Scaling of the CKM Matrix in the 5D MSSM

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Based on: *arXiv:1110.1942 [hep-ph]*

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Outline

Introduction

- 5D $\mathcal{N} = 1$ supersymmetric model compactified on the S^1/Z_2 orbifold to test effects of Extra-Dimension on the quark Yukawa couplings and the CKM matrix observables.
- S_1/Z_2 breaks 5D Lorentz invariance to the usual 4D one.
- $$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$
- Matter fields are in the bulk or localised to the brane.
- β-function can be derived more easily in the superfield formalism.

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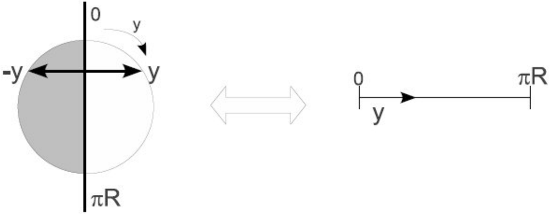
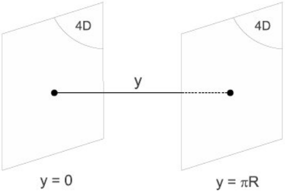
orbifold & brane

- To recover MSSM at low energy, we need **chiral zero modes for fermions** so we compactify the fifth dimention on the orbifold S_1/Z_2.
- Z_2 symmetry identifies $y \to -y$ and we have two orbifold fixed points invariant under Z_2 called branes.





5D MSSM (gauge sector)

- Described by a 5D $\mathcal{N} = 1$ vector supermultiplet which consists (on-shell) of a 5D vector field A^M, a real scalar S and two gauginos λ and λ'.
- 5D $\mathcal{N} = 1$ supersymmetric action :

$$S_g = \int d^5x \frac{1}{2kg^2} \mathrm{Tr}\left[-\frac{1}{2}F^{MN}F_{MN} - D^M S D_M S - i\bar{\lambda}\Gamma^M D_M \lambda \right.$$

$$\left. - i\bar{\lambda}'\Gamma^M D_M \lambda' + (\bar{\lambda} + \bar{\lambda}')[S, \lambda + \lambda']\right]$$

- $D_M = \partial_M + iA_M$, $\Gamma^M = (\gamma^\mu, i\gamma^5)$ and $F^{MN} = -\frac{i}{g}[D^M, D^N]$.

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5D MSSM

- Rearrange fields in terms of a $\mathcal{N}=2$ **4D vector supermultiplet**, $\Omega=(V,\chi)$:
  • V : $\mathcal{N}=1$ vector supermultiplet : A^{μ} and λ.
  • χ : $\mathcal{N}=1$ chiral supermultiplet : λ' and $S'=S+iA^5$.
- 4D $\mathcal{N}=1$ action :

$$S_g=\int \mathrm{d}^5x\mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}\frac{1}{4kg^2}\mathrm{Tr}\left[\frac{1}{4}(W^{\alpha}W_{\alpha}\delta(\overline{\theta}^2)+h.c)+(e^{-2gV}\nabla_y e^{2gV})^2\right]$$

$W^{\alpha}=-\frac{1}{4}\overline{D}^2e^{-2gV}D_{\alpha}e^{2gV}$ and $\nabla_y=\partial_y+\chi$.

- Action of Matter sector and its coupling to gauge sector :
  $S=\int \mathrm{d}^5x\mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}\left[\overline{\Phi}e^{2gV}\Phi+\Phi^ce^{-2gV}\overline{\Phi}^c+(\Phi^c(\nabla_5+m)\Phi\delta(\overline{\theta}^2)+h.c)\right]$

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Fields

- χ-field should be odd under Z_2 symmetry because it appears togother with a derivative ∂_y, whereas V is even.
- Matter superfields : Φ is even, Φ^c is odd. Only the even fields have zero modes.
- The Fourier decomposition of the fields :

$$V(x,y) = \frac{1}{\sqrt{\pi R}}\left[V^{(0)}(x) + \sqrt{2}\sum_{n\geq 1} V^{(n)}(x) cos\left(\frac{ny}{R}\right)\right]$$

$$\chi(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n\geq 1}\chi^{(n)}(x) sin\left(\frac{ny}{R}\right)$$

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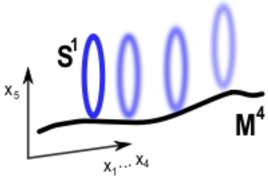
Low energy spectrum



- At energies well below the scale R^{-1}, massive KK states decouple, only the zero modes remain in the spectrum $\Rightarrow$ assume that physics is described by the usual MSSM.
- Brane interactions contain Yukawa-type couplings :

$$S_{brane} = \int \mathrm{d}^8 z \mathrm{d}y \delta(y) \left[(\frac{1}{6} \tilde{\lambda}_{ijk} \Phi_i \Phi_j \Phi_k + \frac{\tilde{k}_{ij}}{4} L_i H_u L_j H_u) \delta(\bar{\theta}) + h.c. \right]$$

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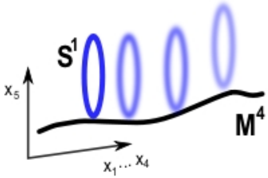

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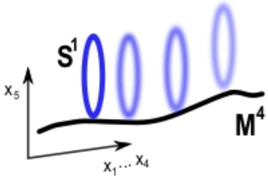

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Equations of gauge couplings

- Our 5D theory is effective up to a scale Λ, then between compactification scale R^{-1} and cut-off scale Λ, there are finite quantum corrections from the ΛR number of KK states.
- $\Rightarrow$ Couplings exhibit power law dependencies on Λ.

$$\beta^{4D} \to \beta^{4D} + (S(\mu) - 1)\,\tilde{\beta}$$

  $\tilde{\beta}$ **is a generic contribution from a single KK level**.
- $S(\mu) = \mu R$, $t = \ln \frac{\mu}{M_Z}$, $S(t) = e^t M_Z R$.
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$$16\pi^2 \frac{dg_i}{dt} = [b_i + (S(t) - 1)\tilde{b}_i]{g_i}^3$$

- $(b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)$ and $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\frac{6}{5}, -2, -6) + 4\eta$;
  η : number of generations of fermions in the bulk.

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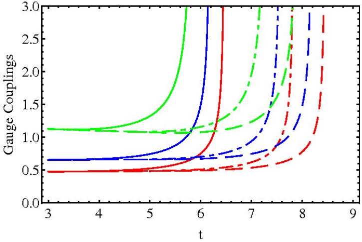
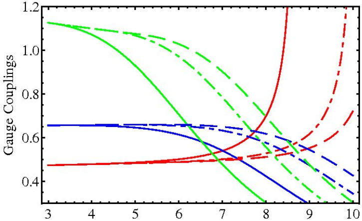
Gauge couplings as a function of the scale parameter t

g_1, g_2, g_3 for 3 different values of the compactification scales : 2 TeV (solid line), 8 TeV (dot-dashed line), 15 TeV (dashed line)

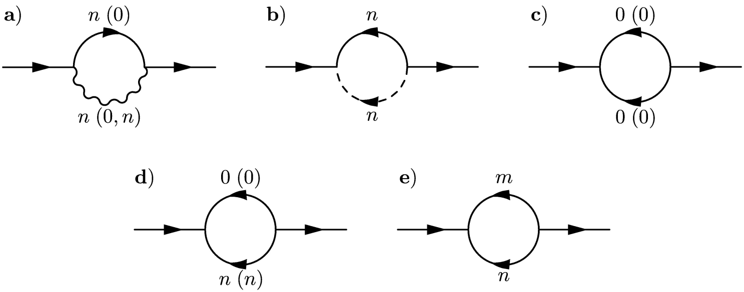
bulk



brane



One-loop diagrams



- **The one-loop diagrams related to the wave-function renormalisation of the matter superfields**.Figs.a-e : matter fields are in the bulk ; whereas Figs.a,c,d are related to the brane localised matter fields case, KK states are labeled by the number inside the bracket.

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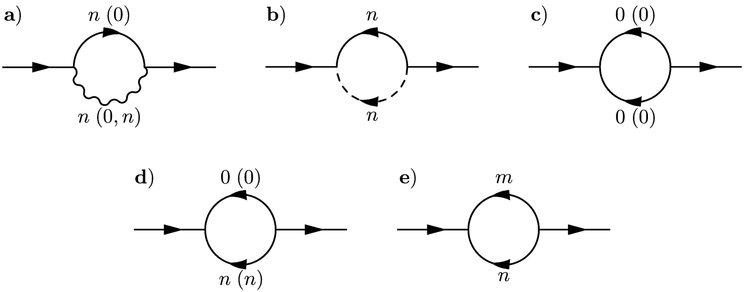

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RGE in 5D MSSM for 3 generations propagating in the bulk

- $$\begin{aligned} 16\pi^2 \frac{dY_d}{dt} &= Y_d(3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e) + 3Y_d^\dagger Y_d + Y_u^\dagger Y_u)\pi S(t)^2 \\ &- Y_d \left(\frac{7}{15} g_1^2 + 3g_2^2 + \frac{16}{3} g_3^2 \right) S(t), \\ 16\pi^2 \frac{dY_u}{dt} &= Y_u(3Tr(Y_u^\dagger Y_u) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d)\pi S(t)^2 \\ &- Y_u \left(\frac{13}{15} g_1^2 + 3g_2^2 + \frac{16}{3} g_3^2 \right) S(t) \end{aligned}$$
- When $E < 1/R$ or $t < \ln(\frac{1}{M_Z R})$ β-functions become those for the usual 4D MSSM.

RGE on the brane

- $$\begin{aligned} 16\pi^2 \frac{dY_d}{dt} &= Y_d(3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e) + (6Y_d^\dagger Y_d + 2Y_u^\dagger Y_u)S(t)) \\ &- Y_d\left(\frac{19}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)S(t)\ , \\ 16\pi^2 \frac{dY_u}{dt} &= Y_u(3Tr(Y_u^\dagger Y_u) + (6Y_u^\dagger Y_u + 2Y_d^\dagger Y_d)S(t)) \\ &- Y_u\left(\frac{43}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)S(t) \end{aligned}$$
- We also found the RGE for leptonic sector $16\pi^2 \frac{dY_e}{dt} = ...$ refer to the paper arXiv :1110.1942 [hep-ph].

CKM matrix (bulk case)

- $\mathrm{diag}\left(f_u^2, f_c^2, f_t^2\right) = U Y_u^\dagger Y_u U^\dagger$; $\mathrm{diag}\left(h_d^2, h_s^2, h_b^2\right) = V Y_d^\dagger Y_d V^\dagger$
- CKM matrix is a result of transition from quark flavor eigenstates to quark mass eigenstates : $V_{CKM} = U V^\dagger$
- $$16\pi^2 \frac{df_i^2}{dt} = f_i^2\left[2(T_u \pi S^2 - G_u) + 6\pi S^2 f_i^2 + 2\pi S^2 \sum_j h_j^2 |V_{ij}|^2\right]$$
  $$16\pi^2 \frac{dh_j^2}{dt} = h_j^2\left[2(T_d \pi S^2 - G_d) + 6\pi S^2 h_j^2 + 2\pi S^2 \sum_i f_i^2 |V_{ij}|^2\right]$$
- Evolution of CKM matrix elements when $E > R^{-1}$:

$$16\pi^2 \frac{dV_{ik}}{dt} = \pi S^2 \left[\sum_{m,j\neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} + \sum_{j,m\neq k} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im}\right]$$

CKM matrix (bulk case)

- $\mathrm{diag}\left(f_u^2, f_c^2, f_t^2\right) = UY_u^\dagger Y_u U^\dagger$; $\mathrm{diag}\left(h_d^2, h_s^2, h_b^2\right) = VY_d^\dagger Y_d V^\dagger$
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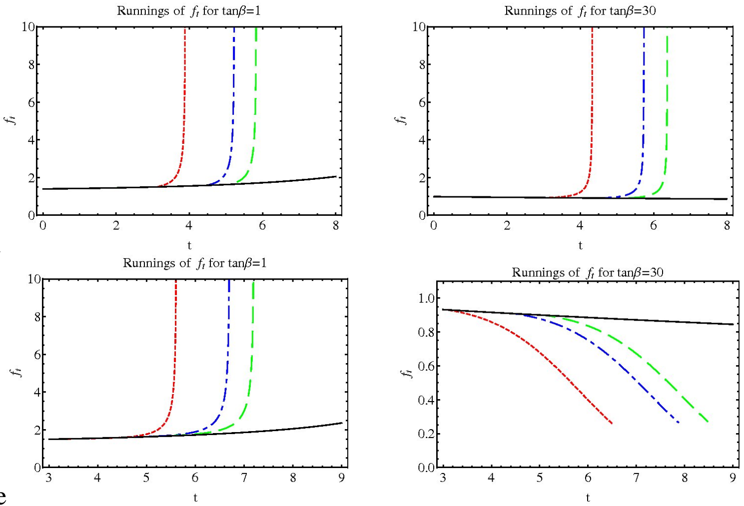
Top Yukawa for R^{-1} :2 TeV (dotted line), 8 TeV (dot-dashed line), 15 TeV (dashed line)

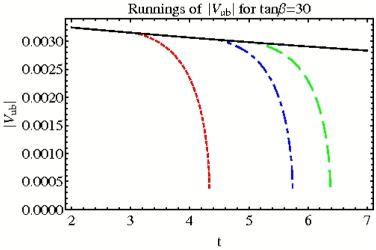


bulk

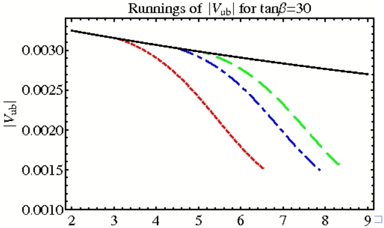
brane

Vub for R^{-1} :2 TeV (dotted line), 8 TeV (dot-dashed line), 15 TeV (dashed line)

bulk

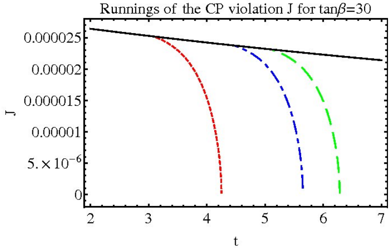


brane



Jarlskog Parameter for R^{-1} :2 TeV (dotted line), 8 TeV (dot-dashed line), 15 TeV (dashed line)

bulk



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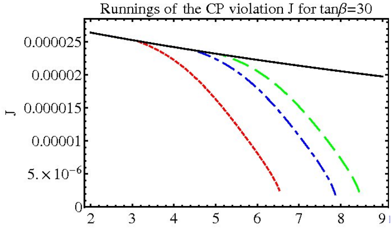

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- Brane case, the dependence on the energy scale is only linear and Yukawa couplings remain perturbative until gauge coupling unification.
- From numerical analysis of CKM parameters, both cases give us a scenario with small or no quark flavour mixings at high energies.
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- Bulk case : quadratic running for yukawa couplings. y_t becomes non-perturbative already at rather low energies. This strongly limits the range of validity of the model.
- Brane case, the dependence on the energy scale is only linear and Yukawa couplings remain perturbative until gauge coupling unification.
- From numerical analysis of CKM parameters, both cases give us a scenario with <u>small or no quark flavour mixings at high energies</u>.
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