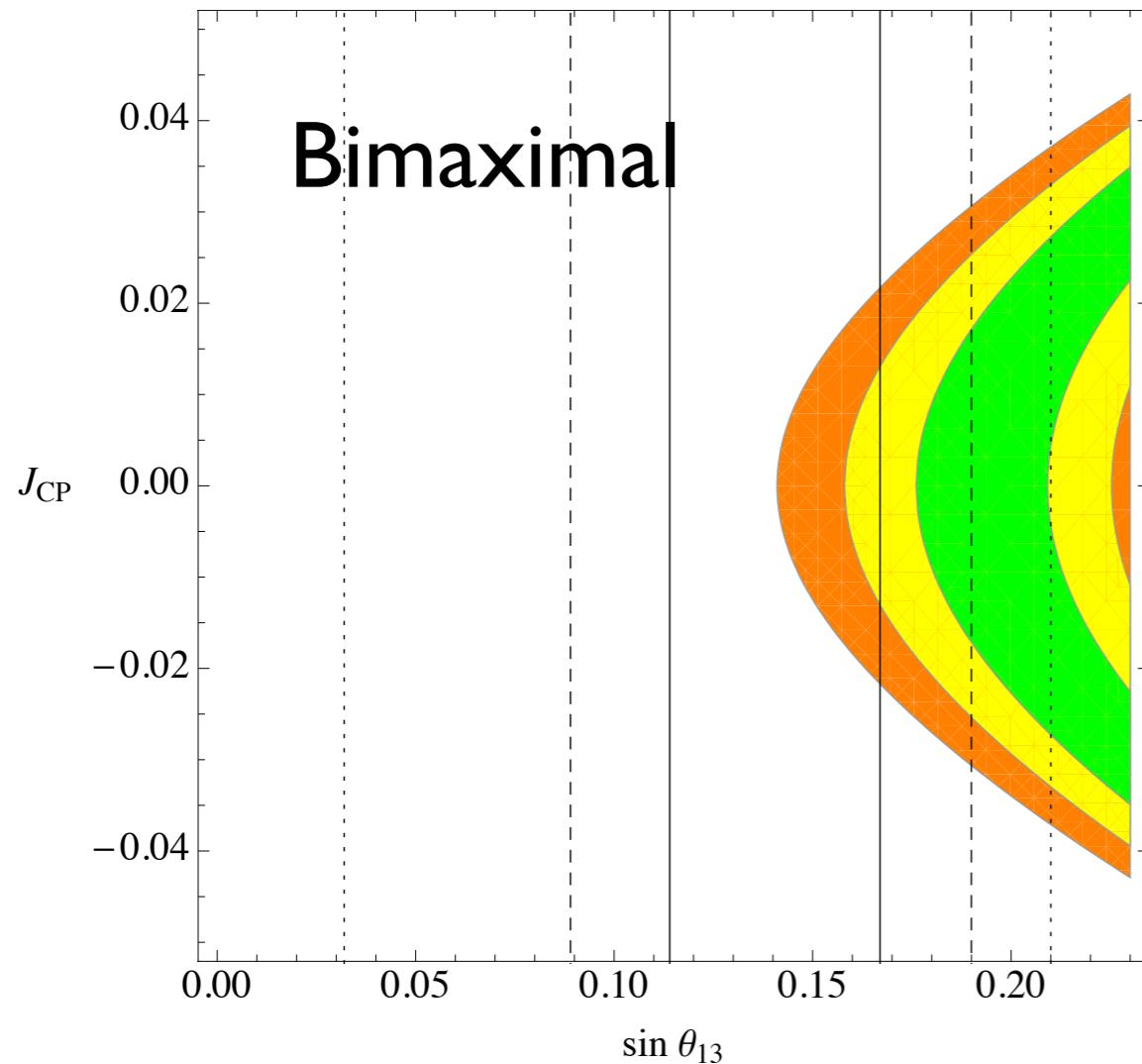
- For Bimaximal mixing:

$$\sin^2 \theta_{12} \approx \frac{1}{2} + \sin \theta_{13} \cos \delta$$

- For Tri-Bimaximal mixing:

$$\sin^2 \theta_{12} \approx \frac{1}{3} + \frac{2\sqrt{2}}{3} \sin \theta_{13} \cos \delta$$

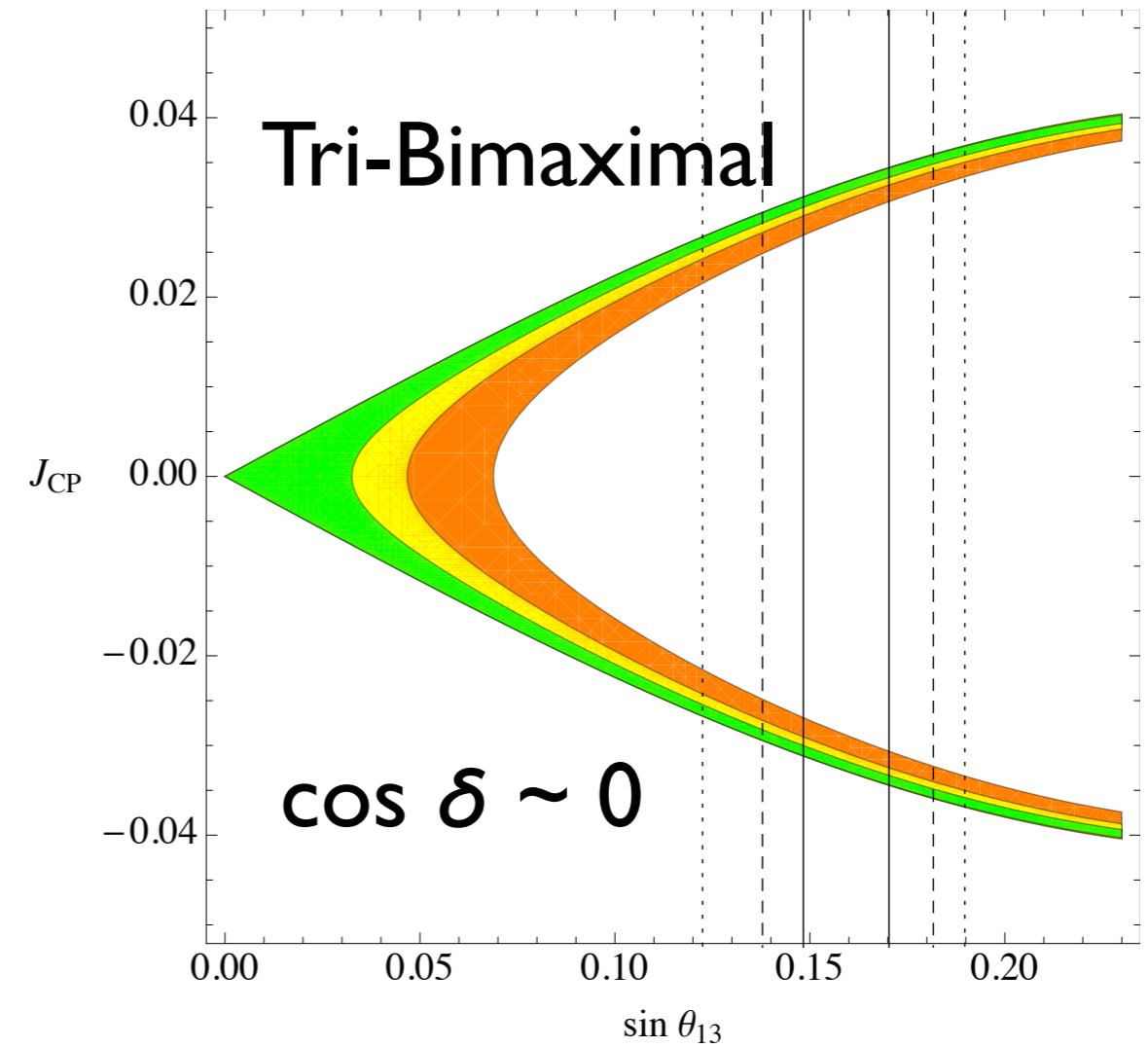
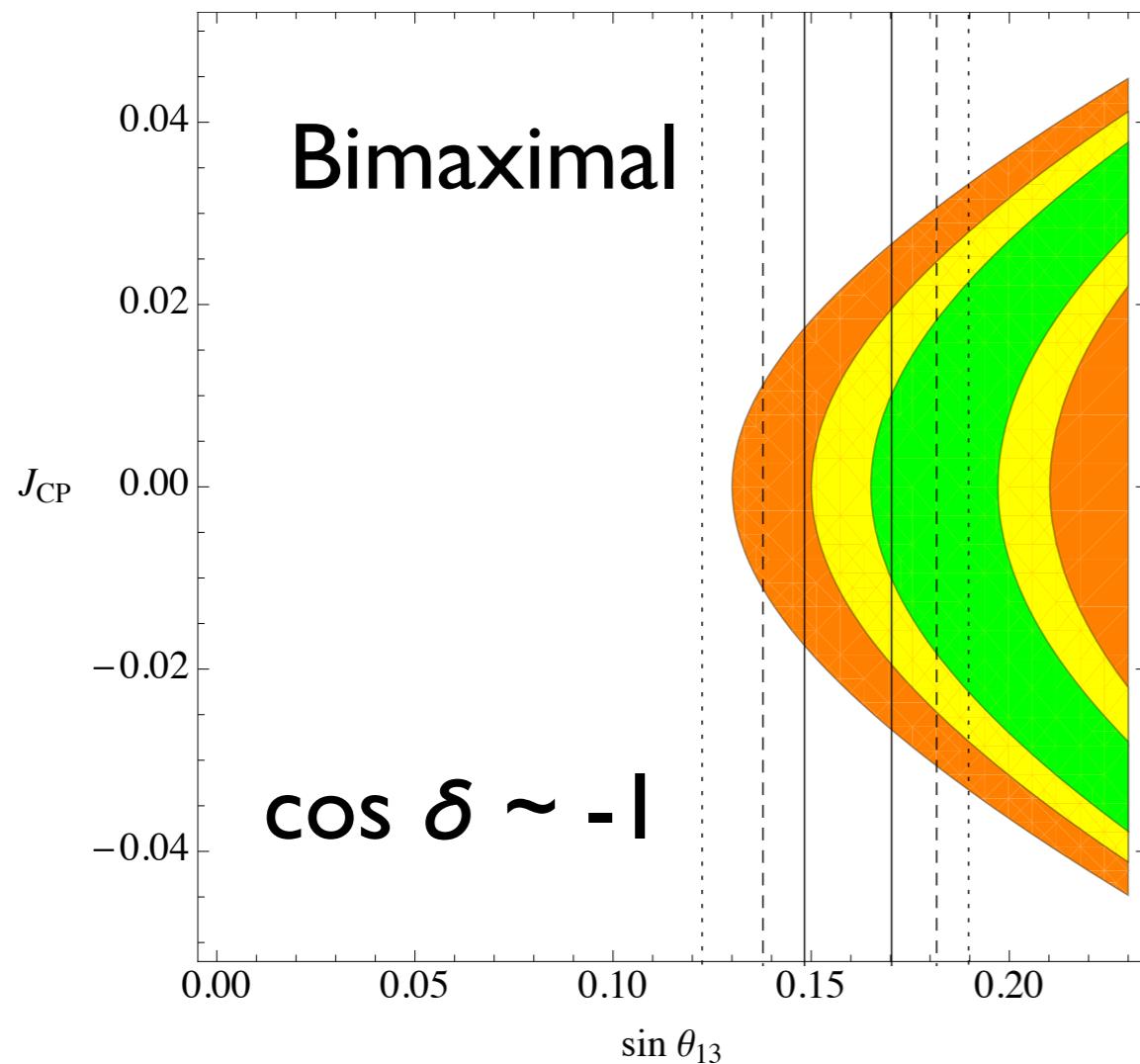
# Implications for $J_{CP}$ (June '11)



[Taken from Marzocca *et al.* '11, based on the global fit by Fogli *et al.* '11]

# Implications for $J_{CP}$

## (May `12)



[Based on the global fit by Tortola, Valle, Vanegas '11, Thanks to D. Marzocca for providing this update]

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# Pattern for Trimaximal Mixing

[Variant we use based on C. S. Lam `06; C. H. Albright, W. Rodejohann `09; C. H. Albright, A. Dueck, W. Rodejohann `10]

- Minimalistic pattern for TBM:

$$Y_\nu = \begin{pmatrix} 0 & b \\ a & b \\ -a & b \end{pmatrix}, \quad M_R = \begin{pmatrix} M_A & 0 \\ 0 & M_B \end{pmatrix}$$

- Alternative possibility:

$$Y_\nu = \begin{pmatrix} 0 & b \\ a & 0 \\ -a & 2b \end{pmatrix}, \quad M_R = \begin{pmatrix} M_A & 0 \\ 0 & M_B \end{pmatrix}$$

[Antusch, King, Luhn, MS `11]

# How to get these alignments?

[Antusch, King, Luhn, MS `11]

- Two sets of flavons:

1st column of  $U_{PMNS}$  (good  $\theta_{12}$ )

$$\langle \phi_1^\nu \rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \langle \phi_2^\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \langle \phi_3^\nu \rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \quad \text{←}$$

$$\langle \phi_1^e \rangle \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \phi_2^e \rangle \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \langle \phi_3^e \rangle \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- Use orthogonality ( $F_O = 0$ ):

$$\mathcal{W} = O_1(\phi_2^e \cdot \phi_{102}) + O_2(\phi_3^\nu \cdot \phi_{102})$$

# Phenomenology I

[Antusch, King, Luhn, MS `11]

- The neutrino mass matrix:

$$M_\nu = m_a \begin{pmatrix} \eta & 0 & 2\eta \\ 0 & 1 & -1 \\ 2\eta & -1 & 1+4\eta \end{pmatrix}, \quad \eta = \epsilon e^{i\alpha}, \quad \epsilon \ll 1$$

- Approximate formulas:

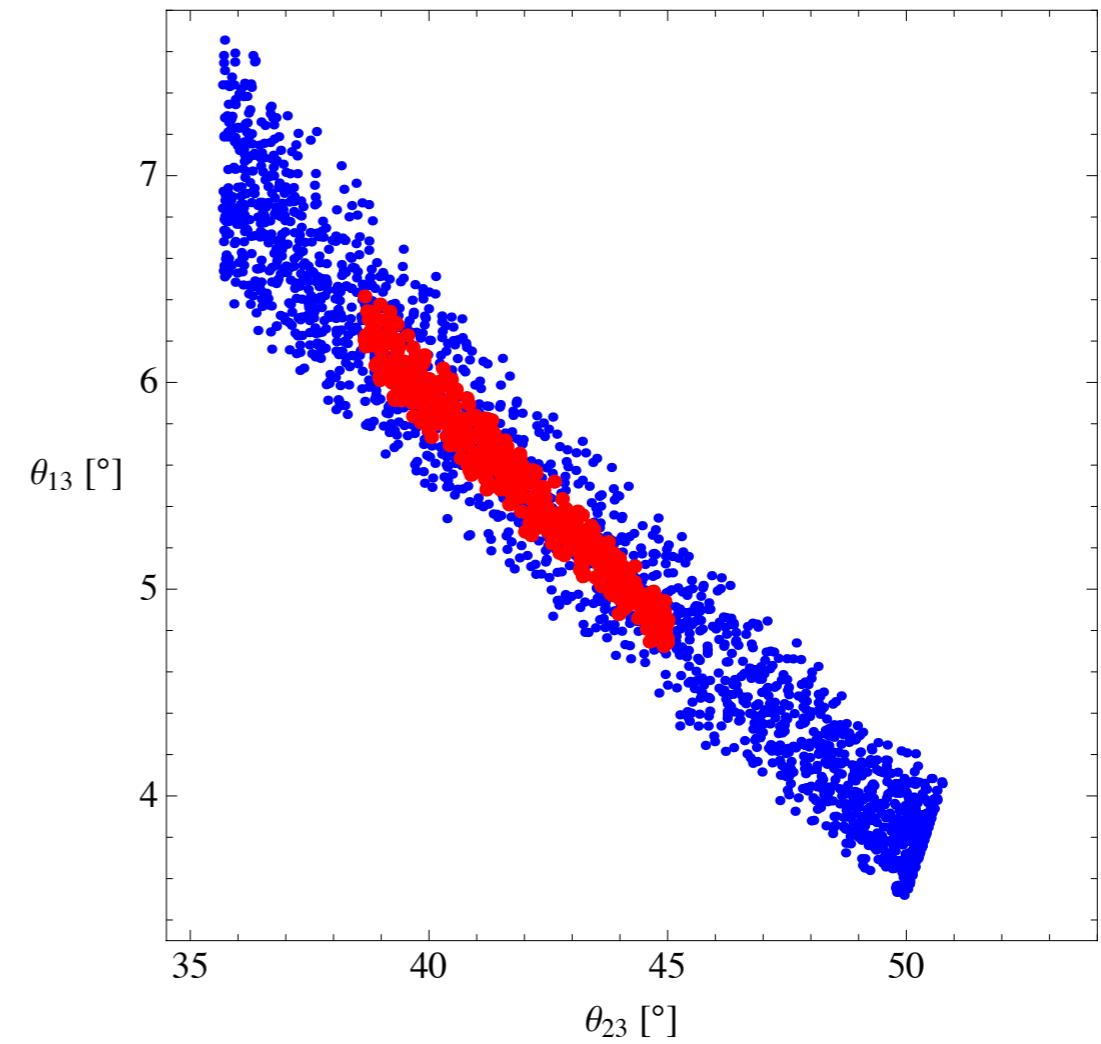
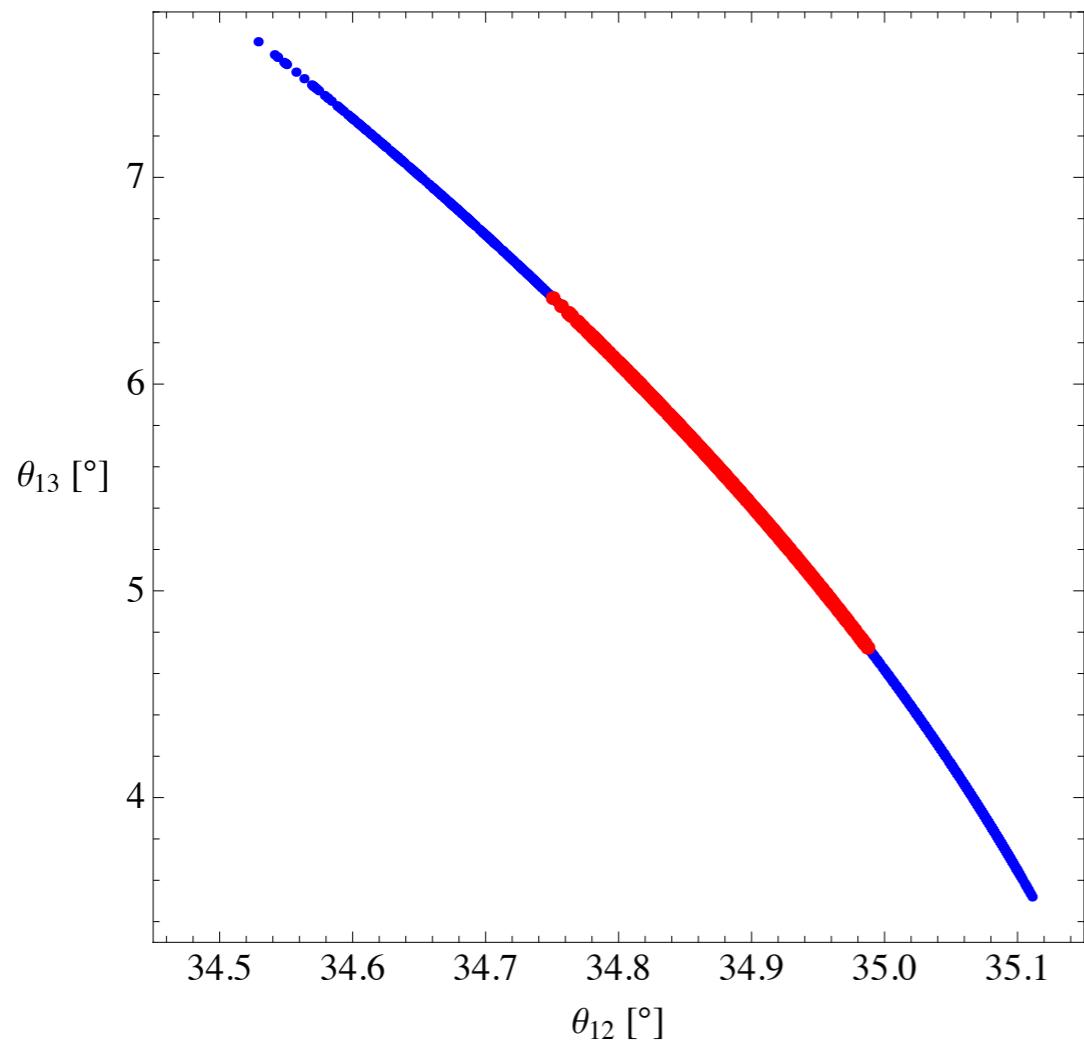
$$m_1^\nu = 0,$$

$$m_2^\nu = [3\epsilon - 3\epsilon^2 \cos \alpha] m_a,$$

$$m_3^\nu = \left[ 2 + 2\epsilon \cos \alpha + \frac{\epsilon^2}{2} (7 - \cos 2\alpha) \right] m_a,$$

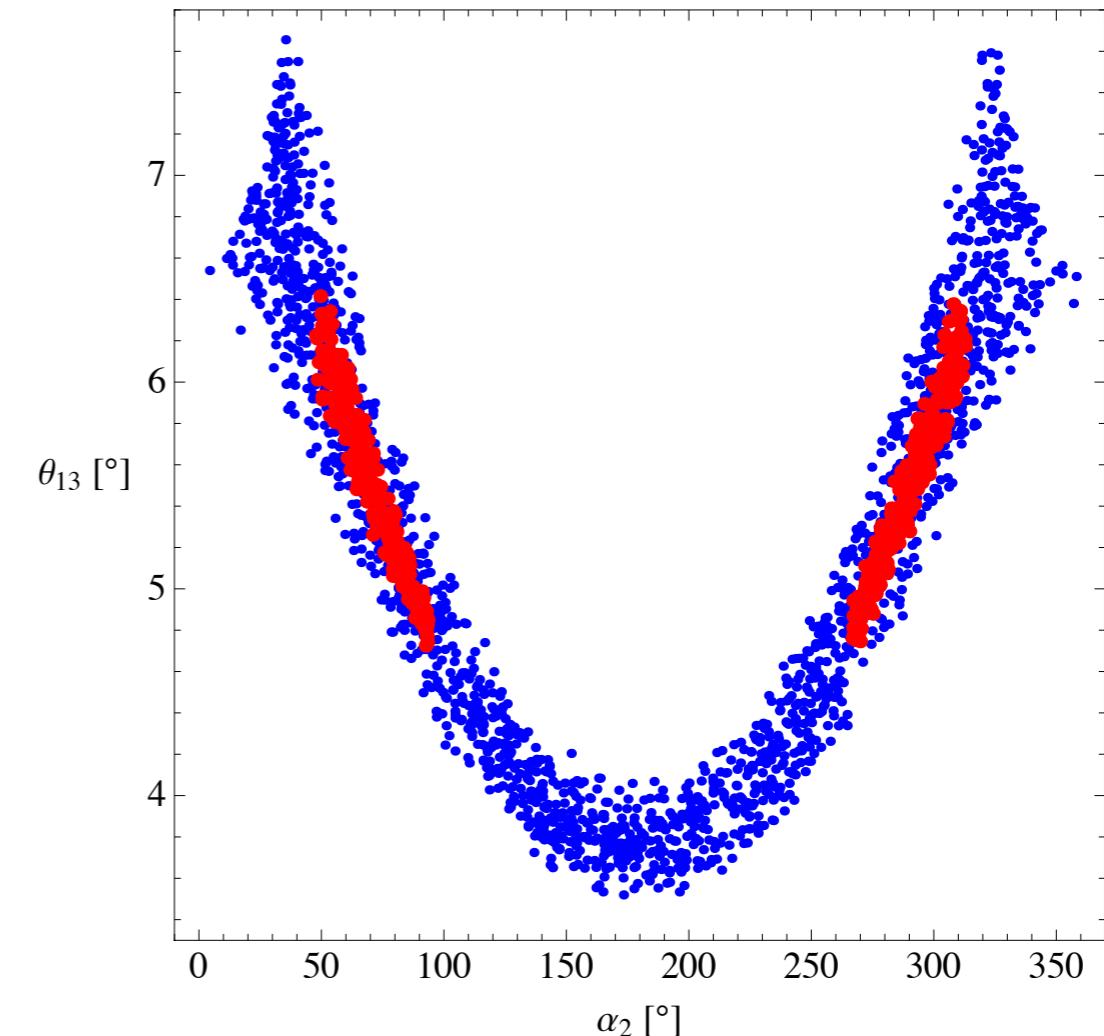
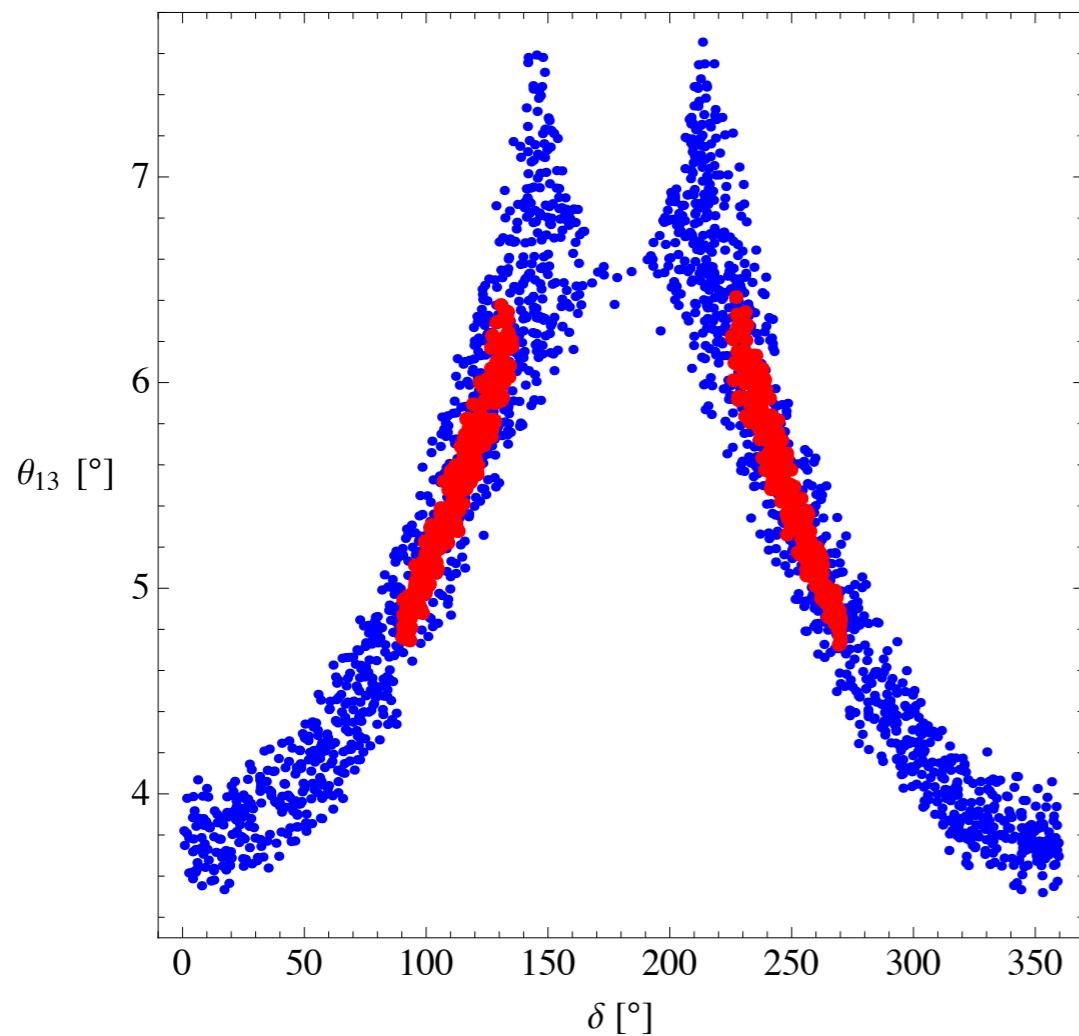
$$\theta_{13} = \frac{\sqrt{2}}{3} \frac{m_2^\nu}{m_3^\nu}$$

# Phenomenology II



[Antusch, King, Luhn, MS`11]

# Phenomenology III



[Antusch, King, Luhn, MS `11]

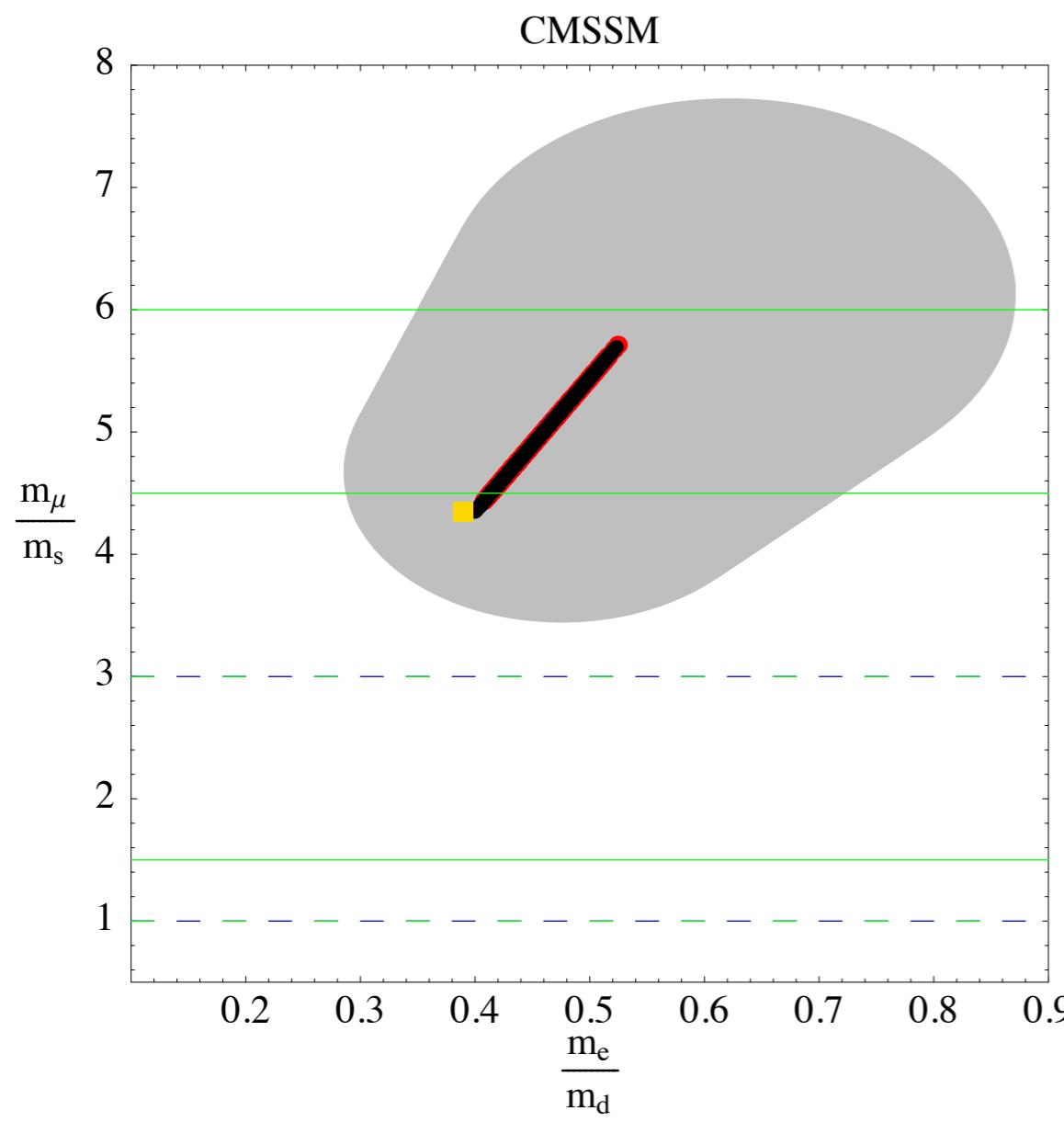
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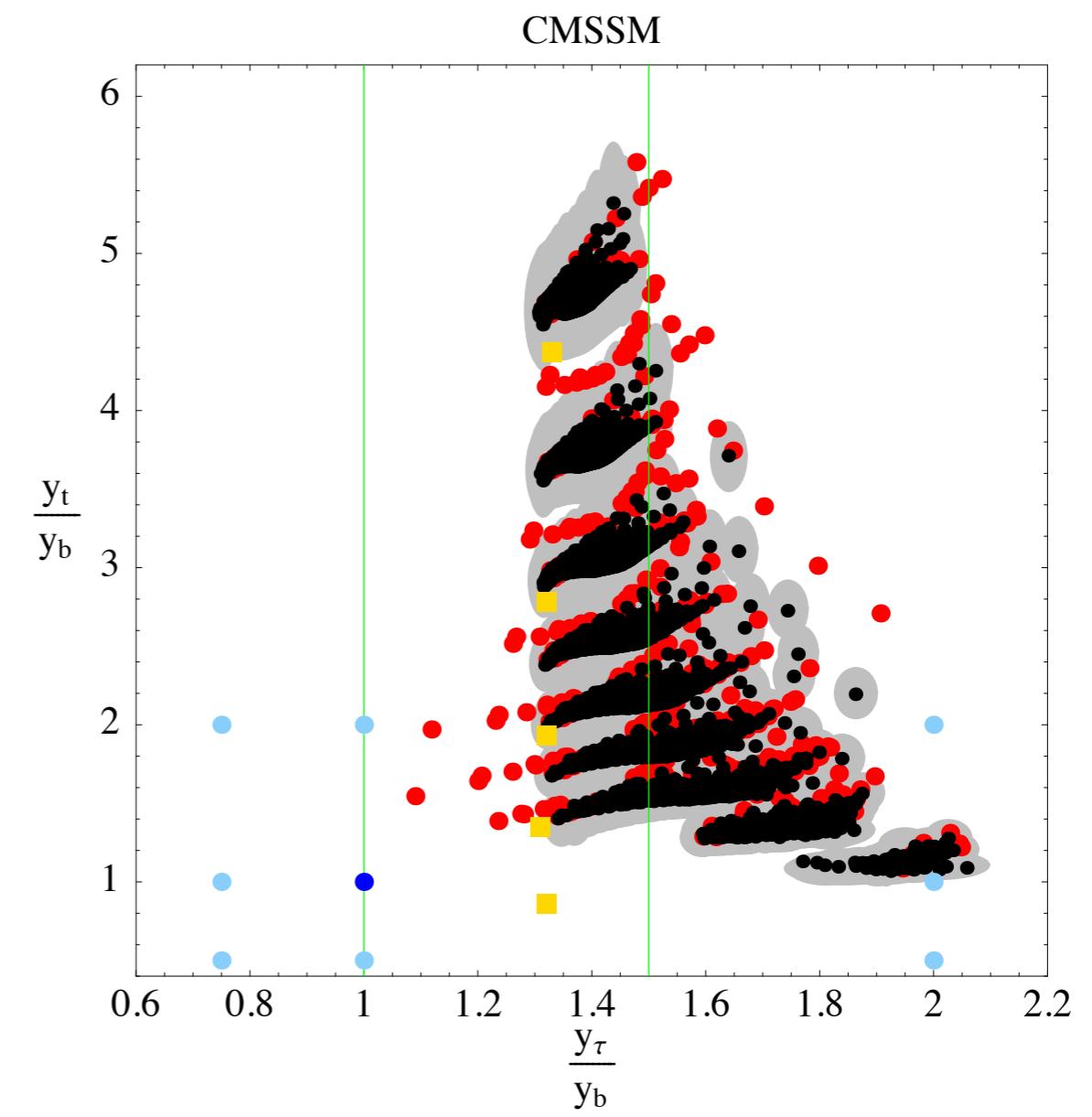
# Summary and Conclusions

- Many Flavour Models ruled out by new experimental data
- Where to go? 2 possibilites (out of many):
  - Charged Lepton Corrections  
[Antusch, Maurer '11; Marzocca, Petcov, Romanino, MS '11]
  - Tribimaximal Mixing  
[Antusch, King, Luhn, MS '11]
  - CP violation?!

**Thanks for your  
attention!**

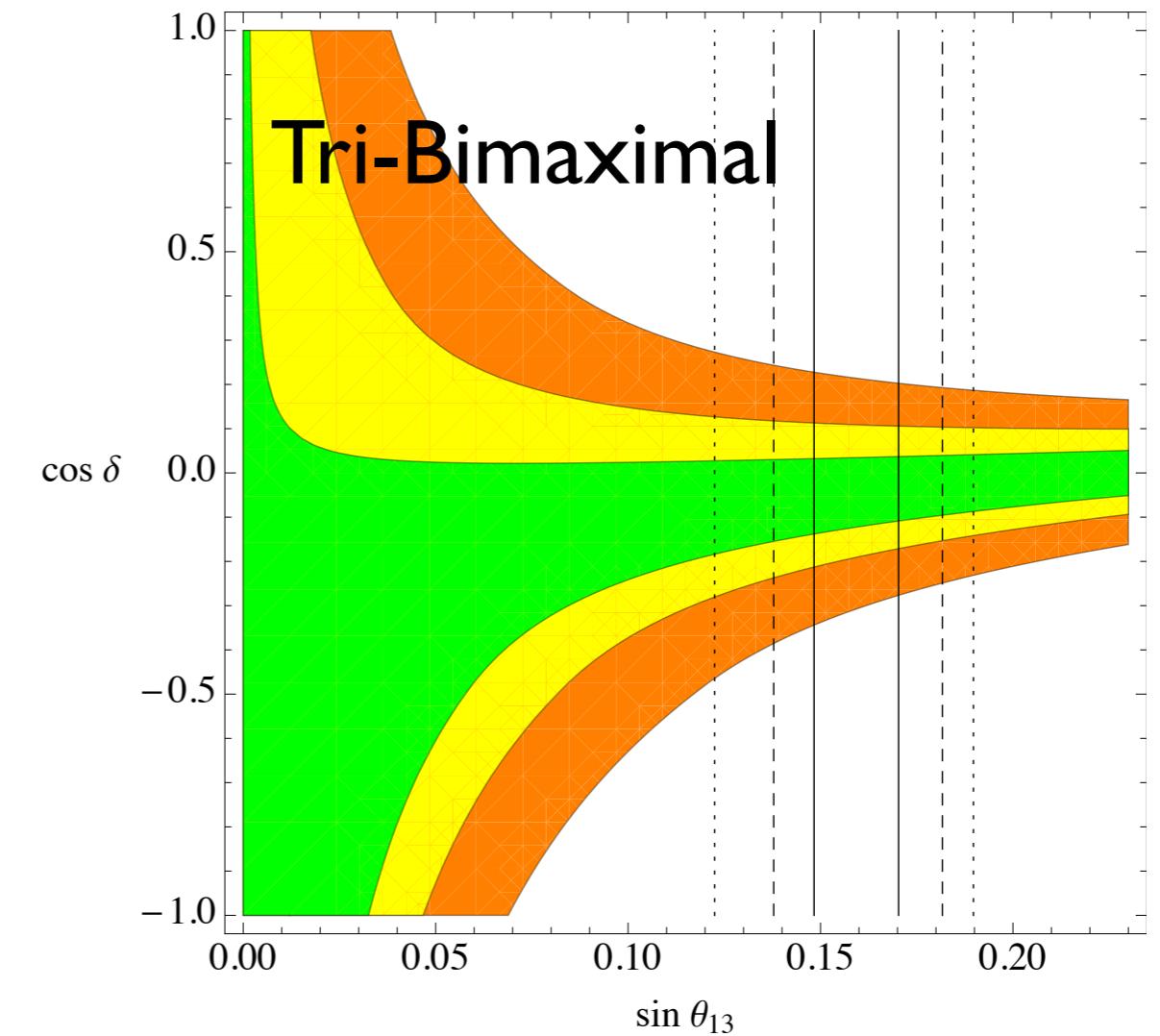
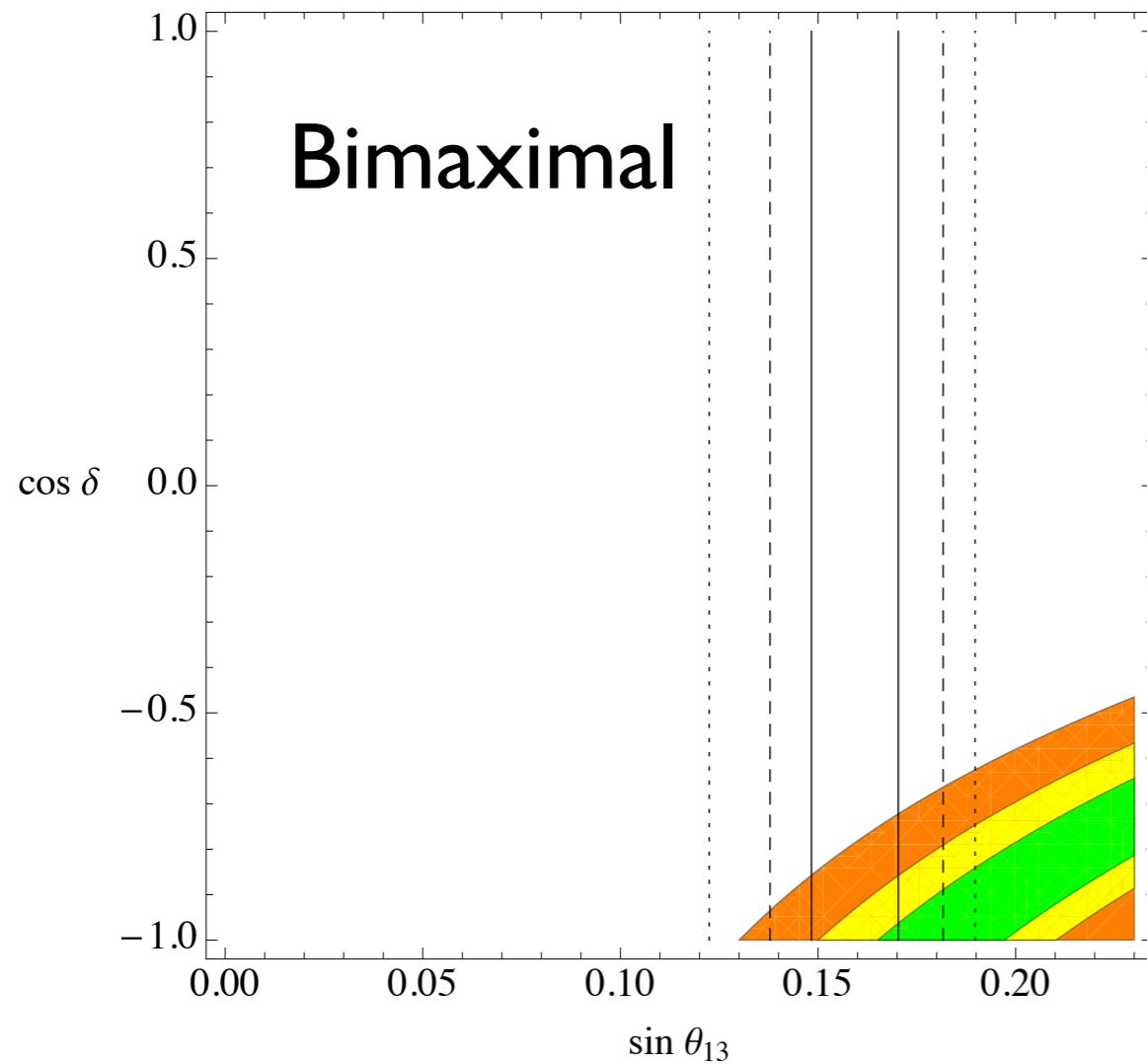


[Antusch, MS '09]



# Implications for $\delta$

## (May `12)



[Based on the global fit by Tortola, Valle, Vanegas '11, Thanks to D. Marzocca for providing this update]

# Backup

[Antusch, King, Luhn, MS`11]

$$\theta_{23} = \frac{\pi}{4} + \epsilon \cos \alpha + \epsilon^2 \left( \frac{3}{2} - \cos 2\alpha \right) ,$$

$$\theta_{12} = \arcsin \frac{1}{\sqrt{3}} - \frac{\epsilon^2}{2\sqrt{2}} ,$$

$$\theta_{13} = \frac{\epsilon}{\sqrt{2}} + \frac{\epsilon^2}{2\sqrt{2}} \cos \alpha ,$$

$$\delta = \alpha - \epsilon \frac{5}{2} \sin \alpha \quad (\text{only up to order } \epsilon) ,$$

$$\alpha_2 = -\alpha + 2\epsilon \sin \alpha - 3\epsilon^2 \sin 2\alpha ,$$

$$\alpha_3 = 0$$